SOME PAGES ARE TEXT CUT

PROBLEMS IN PHYSICS

FOR

TECHNICAL SCHOOLS, COLLEGES, AND UNIVERSITIES

BY

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PREFACE

The problems presented in this text grew out of the author's experience in the teaching of large classes of engineering students in general physics in the University of Michigan. Throughout the course of this work *one hour a week was devoted wholly to the solution of practical problems bearing upon the fundamental principles treated in the lecture room and laboratory.* These exercises are intended to supplement the usual one year's course in general physics with a supply of material of such range and variety as will be likely to stimulate the student's interest and clarify his understanding.

This text represents a thorough revision of the earlier edition, the order and character of the problems being somewhat changed and new subjects added so as to bring the material into conformity with recent advancement in the field of physics. In revising the text three main objects were kept in view, namely, (*a*) to make an application of the fundamental principles of physics by the problem method, (*b*) to grade the problems in terms of the principles and the inherent difficulties involved, and (*c*) to select the quantities employed in such a way as to reduce the actual work involved in the arithmetical computations to a minimum.

The chief characteristics of the text then may be summarized briefly as follows:

1. *Statement of Fundamental Principles.*—Accompanying each set of problems there is a brief statement of the principles involved, and also a large number of illustrative examples which enable the student to proceed with his work with a minimum of time and attention on the part of the instructor.

2. *Character of the Problems.*—The problems are practical, carefully graded, and thoroughly workable.

3. *Range of Problems.*—The problems offer a range and variety which will enable the instructor not only to select examples suitable for special groups of students but also to vary the assignments from year to year.

4. *Data Modern.*—The data presented in connection with these exercises are thoroughly modern and in accordance with the recommendations of the U. S. Bureau of Standards and of our scientific societies and engineering associations.

Questions, suggestions, or criticisms relative to the problems contained in this text by teachers using the same will be welcomed by the author.

W. D. H.

ANN ARBOR, MICHIGAN,
March, 1931.

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PROBLEMS IN PHYSICS

CHAPTER I

INTRODUCTORY

UNITS OF MEASUREMENT

1. Fundamental Units.—The fundamental units of measurement are those of length, mass, and time. In the United States the legal units of length and mass are derived from the standards of the metric system, which was legalized by Act of Congress in 1866. The English units of this country (the foot, the pound, etc.) come historically from the units of Great Britain; *they are, however, defined in terms of the metric system.* The U. S. English units of length and mass, therefore, differ somewhat from the corresponding British units. For example, the U. S. inch is 1/39.37 of a standard meter; the British inch, 1⁄36 of a standard British yard. The U. S. inch is just a little longer than the British inch, as may be seen from the following: 1 U. S. inch = 2.5402 cm; 1 British inch = 2.5399 cm.

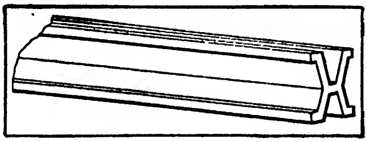

FIG. 1.—Section of U. S. standard meter.

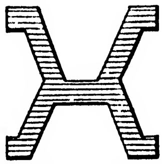

FIG. 2.—Cross-section of U. S. standard meter, actual size.

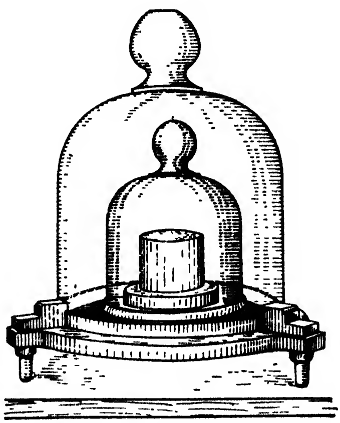

FIG. 3.—U. S. Standard kilogram.

2. Metric Standards.—The International metric standards of length and mass are kept at the International Bureau at Sèvres, near Paris, France. The U. S. metric standards are the standard meter and kilogram, kept at the Bureau of Standards, Washington, D. C. Our national metric standards

are, as nearly as possible, exact copies of the international standards. There is shown in Fig. 1 a reproduction of a photograph of the U. S. standard meter; in Fig. 2, a cross-section of the national meter (actual size); in Fig. 3, the U. S. standard kilogram.

3. Abbreviation of Units.—In accordance with recommendations made by the U. S. Bureau of Standards, *Circular* 47, July 1, 1914, the period is omitted after abbreviations of metric units, while it is used after those of the customary English system. For example, we write *cm* for centimeter, *mm* for millimeter, *m* for meter, and so on; in the case of units of the English system, however, we write *ft.* for foot, *lb.* for pound, *cu. ft.* for cubic foot, etc., using the same abbreviation for both singular and plural in both cases. Also, the exponents ^{2} and ^{3} are used to signify area and volume respectively, in the case of metric units, instead of the more cumbersome terms sq. cm. and cu. cm. In accordance with this rule, therefore, we write cm^2 for square centimeter, and cm^3 for cubic centimeter. In the English system, however, the terms sq. ft., cu. ft., etc., are retained.

4. Units of Length.—The metric unit of length is the meter. A *meter* is the distance between two points on a platinum iridium bar kept at the Bureau of Standards at Washington, the measurement being made at 0°C. In designating fractions of the meter we use the Latin prefixes, *deci, centi, milli;* multiples, the Greek prefixes, *deka, hecto, kilo.* The divisions and multiples of the meter are given in the following tables:

METRIC UNITS OF LENGTH

Fractions			Multiples		
1 decimeter (dm)	= $\frac{1}{10}$	meter	1 dekameter (dkm)	= 10	meters
1 centimeter (cm)	= $\frac{1}{100}$	meter	1 hectometer (hm)	= 100	meters
1 millimeter (mm)	= $\frac{1}{1000}$	meter	1 kilometer (km)	= 1000	meters

The following conversion tables contain the metric units of length and their legal U. S. English equivalents.

CONVERSION TABLES

Metric to English		English to Metric	
1 kilometer	= 0.621,37 mile	1 mile	= 1 609,35 kilometers
1 meter	= 3.280,83 feet	1 mile	= 1609.35 meters
1 meter	= 39.37 inches	1 foot	= 9.3048 meters
1 centimeter	= 0.3937 inch	1 inch	= 2.54 centimeters

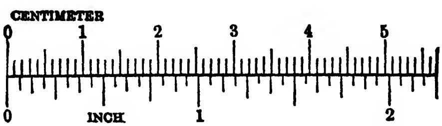

FIG. 4.—Relation of inch to centimeter.

The relation between centimeters and inches is shown in Fig. 4.

The following values expressed in English units are convenient for reference:

Length

12 inches	= 1 foot	1000 mils	= 1 inch
16.5 feet	= 1 rod	4 inches	= 1 hand
320 rods	= 1 mile	9 inches	= 1 span
3 statute miles	= 1 league	6 feet	= 1 fathom
1 mile	= 5280 feet		

Area

144 square inches	= 1 square foot	160 square rods	= 1 acre
9 square feet	= 1 square yard	640 acres	= 1 square mile
30¼ square yards	= 1 square rod	36 square miles	= 1 township

5. Units of Volume and Capacity.—In general we use the term "volume" to refer to the size of a body, while, on the other hand, when we use the term "capacity" we think of how much a vessel will hold. For example, we speak of the volume of a cylinder or sphere, and of the capacity of a beaker or Florence flask.

The metric unit of volume is the cubic meter (m^3). The fractional units of volume are the cubic decimeter (dm^3) and the cubic centimeter (cm^3).

The metric unit of capacity is the liter. A liter is the volume of 1 kg of air-free distilled water at 4°C. The reason for defining the liter in terms of the volume of a kilogram of air-free distilled water at 4°C., instead of defining it directly as 1000 cc, is because of the convenience in calibrating glass flasks and similar vessels.

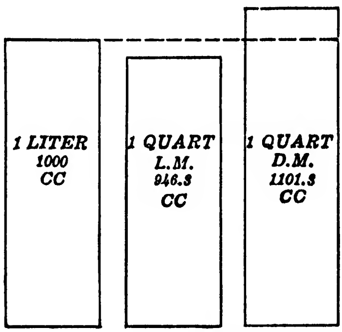

Fig. 5.—Outline showing comparative volumes of liter, liquid quart, and dry quart.

$$1 \text{ liter} = 1000 \text{ cc} = 1{,}000.027 \text{ cm}^3$$
$$1 \text{ cc} = \tfrac{1}{1000} \text{ liter} = 1 \text{ milliliter (ml)}.$$

Since a kilogram of water at 4°C. has a volume which is nearly equal to a cubic decimeter, a liter will be considered as equivalent to 1000 cm^3, unless specifically stated to the contrary; that is, 1 cc and 1 ml will be used as equivalent terms.

Conversion Tables

1 liter	= 0 035,32 cubic foot	1 cubic foot	= 28 317,01 liters
1 liter	= 0.908 quart (dry)	1 quart (dry)	= 1 101 liters
1 liter	= 1.0567 quarts (liquid)	1 quart (liquid)	= 0 946 liter

A comparison of the relative volumes of the liter and the liquid quart and the dry quart is given in outline in Fig. 5.

U. S. ENGLISH UNITS OF CAPACITY

Dry Measure		Liquid Measure	
2 pints	= 1 quart	2 pints	= 1 quart
8 quarts	= 1 peck	4 quarts	= 1 gallon
4 pecks	= 1 bushel	1 gallon	= 231 cubic inches
1 bushel	= 2150.42 cubic inches	31½ gallons	= 1 barrel

6. Units of Mass.—The U. S. standard of mass is the kilogram. A *kilogram* is a mass equivalent to the National Standard Kilogram. A *gram* is $\frac{1}{1000}$ kilogram. For all practical purposes a gram may be considered as equivalent to the mass of a cubic centimeter of air-free distilled water. The divisions of the kilogram and gram are given in the following tables:

METRIC UNITS OF MASS

1 gram (g)	= $\frac{1}{1000}$ kilogram (kg)
1 decigram (dg)	= $\frac{1}{10}$ gram
1 centigram (cg)	= $\frac{1}{100}$ gram
1 milligram (mg)	= $\frac{1}{1000}$ gram

CONVERSION TABLES

1 kilogram	= 2 204,622 pounds	1 pound	= 0 453,59 kilogram
1 gram	= 15 432 grains	1 grain	= 0 064,80 gram

ENGLISH UNITS, AVOIRDUPOIS WEIGHT

437 5 grains	= 1 ounce	1 pound	= 7000 grains
16 ounces	= 1 pound	2000 pounds	= 1 ton

U. S. COINS

According to the rules of the United States Mint, the following pieces of money, as legalized by Act of Congress, are coined in terms of the metric units of mass. The mass of a

5-cent piece (nickel)	= 5.0 grams
10-cent piece (dime)	= 2.5 grams
25-cent piece (quarter)	= 6 25 grams
50-cent piece (half dollar)	= 12.5 grams

KARAT AND CARAT

The fineness of gold and the weight of precious stones are measured in terms of the "karat" and the "carat," respectively.

1 karat (fineness of gold)	= $\frac{1}{24}$ by weight of pure gold
1 carat (of precious stones)	= 200 milligrams

For example, a ring of 18 karats fineness contains $\frac{18}{24}$ of pure gold; 24 karats signifies pure gold. On the other hand, a 5-carat diamond is one that has a mass of 5×200 mg = 1 g.

7. Unit of Time.—The unit of time is the second. A *second* is 1/86,400 of a mean solar day. A solar day is the interval between two successive

passages of the sun across a given meridian. Solar days vary in length throughout the year. A mean solar day is the average length of all the solar days taken throughout the year. The time recorded by watches and clocks is expressed in mean solar time.

8. Angular Measure.—An angle may be measured in degrees or in radians. In a circle 360 degrees = 2π radians. Then

$$\begin{aligned} 360° &= 2\pi \text{ radians} \\ 1° &= 2\pi/360 = 0.017{,}45 \text{ radian} \\ 1 \text{ radian} &= 360/2\pi = 57.296° \end{aligned}$$

9. Dimensional Formulae.—In general, physical quantities such as density, velocity, acceleration, force, momentum, work, power, etc., may be expressed in terms of length L, mass M, and time T. For example, the dimensional formula for area (a length multiplied by a length) is L^2; likewise the dimensional formula for volume is L^3. Density is equal to mass per unit volume; that is, $D = M/V$. The dimensional formula for density, then, is $M/L^3 = ML^{-3}$.

10. Equations and Symbols.—The following equations and equivalents are given for reference in connection with the solution of problems. The terms given in parentheses are values usually used.

EQUATIONS AND EQUIVALENTS

$\pi = 3.1416$; $\pi^2 = 9.8696 = (9.87)$; $\sqrt{\pi} = 1.7724$; $\log \pi = 0.497{,}15$

Conversion factor between common and natural logs = 2.302,58 = (2.3)

$\text{Log}_{10}\, n = \log_e n/2.3$; $\log_e n = \log_{10} n \times 2.3$

Circle: *circumference* = $2\pi r$; *area* = πr^2

Cylinder: *lateral area* = $2\pi rh$; *volume* = $\pi r^2 h$

Cone: *lateral area* = $\frac{1}{2}$(circumference $\times$ slant height); *volume* = $\frac{1}{3}$ (area base $\times$ height)

Sphere: *area* = $4\pi r^2$; *volume* = $\frac{4}{3}\pi r^3$

Mass of 1 cm^3 of pure water at 4°C. = 1 g

Mass of 1 cu. ft. of pure water at 4°C. = 62.3565 lb. = (62.4 lb.)

Mass of 1 gal. of pure water at 4°C. = 8.335,85 lb. = (8.3 lb.)

For Greek-letter symbols, see Table III, Appendix.

Problems

NOTE.—In the solution of all problems involving the ratio 3.1416, the symbol π should be used, unless stated to the contrary.

Example.—Find the area of a circle the radius of which is 10 in.

Solution: Area = $\pi r^2 = \pi \times 10^2 = 100\pi$ sq. in.

1. (*a*) Find the value of 2.6 km in meters, centimeters, millimeters. (*b*) Find the value of 104 cm in millimeters, meters, kilometers.

2. In 10 miles there are how many (*a*) rods; (*b*) feet; (*c*) kilometers?

3. The ordinary country highway is 4 rd. wide. This is equivalent to how many (*a*) feet; (*b*) meters?

4. At the 1912 Olympiad held in Stockholm, Sweden, the "100-m dash" was won in the record time of 10.6 sec. At this rate, find the average speed of the runner in (*a*) feet per second; (*b*) miles per hour.

5. The great cannon used by the Germans in the reduction of the Belgium forts in 1914 were known as "42-cm guns." Find the diameter of the bore of these guns in inches.

6. The train known as the "Twentieth Century Limited" of the New York Central R. R. is scheduled to run from Chicago to New York, a distance of 1003 miles, in 20 hr. Find the average speed of this train in (*a*) miles per hour; (*b*) kilometers per hour.

7. In the Berlin-Zossen tests of electric cars, a rate of 210 km per hr. was attained. Find the speed attained by this car in miles per hour.

8. According to Act of Congress, 1915, the diameter of the "head" of a standard barrel for "fruits, vegetables, and other dry commodities" shall be $17\frac{1}{8}$ in. Find (*a*) the circumference of the head of this barrel, and (*b*) the area, in square inches.

9. The circumference (inside measurement) of a water tank is 20π ft. and its volume is 2000π cu. ft. Find (*a*) the cross-sectional area of the tank, and (*b*) its height.

10. Find the capacity of the tank (problem 9) in (*a*) gallons (liquid measure); (*b*) kiloliters.

11. A cubic foot of water weighs 62.4 lb. Find the weight of the water in the tank (problem 9) when it is full (*a*) in pounds; (*b*) in kilograms.

12. What is (*a*) the area and (*b*) the volume of the largest sphere that can be cut from a cubical block 1 ft. 6 in. on each edge?

13. The volume of a sphere is 2304π cu. in. Find in square feet (*a*) the area of the sphere; (*b*) the area of a great circle of the sphere.

14. The radius of the base of a right cone is 4 ft. Its height is 10 ft. Find (*a*) the lateral area of the cone; (*b*) the total area; (*c*) the volume.

15. The spherical bulb of a thermometer is 0.6 cm in diameter (inside measurement). It is filled with mercury, density 13.59 g per cm^3. Find in grams the mass of the mercury in the bulb.

16. Twelve bicycle balls, each having a diameter of 1 cm are dropped into a liquid having a density of 0.8 g per cm^3. Find (*a*) the volume of the liquid displaced; (*b*) the mass in grams of the liquid displaced.

17. An overflow vessel is full of distilled water having a temperature of 4°C. Into this vessel there are dropped six spherical metal balls of uniform size, causing a displacement of 13.824π g of water. Find the diameter of one of the balls.

18. Out of a circular piece of metal of radius 10 in. there is cut a sector having an arc of 1 ft. Find the area of the sector.

19. According to the rules of the U. S. Mint, a 5-cent nickel coin has a mass of 5 g; a 10-cent piece, a mass of 2.5 g; a 25-cent piece, a mass of 6.25 g. Find the weight in pounds of $10 in (*a*) nickels; (*b*) dimes; (*c*) quarters.

20. Velocity v = length/time. Show that the dimensional formula for velocity is LT^{-1}.

21. Acceleration a = velocity/time. Show that the dimensional formula for acceleration is LT^{-2}.

22. Force F = mass $\times$ acceleration. Show that the dimensional formula for force is MLT^{-2}.

23. Momentum mv = mass $\times$ velocity. Write the dimensional formula for momentum.

CHAPTER II

MECHANICS OF SOLIDS

VECTOR QUANTITIES

11. Vectors and Scalars.—A *vector* quantity is one that has magnitude and direction; a *scalar* quantity has magnitude only. Forces, velocities, and accelerations are examples of vector quantities. On the other hand, mass, volume, density, and heat are scalar quantities because they possess magnitude only. A vector may be represented by a straight line whose direction is that of the quantity (force, velocity, or acceleration) and whose length is chosen to represent the magnitude of the quantity. In general an arrow is used to indicate direction. Scalars may be added algebraically, but vectors must be added geometrically.

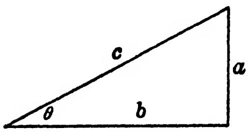

FIG. 6.—Functions of the angle θ.

12. Trigonometric Functions.—The trigonometric functions, *sine, cosine,* and *tangent,* have to do primarily with the relation of the sides of a right angled triangle to the direction angle θ (Fig. 6).

$$\sin\theta = \frac{a}{c};\ \cos\theta = \frac{b}{c};\ \tan\theta = \frac{a}{b}$$

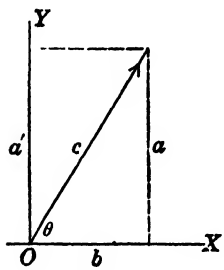

FIG. 7.—Projection on X and Y axes.

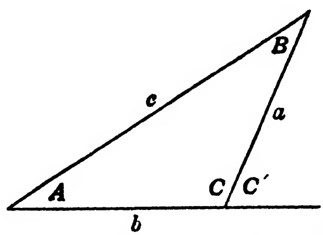

FIG. 8.—Relation between the sides and angles of a triangle.

SINE AND COSINE VALUES

$\sin 0° = 0.0$	$\cos 0° = 1.0$
$\sin 30° = 0.5$	$\cos 30° = 0.866$
$\sin 45° = 0.707$	$\cos 45° = 0.707$
$\sin 60° = 0.866$	$\cos 60° = 0.5$
$\sin 90° = 1.0$	$\cos 90° = 0.0$

13. Projection on Rectangular Axes.—It is frequently of importance to get the projection of a given vector quantity upon the x- and y-axes. For example, the projections of the vector c (Fig. 7) upon the x- and y-axes are represented by the quantities b and a', in which $a' = a$; that is,

$$a = c \sin \theta, \text{ and } b = c \cos \theta.$$

14. Sine and Cosine Laws.—In solutions involving the sides and angles of a triangle (Fig. 8) the following equations are of fundamental importance and should be memorized by the student:

$$\sin C = \sin C'$$
$$\cos C = \cos (180° - C') = -\cos C'$$

LAW OF SINES

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\cos C}$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

The student should note that in the case of obtuse angles, as C in Fig. 8, $\cos C = -\cos C'$, and the equation becomes $c^2 = a^2 + b^2 + 2ab \cos C'$.

Example.—We wish to find the magnitude and direction of c (Fig. 8), the factors a, b, and C being given. *Solution:* (*a*) Let $a = 20$, $b = 20$, and $C = 120°$. Angle $C' = 180° - 120° = 60°$, and $\cos 60° = 0.5$. Then $c^2 = 20^2 + 20^2 + (2 \times 20 \times 20 \times 0.5) = 8800$, from which $c = 34.64$. (*b*) We now wish to find the angle A, which determines the direction of c.

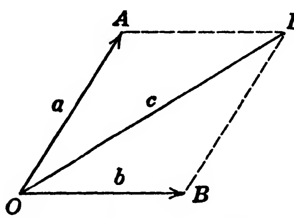

FIG. 9.—Parallelogram method.

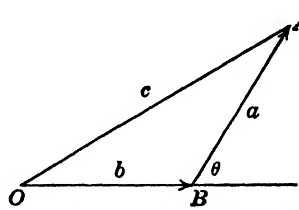

FIG. 10.—Triangle method.

Using the equation $c/\sin C = a/\sin A$, we have $34.64/0.866 = 20/\sin A$, from which $\sin A = 0.5$ and consequently the angle $A = 30°$.

15. Geometric Sum of Vectors.—The resultant or geometric sum of two or more vector quantities may be formed in two ways, one known as the "parallelogram method" and the other as the "triangle method." Suppose, for example, a boat, acted upon by wind and ocean current, has imparted to it two velocities, a in the direction OA and b in the direction OB, the angle between them being $AOB = \theta$. We desire to find the resultant velocity c. *Parallelogram method:* Using OA and OB as sides, complete the parallelogram (Fig. 9). The diagonal OD is the resultant required. *Triangle*

method: Starting with vector *b*, place *a* and *b* end to end, as shown in Fig. 10, keeping their relative directions unchanged. Complete the triangle by drawing the line *c*. This is the resultant or geometric sum of the vectors *a* and *b*.

Problems

24. A man rows a boat in a northeasterly direction OA (Fig. 11) with a velocity of 3 miles per hr. At the same time the boat drifts, due to the wind, in the direction OB with a velocity of 2 miles per hr. The angle AOB is 30°. Complete the parellogram for the sides AO and OB and draw the diagonal representing the resultant. Find the magnitude of the velocity of the boat.

25. Find the direction in which the boat actually moves in relation to the east-west line OE, the angle AON being 45°.

26. Find (problem 24) (*a*) the easterly, and (*b*) northerly velocities of the boat.

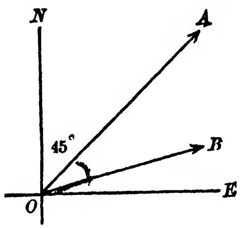

Fig. 11.

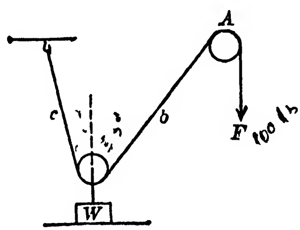

Fig. 12.

27. Suppose that the man (problem 24) rows the boat in the direction OA with a velocity of 3 miles per hr., and the wind and currents carry him westward at the rate of 3 miles per hr. How far west of line ON (Fig 11) will he be in 1 hr.?

28. A hill used for coasting in winter has a rise of 1 ft. in 4; that is, a vertical rise of 1 ft. for a horizontal distance of 4 ft. A boy sliding down the hill has at a given instant a velocity of 20 miles per hr. Find (*a*) his vertical velocity, and (*b*) his horizontal velocity.

29. Suppose that a force F, of 100 lb. (Fig. 12), acts vertically on the rope passing over the pulley A. The rope b makes an angle with the vertical at W of 30°, and c makes an angle of 10°. Neglect the friction of the pulleys and assume that the force at F is transmitted undiminished to all parts of the rope; that is, each

rope, b and c, exerts a force of 100 lb. on the pulley attached to W. Find the force with which W is lifted vertically.

30. Find the magnitude of the horizontal component of the force on W (Fig. 12). In which direction does W tend to move, to the right or left?

31. Two forces, F and F', act on a body at the point O, F to the eastward (right) and F' to the northeast, making an angle of 45° with F. The magnitude of F is 30 lb.; that of F', 20 lb. Find the magnitude and direction of the resultant R.

32. Find the vertical and horizontal components of the resultant R (problem 31).

33. Suppose that a body at O has impressed upon it a force OA of 120 lb., and a force OB of 80 lb., the two making an angle of 120°. Force OA makes an angle of 30° with OX. Find (a) the magnitude and (b) the direction of the resultant R, with reference to OX.

34. The resultant of two forces OA and OB has a magnitude of 600 lb., and makes an angle of 30° with OA, the value of which is 300 lb. Find the magnitude of OB.

VELOCITY AND ACCELERATION

16. Speed and Velocity.—*Speed* is the space passed over per unit of time, without reference to direction. We speak of the speed of an automobile going around a curve or the speed of a particle moving uniformly around a circle, because in each case the direction of motion changes at every instant. *Velocity*, on the other hand, is the space passed over per unit of time in a definite direction, that is,

$$\textit{linear velocity} = v = \frac{s}{t}$$

17. Acceleration.—*Acceleration* is the change (increase or decrease) of velocity per unit of time. When the acceleration is positive we use the symbol $+a$; when negative, $-a$. Let v' be the initial velocity a body has at a given instant; v, the final velocity at the end of the time t; and a, the acceleration. Let s be the space passed over in the time t. We may write the fundamental equations connecting v, v', a, s, and t, as follows:

$$\begin{aligned} v &= v' \pm at, \\ s &= v't \pm \tfrac{1}{2}at^2, \\ v^2 &= v'^2 \pm 2as. \end{aligned}$$

In the case of falling bodies, the acceleration is represented by the symbol g. In general the numerical value of the acceleration due to gravity, employed in the solution of problems in this text, is g = 32 ft. per sec. per sec. = 980 cm per sec. per sec.

VALUES OF g IN CM/SEC./SEC.

Boston, Mass.	980.38	Washington, D. C.	980.10
Ithaca, N. Y.	980 29	Cincinnati, Ohio	979.99
Chicago, Ill	980.26	Charlottesville, Va.	979.92
Cleveland, Ohio	980.23	Denver, Colo	979.60
Philadelphia, Pa.	980.18	Pike's Peak, Colo.	978.94

18. Motion on an Inclined Plane.—There are two cases which come under this head, namely, (*a*) the motion of a body sliding without friction or rotation, and (*b*) the motion of a body (cylinder or sphere) rolling without slipping. This latter case will be discussed later on. Let us consider case (*a*). Given a body S (Fig. 13) sliding without friction down the incline AC, the direction angle of which is θ. The acceleration of S down the incline is

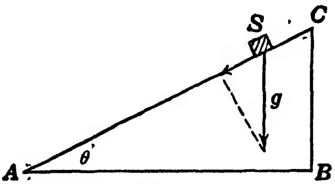

FIG. 13.—Sliding motion on an inclined plane.

$$a = g \sin \theta.$$

Problems

35. A body starts from rest and slides without friction down a long inclined plane with uniformly accelerated motion. At the end of the first second its velocity is 10 cm per sec.; at the end of the second second, 20 cm per sec.; and the third, 30 cm per sec., and so on. (*a*) What is the acceleration; is it positive or negative? (*b*) What is the velocity at the end of 10 sec.? (*c*) The space passed over in 10 sec.?

36. Consider the body (problem 35) moving down the inclined plane for 10 sec., with an acceleration of 10 cm per sec. per sec. (*a*) What is the average velocity during the first 5 sec.; during the last 5 sec. (sixth to tenth seconds inclusive)? (*b*) How far does the body move during the first 5 sec.; during the last 5 sec.?

37. A body falls from rest under the force of gravity for 10 sec. Find the final velocity, and the space passed over in (*a*) centimeters; (*b*) feet.

38. A body is thrown vertically downward with an initial velocity of 10 ft. per sec. How far will it fall in (*a*) 5 sec.; (*b*) during the fifth second?

39. A body is projected vertically upward with an initial velocity of 320 ft. per sec. (*a*) In what time will it come to rest? (*b*) How high will it rise?

40. A body is projected vertically upward with an initial velocity of 8820 cm per sec. How high above the starting point will it be in (*a*) 6 sec.; (*b*) 9 sec.; (*c*) 12 sec.?

41. A body is projected vertically upward with an initial velocity of 6860 cm per sec. Find in what time it will be 22,050 cm above the starting point. How do you account for the two values of t?

42. A body starting from rest is acted upon by a constant force. At the end of the first second its velocity is 5 ft. per sec. (*a*) Find velocity at end of the tenth second. (*b*) How far did it travel during the tenth second? (*c*) If all force had ceased to act on it at the end of the tenth second, how far would it have traveled the eleventh second?

43. A train running at the rate of 36 miles per hr. comes to rest in 10 sec. (*a*) Find acceleration in feet per second per second. (*b*) What distance did it travel during the 10 sec.?

44. A sled starting from rest runs down hill with a uniformly accelerated motion. Its velocity at the end of the fourth second is 20 ft. per sec. Find (*a*) the acceleration; (*b*) the velocity at the end of the tenth second; (*c*) the space passed over during the 10 sec.; (*d*) space passed over during the eighth, ninth, and tenth seconds.

45. A bullet is shot horizontally from the top of a lighthouse tower 64 ft. above the water level with a velocity of 850 ft. per sec. Neglecting air resistance, how far from the tower will the bullet strike the water?

46. A body A is projected vertically upward with an initial velocity of 192 ft. per sec; 4 sec. later a second body B is projected upward with the same initial velocity. How high above the starting position will they pass each other?

47. A stone is dropped from the top of a cliff at the same instant another stone is projected vertically upward with an initial velocity of 128 ft. per sec. In t sec. they meet each other half way. How high is the cliff?

8. A body starts from rest and slides without friction down an incline (Fig. 13). In 4 sec. it travels a distance of 64 ft. What is the value of the direction angle θ?

49. The base AB of a given inclined plane is 18.65 ft., and the height BC is 5 ft. (*a*) What is the direction angle? (*b*) What is the acceleration down the plane incline?

50. Given a smooth inclined plane, with the base AB lying in a horizontal position. The altitude BC is 4 ft., length of incline AC 16 ft., and the direction angle 14° 29′. We wish to compare the velocities of two bodies starting at rest from C, S sliding without friction down the incline from C to A, and S' falling vertically from C to B. (*a*) What time will be required for S to slide from C to A, and what will be its velocity at C? (*b*) What time will be required for S' to fall vertically from C to B, and what will be its velocity at B?

FORCE AND LINEAR MOTION

19. Force and Pressure.—*Force* is that which produces or tends to produce motion. Force may be measured in two ways: First, by the push or pull which it exerts; second, by the acceleration which it imparts to a given mass. The pull which a force exerts may be measured by means of a spring balance (dynamometer); in the second case the magnitude of the force may be measured in terms of the product of the mass and the acceleration, as represented by the equation, $F = ma$.

Pressure is force per unit of area; that is,

$$P = \frac{F}{A}.$$

20. Units of Force.—There are two sets of units of force, known as (*a*) gravitational or practical units, and (*b*) absolute units. Practical and absolute units may be expressed in both the English and metric systems. The relation of the two sets of units is shown in the following outline:

$$\textit{Units of force}\begin{cases}\textit{Gravitational}\begin{cases}\textit{English} = \begin{cases}\textit{pound of force} = \textit{weight of a} \\ \quad \textit{pound}\end{cases} \\ \textit{Metric} = \begin{cases}\textit{gram of force} = \textit{weight of a} \\ \quad \textit{gram}\end{cases}\end{cases} \\ \textit{Absolute}\begin{cases}\textit{English} = \textit{poundal} \\ \textit{Metric} = \textit{dyne}\end{cases}\end{cases}$$

A *pound of force* (the force of a pound) is a force equivalent to the attraction of gravity for a pound mass at sea level, 45° N. latitude. A *gram of force* (force of a gram) is equivalent to the attraction of gravity for a gram mass. A *force of a kilogram* is equal to 1000 g of force.

A *poundal* is a force that will give to a mass of 1 lb. an acceleration of 1 ft. per sec. per sec. A *dyne* is a force that will give to a mass of 1 g an acceleration of 1 cm per sec. per sec.

The relation of gravitational to absolute units is:

$$1 \textit{ pound of force} = 32 \textit{ poundals}$$
$$1 \textit{ gram of force} = 980 \textit{ dynes}$$

21. Weight.—The term "weight" is used in two entirely different senses. It may refer either to an *object*, or to a *force*. For example, we say, "Lift the weight from the floor to the table," or "Put the weight on the scale pan,"

using the term in both cases to refer to an object. On the other hand we may speak of the weight of a body, meaning thereby the force by which it is attracted to the earth. In this sense *weight is a force.* When we say that a stone weighs 10 lb., we mean that it is attracted to the earth by a force of 10 lb.

Example 1.—(*a*) A force of 10 lb. will impart what acceleration to a mass of 10 lb.? (*b*) A force of 10 poundals will impart what acceleration to a mass of 10 lb.? *Solution:* (*a*) In this case the force (10 lb.) is given in gravitational units. But since we wish to express the result in terms of acceleration we must change gravitational units to absolute so as to make use of the equation $F = ma$. A force of 10 lb. $= 10 \times 32 =$ poundals. Then $320 = 10 \times a$, from which $a = 32$ ft. per sec. per sec. (*b*) Here F is expressed in absolute units, hence $10 = 10 \times a$, and $a = 1$ ft. per sec. per sec.

Example 2.—(*a*) A force of 10 g will impart what acceleration to a mass of 10 g? (*b*) A force of 10 dynes will impart what acceleration to a mass of 10 g? *Solution:* (*a*) $F = ma = 10 \times 980 = 10 \times a$, hence $a = 980$ cm per sec. per sec. (*b*) $10 = 10 \times a$, hence $a = 1$ cm per sec. per sec.

22. Momentum.—*Momentum* is mass times velocity $= mv$. The expression $F = ma$ may be deduced from Newton's second law of motion. Force is always dual in its nature; that is, if it acts on one body it reacts on another. Now if we multiply the equation $F = ma$ by t, we have $Ft = mat = mv$, in which the term Ft is called the impulse. If, therefore, action and reaction occur between two bodies of masses m and m' for the time t, we have the equation

$$mv = m'v'.$$

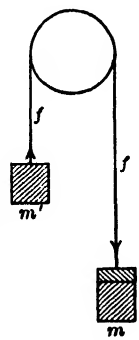

FIG. 14.—Motion of connected bodies in vertical direction.

23. Motion of Connected Masses.—In all cases in the derivation of equations relating to the motion of connected masses as illustrated in Figs. 14, 15, and 16, we assume that the mass and frictional effects of the pulleys are negligible. Consider two masses m and m' to be connected by a flexible cord hung over a vertical fixed pulley (Fig. 14) as illustrated by the case of the Attwood machine. Let $m' < m$, in which case m' will move upward and m will move downward. If a be the acceleration of the system, then

$$(m + m')a = mg - m'g = (m - m')g.$$

If we let f be the force (tension) in the string, we have

$$f = (mg - ma) = (m'g + m'a) = \frac{2gmm'}{(m + m')},$$

it being understood that the sign or sense of f depends on whether we consider the body as moving upward or downward.

If the two masses move on an inclined plane (Fig. 15) our equations become

$$(m + m')a = (m \sin \theta - m' \sin \theta')g,$$

and

$$f = (mm'g)\left(\frac{\sin \theta + \sin \theta'}{m + m'}\right).$$

If one of the masses moves in a vertical direction and the other moves in a horizontal direction (Fig. 16) $\sin\theta = 1$, and $\sin\theta' = 0$, and the equations become

$$(m + m')a = mg,$$

$$f = \frac{mm'g}{(m + m')}.$$

Example.—Given masses m and m' on inclines, as shown in Fig. 15. The angle θ is 60° and θ' is 30°. The mass of m' is 120 g. (*a*) Neglecting all friction in the moving system, find what mass m will be required to give an

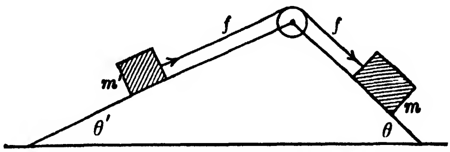

FIG. 15.—Motion of connected bodies on inclines.

acceleration of 10 cm per sec. per sec. (*b*) Find the pull in the string. *Solution:* (*a*) $10m + 10 \times 120 = (m \times 0.5 - 120 \times 0.866)980$, whence $m = 214.7$ g and (*b*) $f = 103{,}046$ dynes.

24. Change of Force with Change of Acceleration.—Force is measured in terms of mass and acceleration, $F = ma$, therefore a change (increase or decrease) in acceleration indicates a corresponding change in the force exerted. This may be illustrated by means of the following simple experiment. Suspend from the hook of a spring balance a given weight (mass) of 5 lb., say, the other end of the balance being held in the hand. So long as the system is at rest, the pointer indicates a force of 5 lb., or in absolute units, $mg = 5 \times 32 = 160$ poundals. Now suddenly jerk the hand upward. The pointer quickly moves downward, indicating an increase in the force acting upon it equivalent to $mg + ma$ poundals, in which a is the acceleration imparted by the upward jerk of the hand. If, on the other hand, the system be suddenly moved downward, the pointer will indicate less than 5 lb. or 160 poundals, the force now being $mg - ma$. If the system be moved upward or downward with an absolutely uniform velocity the force remains uniform at 5 lb.

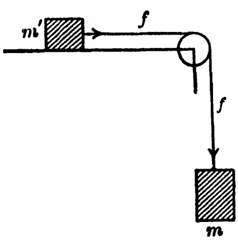

FIG. 16.—Motion of connected bodies at right angles.

25. Frictional Forces.—In all the examples considered thus far the friction factor has been neglected. Let us consider now a problem in which frictional forces are taken into account.

Example.—A box weighing 800 lb. is dragged along over the concrete floor of a warehouse, the force being applied parallel to the floor. The frictional force between the box and the floor is 200 lb. (*a*) Find in gravitational and absolute units the force required to give the box uniform motion. (*b*) Find the force required to change the velocity of the box by 20 ft. in 5 sec., equivalent to an acceleration of 4 ft. per sec. per sec. *Solution:* (*a*) For uniform

motion $F = 200$ lb. or $200 \times 32 = 6400$ poundals. (*b*) Accelerated motion, $F = 800 \times 4 = 3200$ poundals, which added to the frictional force gives the total force required. But before we can add these terms both must be reduced to the same units. Hence the total force in this case is $\frac{3200}{32} + 200 = 300$ lb. or $3200 + 200 \times 32 = 9600$ poundals.

Problems

51. Define gravitational and absolute units of force, in both the English and metric systems.

52. Write the dimensional formulae for force and pressure, and explain wherein they differ in terms of the fundamental units.

53. What force in (*a*) dynes, (*b*) grams, will give a mass of 10 g and acceleration of 10 cm per sec. per sec.?

54. What force in (*a*) poundals, (*b*) pounds, will give a mass of 10 lb. and acceleration of 10 ft. per sec. per sec.?

55. A mass of 10 lb. lies on the table. It is acted on by a force (gravitation) that tends to give it an acceleration of 32 ft. per sec. per sec. Find the force which it exerts upon the table in (*a*) pounds; (*b*) poundals.

56. A 10-kg weight lies upon the table. Assuming that the term "weight" refers to a body having a mass of 10 kg, find the force which it exerts upon the table in (*a*) gravitational units; (*b*) absolute units.

57. A mass of cement exerts a force of 1000 lb. on a surface 5 by 10 ft. Find the pressure in (*a*) pounds; (*b*) poundals.

58. A force of 1 kg is exerted on a surface 5 by 10 cm. Find the pressure in (*a*) grams; (*b*) dynes.

59. Define: gram mass, pound mass, gram weight, force of a gram, gram of force, pound weight, force of a pound, poundal, dyne.

60. When the mass is given in grams and the acceleration in centimeters per second per second, how is (*a*) F expressed in the equation $F = ma$; (*b*) W, in the equation $W = mg$?

61. When the mass is given in pounds and the acceleration in feet per second per second, in what units is (*a*) F expressed; (*b*) W?

62. A mass of 220.4622 lb. lies on the floor. Find the force which it exerts upon the floor in (*a*) pounds; (*b*) poundals; (*c*) kilograms; (*d*) dynes.

63. A mass of 10 kg is acted upon by a force which imparts to it an acceleration of 10 ft. per sec. per sec. Find the force in (*a*) poundals; (*b*) pounds; (*c*) dynes; (*d*) grams.

64. A metal cylinder having a radius of 10 cm, height 20 cm, and density 8 g per cm^3 rests on one end upon a table. Find the pressure which it exerts upon the table in (*a*) grams; (*b*) dynes.

65. A mass of 10 g is moving with a velocity of 10 cm per sec. It is then acted upon by a force for 4 sec., after which it has a velocity of 50 cm per sec. Find the force in dynes.

66. A mass of 5 lb. at rest is acted upon by a force for 5 sec., iving it a velocity of 100 ft. per sec. Find the force in (*a*) poundals; (*b*) pounds.

67. A mass of 5 tons is acted upon by a force which imparts to it a change in velocity of 8 ft. per sec. in 4 sec. Find the force in pounds.

68. A mass of 20 g has an initial velocity of 10 cm per sec. It is acted upon by a force of 120 dynes for 5 sec. Find (*a*) its velocity at the end of 5 sec.; (*b*) the change of velocity; (*c*) the acceleration imparted.

69. A mass of 20 g is moving with a velocity of 5 cm per sec. It is acted upon by a force for 2 sec. after which it has a velocity of 15 cm per sec. Find the force in (*a*) dynes; (*b*) grams.

70. A force of 20 dynes acting on a mass of 10 g will impart to it (*a*) what acceleration in 1 sec.; 10 sec.? (*b*) What will be its velocity in 1 sec.; 10 sec.?

71. Given a mass of 100 lb. at sea level. What is its weight in (*a*) pounds; poundals? (*b*) If the body be taken to Denver, Colo., say, how will its mass be affected; its weight?

72. Given a 10-lb. weight (mass of 10 lb.) at sea level. (*a*) What is its weight in pounds? (*b*) If it be carried to a point below sea level, how will its weight be affected?

73. A mass of 6 kg is acted upon by a force which imparts to it a change of velocity of 8 m per sec. in 4 sec. Find the force in (*a*) dynes; (*b*) grams.

74. In problem 73 substitute tons and inches for kilograms and meters, and solve for the force in pounds.

75. A mass of 10 g is suspended by means of a string. Find the force in dynes exerted on the string (*a*) when the system is at rest; (*b*) when it is drawn upward with a uniform velocity of 10 cm per sec.; (*c*) when it descends with a uniform velocity of 10 cm per sec.; (*d*) ascends with a uniform acceleration of 10 cm per sec. per sec.; (*e*) descends with a uniform acceleration of 10 cm per sec. per sec.; (*f*) descends with an acceleration of 980 cm per sec. per sec.

76. A mass of 10 lb. is suspended by means of a spring balance from the roof of an elevator. The spring balance is calibrated to give readings in absolute units (poundals). Consider the value of g to be 32 ft. per sec. per sec. What is the reading of the spring balance when the elevator is (*a*) at rest; (*b*) ascending with the uniform velocity of 10 ft. per sec.; (*c*) descending with a uniform velocity of 10 ft. per sec.; (*d*) ascending with a uniform acceleration of 10 ft. per sec. per sec.; (*e*) descending with a uniform acceleration of 10 ft. per sec. per sec.; (*f*) descending with a uniform acceleration of 32 ft. per sec. per sec.?

77. The spring balance (problem 76) is calibrated to give readings in gravitational units (pounds). Find the reading of the balance in each of the cases given in problem 76.

78. A man weighing 160 lb. (gravitational units) stands on the floor of an elevator. What is his weight with reference to the floor of the elevator in pounds when the elevator is (*a*) at rest; (*b*) moving with a uniform velocity; (*c*) ascending with a uniform acceleration of 8 ft. per sec. per sec.; (*d*) descending with a uniform acceleration of 8 ft. per sec. per sec.?

79. A body having a weight of 1 ton is suspended by means of a rope. The body is pulled upward with an initial acceleration of 4 ft. per sec. per sec. What is the force in pounds sustained by the rope?

80. Two equal masses of 100 g are hung by a flexible cord over a frictionless pulley. A mass of 10 g is placed upon one of the 100-g masses (Fig. 14). Find (*a*) the acceleration of the system; (*b*) the force exerted on the cord in dynes.

81. A mass of 100 g hanging by a flexible cord (Fig. 16) drags a mass of 96 g along the top of a smooth table. Neglecting frictional forces, find (*a*) the acceleration of the system, and (*b*) the stretching force in the cord.

82. A mass of 50 lb. rests upon a smooth horizontal plane. A string fastened to this mass passes over a frictionless pulley and supports vertically a mass of m' lb. The force on the string is 600 poundals. Find the value of m'.

83. A mass of 30 lb. rests upon a smooth plane which is inclined 30° to the horizontal. A string fastened to this mass passes to the top of the plane, over a frictionless pulley, and has a mass of 50 lb. suspended from it. Neglecting friction, find (*a*) the acceleration of the masses; (*b*) the force in the string.

84. According to the law of action and reaction, when a bullet is fired from a gun the momentum of the gun is equal to that of the bullet, that is, $mv = m'v'$, in which the various factors used are expressed in the same kind of units. Suppose that a given force acts simultaneously upon two bodies, m and m', which are free to move. The mass of m is 10 lb. and that of m' 10 g. The velocity imparted to m' is 10 m per second. What is the velocity of m?

85. A 5-ton safe is drawn with uniform motion up an incline, which makes an angle of 30° with the horizontal. The frictional force is 200 lb. Find the total force required to move the safe.

86. What force in absolute and gravitational units will be required to give a car having a mass of 40,000 lb. an acceleration of 0.8 ft. per sec. per sec. on a horizontal track where the frictional resistance is 800 lb.?

87. What force will be required to move the car (problem 86) up a 1 per cent grade (rise of 1 ft. in 100) at a uniform speed, the frictional resistance being, as before, 800 lb.?

CURVILINEAR MOTION

26. Curvature.—*Curvature* is the space rate of change of direction. If a particle A (Fig. 17) moves from A to A' over the arc s, θ represents the change of direction of motion and θ/s is the curvature. But $\theta = s/r$ radians, therefore we may write

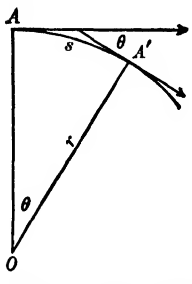

FIG. 17.—Curvature.

$$curvature = \frac{\theta}{s} = \frac{1}{r}$$

27. Angular Velocity and Acceleration.—*Angular velocity*, usually represented by ω, is the angle swept out per unit of time, that is,

$$angular\ velocity = \frac{\theta}{t} = \frac{2\pi}{T} = \omega$$

in which ω may be represented in degrees per second or in radians per second. If we consider A as moving with uniform speed over the arc s (Fig. 17), linear speed of A is $v = s/t$ or $s = vt$. Substituting this value for s in the equation for curvature, we have

$$\frac{\theta}{t} = \omega = \frac{v}{r},$$

and hence

$$v = \omega r,$$

in which v is the linear speed, ω the angular velocity, and r the radius of curvature.

When the rotation of a body is not uniform, that is, when it increases or decreases by a definite amount per unit of time, we have to deal with angular

vibration. *Angular acceleration* α is the time rate of change of angular velocity; that is,

$$\alpha = \frac{\omega}{t}.$$

In the case of rotating bodies, it is important to note the relation between linear acceleration a and angular acceleration α. If we divide the equation $v = \omega r$ by t we have $v/t = \omega r/t$. But $v/t = a$ and $\omega/t = \alpha$, and consequently

$$a = \alpha r.$$

28. Centrifugal and Centripetal Forces.—Consider a body of mass m, attached to a string of length r, whirling around in a circle with a uniform velocity about a given point. In accordance with the first law of motion, the body tends at every instant to fly off in a straight line. It is constrained to travel in a circular path by a force, the normal component of which is called the *centripetal force.* A force equal and opposite to the centripetal force is called the *centrifugal force.* The centripetal force acts *on* the body, and is directed from the circumference toward the center of the circle; the centrifugal force is exerted *by* the body, and is directed from the center toward the circumference.

It may be shown that in the case of a body moving with a uniform velocity around a circle, the acceleration a toward the center is

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \omega^2 r$$

where v = linear velocity; T = period (time of one revolution); ω = angular velocity.

Centripetal and centrifugal forces may be represented by the equation,

$$F = ma = m\frac{v^2}{r} = m\omega^2 r$$

where F = force in absolute units; m = mass of the body; and a = acceleration toward the center.

Problems

88. Show that the dimensional formula for curvature $(\theta/s) = L^{-1}$.

89. A body moving in a circle passes over an arc of 20 cm in a given time. Find the curvature when the angle swept out is (*a*) 40°; (*b*) $\pi/8$ radians. Be sure to name the units in which your answers are expressed.

90. Find the curvature of a body moving in a circle when the radius is (*a*) 12 cm; (*b*) 12 in.; (*c*) 1 ft.

91. A body moves in a circle, through an arc of 2 ft., sweeping out an angle of 30°. Find the curvature.

92. A body of mass 100 g moves uniformly around a circle of radius 10 cm $60/\pi$ times per minute. Find (*a*) the period T; (*b*) the linear speed v in centimeters per second; (*c*) the angular

velocity ω; (*d*) the acceleration a; (*e*) the centrifugal forben a.. absolute units; (*f*) centrifugal force in gravitational units.

Ans. (*a*) $T = \pi$ sec.; (*b*) $v = 20$ cm per sec.; (*c*) $\omega = 2$ radians per sec.; (*d*) $a = 40$ cm per sec. per sec.; (*e*) $F = 4000$ dynes; (*f*) 4.08 *g*.

93. A mass of 2 lb. attached to a string 4 ft. in length is whirled around, making 30 r.p.m. Find (*a*) the period T; (*b*) the linear speed v; (*c*) the angular velocity ω; (*d*) the centrifugal force in absolute units; gravitational units.

94. A mass of 32 lb. is attached to a string and is whirled around with a uniform speed, making 15 r.p.m. The centrifugal force is $0.5\pi^2$ lb. Find the length of the string.

95. A body m attached to a string 2 ft. in length moves in a circle with a linear speed of 8 ft. per sec. The centripetal force exerted upon m is 64 poundals. (*a*) Find the mass m. (*b*) What is the centrifugal force exerted by the body?

96. A stone on the end of a string 2 ft. long revolves in a vertical circle. Find the least velocity it could have so that it will maintain its path at the highest point of the circle against the force of gravity.

97. A body of 10-lb. mass moves around a circle of 2-ft. radius with an angular velocity of 5 radians per sec. Find (*a*) the centrifugal force in pounds; (*b*) the centripetal force.

98. A car of mass 40,000 lb. runs around a curve of radius 200 ft. with a speed of 20 ft. per sec. Find the horizontal thrust in pounds exerted on the outer rail.

99. (*a*) Find in pounds the centrifugal force exerted by a mass of 32 lb. at the equator, the radius of the earth being taken as 4000 miles. (*b*) What is the weight of this body, if measured by a spring balance?

SIMPLE HARMONIC MOTION

29. Illustration of S.H.M.—A body represented by the light figures A, B, C, etc., moves with a uniform velocity around the circle (Fig. 18). Consider the motion of the projection of this body on an axis of the circle, the x-axis, say. The projections of A, B, C, as the body moves around the circle in a positive (counterclockwise) sense, are represented by the heavy figures **a**, **b**, **c**, and so on.

The vibratory motion of the dark figure on the x-axis is an illustration of linear simple harmonic motion.

30. Characteristics of S.H.M.—The main characteristics of the S.H.M., as represented on the x-axis (Fig. 18) are as follows: (*a*) The motion is

vibratory. (*b*) The body executing S.H.M. has its *maximum velocity* at the middle of its path (at the point *a*), and has zero velocity at the extremities of its path (points *C* and *G*). (*c*) The body has its *maximum acceleration* at the extremities of its path (*C* and *G*), and zero acceleration at the middle point.

31. Definitions.—(*a*) The circle drawn around the diameter *GC* is called the *circle of reference*. (*b*) The radius of the circle of reference is the *amplitude* of vibration. (*c*) The time required for the body executing S.H.M. to make one complete vibration (that is, the time required for the body on the circle of reference to make one complete revolution) is the *period* T. (*d*) *Phase* is the time which has elapsed since the body executing S.H.M. last passed through the middle point, going in the positive sense. For example, when the body is at *a*, going toward the right, the phase is zero; when it is at *b*, the phase is one-eighth of a period (that is, $T/8$), and the corresponding phase angle (ωt) is AaB, in this case 45°. When the body has reached the extremity of its path (that is at *C*), the phase is $T/4$, and the corresponding phase angle is 90°, and so on. The maximum value of the phase angle is, of course, 360°, or 2π radians. The terms "phase" and "phase angle" are sometimes used interchangeably. (*e*) The *time angle* is the angle swept out on the circle of reference in the time t. Suppose that we begin to count time when the body executing S.H.M. (Fig. 18) is at *b* (*B* on circle of reference) and stop at *C*. The time angle is BaC. The phase angle, however is AaC. If we should begin to count time when the body is at its middle point going in the positive sense, as at *a*, the time angle and the phase angle would coincide.

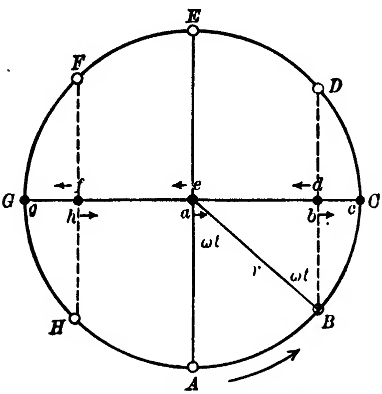

Fig. 18.—Simple harmonic motion and circle of reference.

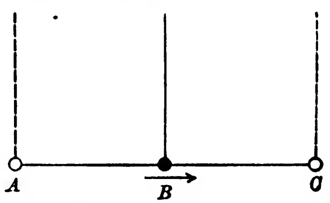

Fig. 19.—S. H. M. of long pendulum.

32. Examples of S.H.M.—(*a*) The motion of the bob of a very long pendulum (Fig. 19) in which the arc *ABC* is practically a straight line is an illustration of S.H.M. In this case the arc *ABC* represents the x-axis diameter of the circle of reference. (*b*) The vibration of a body attached to a spiral spring (Fig. 20) is an example of S.H.M. on the y-axis. The line *DE* represents the y-axis diameter of the circle of reference. (*c*) The vibratory motion of a torsional pendulum (Fig. 21) is an example of angular S.H.M.

33. Equations of S.H.M.—The condition of a body executing S.H.M. is determined at any time t by three defining equations, which determine the displacement x, the velocity v_x, and the acceleration a_x of the moving

body. These fundamental equations may be derived geometrically or by means of the calculus. These equations are, for the x-axis,

$$x = r \sin(\omega t),$$
$$v_x = \omega r \cos(\omega t),$$
$$a_x = -\omega^2 r \sin(\omega t) = -\omega^2 x,$$

in which r = amplitude of vibration; ω = angular velocity ($\omega = 2\pi/T$); t = time which has elapsed since we began to count time; ωt = phase angle; x = displacement from the middle point in time t; v_x = velocity of the particle in time t; and a_x = acceleration.

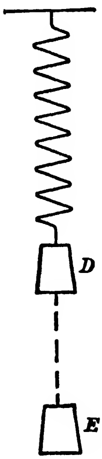

FIG. 20—S. H. M. of spiral spring.

The equation $a_x = -\omega^2 x$ gives us the basis for the fundamental definition of S.H.M., namely: *simple harmonic motion is a vibratory motion of such a nature that the acceleration is proportional to the displacement and opposite in sign.*

It should be noted that in all problems in this text it is understood that we begin to count time when the body is passing through its zero phase, going in the positive sense. In each case then, ωt represents the *phase angle.*

Example.—Consider Fig. 18 in which a body is executing S.H.M. on the x-axis. At a given instant the body is at the point d, going in a negative sense. The corresponding body on the circle of reference is at D. The period T is 4 sec.; the amplitude r, 2 ft. Find the displacement x, the velocity v_x, and the acceleration a_x, in 1.5 sec. after passing through the middle point, going in the positive sense. *Solution:* In this case $\omega = 2\pi/T = \pi/2$ radians per sec.; $t = 1.5$ sec. The phase angle $\omega t = (\pi/2) \times 3/2 = 3\pi/4$ radians $= (3/4)(180°) = 135°$. Now sin 135° = sin 45° = 0.707; also cos 45° = 0.707. It follows that $x = 2 \times 0.707 = 1.414$ ft. from middle point; $v_x = (\pi/2) \times 2 \times 0.707 = 0.707\pi$ ft. per sec.; $a_x = (\pi^2/4) \times 2 \times 0.707 = 0.354\pi^2$ ft. per sec. per sec.

Example.—A body executing S.H.M. has an amplitude $r = 10$ cm. and a period $T = 4$ sec. In what time t after passing through the middle point of its path will it have a displacement $x = 7.07$ cm? *Solution:* The angular velocity $\omega = 2\pi/T = \pi/2$ radians per sec. Then from the equation $x = v \sin(\omega t)$, we have $7.07 = 10 \sin(\omega t)$, from which $\sin(\omega t) = 0.707$. This means that (ωt) is an angle such that its sine = 0.707. Referring to the sine table (Art. 12) we find that this angle is $45° = \pi/8$ radian. Since $\omega = 2\pi/T = \pi/2$, we have $(\pi/2)t = \pi/8$, and hence $t = 1/4$ sec.

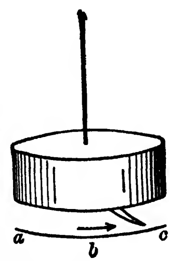

FIG. 21.—Torsional S. H. M.

Problems

100. Consider Fig. 18. What are the phase and the phase angle when the body executing simple harmonic motion on the x-axis is at a; b; c; d; f; g?

101. Suppose we start to count time when the body is at h (Fig. 18), going in the positive sense. (*a*) What is the time angle when it reaches d? (*b*) What is its phase? (*c*) What is the phase angle?

102. What is the maximum value which the phase angle may have, measured in (*a*) degrees; (*b*) radians? Suppose that we start to count time at D (Fig. 18), and stop at B. (*c*) What is the time angle in this case? (*d*) What is the phase angle?

103. A body moves around in a circle of radius 10 cm with a uniform linear speed (v) of 20 cm per sec. Find (*a*) the period T; (*b*) the angular velocity; (*c*) the position of the body with reference to the x-axis $\pi/8$ sec. after passing the middle point in a positive sense.

104. Consider the simple harmonic motion of the projection of this body (problem 103) on an x-axis passing through the center of the circle. Find the displacement x when the phase angle is (*a*) 45°; (*b*) 90°; (*c*) 180°; (*d*) 225°.

105. Find the velocity v_x of the body (problem 103) when the phase angle is (*a*) 45°; (*b*) 90°; (*c*)180°; (*d*) 225°. Where does the velocity have its minimum and maximum values? What is the meaning of the minus signs which appear in the answers to (*c*) and (*d*)?

106. Find the acceleration a_x of the body (problem 103) when the phase angle is (*a*) 45°; (*b*) 90°; (*c*) 180°; (*d*) 225°. What is the meaning of the minus sign that appears in the equation for acceleration, Art. 33?

107. A body attached to a spiral spring (Fig. 20) executes simple harmonic motion. The amplitude of its vibration is 10 cm. Its period T is 4 sec. Referring this motion to the y-axis of a circle of reference, (*a*) find the angular velocity ω in radians, degrees; and (*b*), counting time from the middle point in the path of motion, find the displacement y in 0.5 sec., 1 sec., 2 sec.

108. Find the velocity v_y for conditions as given in problem 107.

109. Find the acceleration a_y, conditions as in problem 107.

110. Find the time required for the body (problem 107) to travel 5 cm from the middle point.

111. Consider that a particle P moves with a uniform speed around a circle of radius 10 in. with a linear speed of 5π in. per sec. Find (*a*) the period T; (*b*) the angular velocity ω; (*c*) the phase angle in degrees, assuming that we start to count time at the middle point, in positive sense, and t is 10 sec.

112. Consider the S.H.M. of the projection of the particle P (problem 111) on the x-axis. Consider that we count time from the middle point. Find the displacement under the following conditions: (*a*) $t = 1$ sec.; (*b*) 2 sec.; (*c*) 3 sec.; (*d*) 4 sec.

113. Find the displacement (problem 112) when the phase angle is (*a*) 45°; 135°; 225°; 315°; (*b*) $\pi/4$ radians; $\pi/2$ radians; π radians.

114. A particle executing S.H.M. has a period of π sec. Its amplitude is 4 ft. Find its velocity when the phase angle is π radians.

115. In a S.H.M., find the period T when the acceleration at a distance of 0.5 ft. from the center is 2 ft. per sec. per sec., the amplitude being 2 ft.

116. The bob of a very long pendulum executes S.H.M.; that is, its motion is practically straight-line motion. Its amplitude of vibration is 4 ft., its period 10 sec. Find the time required for the bob to travel 2 ft. from the middle point, going in the positive sense.

117. Given a S.H.M. of amplitude 8 in. and a period of 4 sec., find the velocity of the body 0.5 sec. after leaving one extremity of its path.

118. A body executing S.H.M. has an acceleration of $\pi^2/4$ ft. per sec. per sec. when the displacement is 1 ft. Find its period.

119. A body of mass 64 lb. executes S.H.M. in a horizontal line, its amplitude being 2 ft. and its period 4 sec. Find the horizontal force ($F = ma_x$) acting upon the body at the end of its path.

120. The piston head of a steam engine, making 240 r.p.m., executes simple harmonic motion. The stroke of the piston is 2 ft. Find the velocity v_x of the piston head at the instant it passes through its middle point.

CHAPTER III

MECHANICS OF SOLIDS (*Continued*)

WORK AND POWER

34. Work and Energy.—*Work* is the product of force times displacement, the displacement being measured in the direction of the force, that is

$$work = W = Fs.$$

In the case of an expanding gas, work may be defined as the product of pressure and volume or, in equational form,

$$W = pv,$$

in which p is pressure (F/A) and v is the volume displacement.

Energy is the capacity which a body has for doing work. Energy is of two kinds, potential and kinetic. *Potential energy* (*P.E.*) is energy of position; *kinetic energy* (*K.E.*) is energy of motion. Both *P.E.* and *K.E.* are measured in terms of work; that is,

$$P.E. = W = Fs,$$
$$K.E. = W = \tfrac{1}{2}mv^2.$$

35. Units of Work.—With reference to the units of force employed, the units of work are of two kinds, namely, *gravitational* and *absolute;* with reference to the system considered, units of work are also of two kinds, *English* and *metric.*

$$\text{Units of work}\begin{cases}\text{Gravitational}\begin{cases}\text{English} = \text{foot-pound}\\ \text{Metric} = \text{gram-centimeter}\end{cases}\\ \text{Absolute}\begin{cases}\text{English} = \text{foot-poundal}\\ \text{Metric} = \text{erg}\end{cases}\end{cases}$$

The *foot-pound* is the work done by the force of a pound acting through the space of a foot. Foot-pounds = pounds of force × feet. The *gram-centimeter* is the work done by the force of a gram acting through the space of a centimeter. A *kilogrammeter* is the work done by the force of a kilogram acting through the space of a meter. One kilogrammeter = 10^5 gram-centimeters.

The *foot-poundal* is the work done by the force of a poundal acting through the space of a foot. Foot-poundals = poundals × feet. The *erg* is the work done by the force of 1 dyne acting through the space of 1 cm. Ergs = dynes × centimeters. One *joule* = 10^7 ergs.

In the derivation of the equation for kinetic energy, we use the equation $F = ma$, thus expressing F in absolute units. The equation $K.E. = \tfrac{1}{2}mv^2$, therefore gives results in absolute units, that is, in foot-poundals or ergs.

1 *foot-pound* = 32 *foot-poundals*
1 *gram-centimeter* = 980 *ergs*

36. Units of Power.—*Power* is the time rate of doing work. The absolute unit of power is the watt. A *watt* is 10^7 ergs per sec., that is, 1 joule per sec. According to the Bureau of Standards definition,

$$1 \; horsepower \; (hp.) = 746 \; watts$$

According to this definition, 746 watts (1 hp.) are equivalent to 550 ft.-lb. per sec., or 33,000 ft.-lb. per min., at 50° N. latitude, and at sea level. In this text, for all problems relating to power, it is assumed that the following equations hold:

$$watts = \frac{ergs}{10{,}000{,}000 \times time \; in \; seconds}$$

$$hp. = \frac{foot\text{-}pounds}{33{,}000 \times time \; in \; minutes} = \frac{foot\text{-}pounds}{550 \times time \; in \; seconds}$$

In connection with the solution of power problems, the student should note that the watt is expressed in absolute units, involving the factors $F = ma = mg$, while horsepower, on the other hand, is expressed in gravitational units, the force F being given in pounds.

Problems

121. Define: foot-pound, foot-poundal, gram-centimeter, kilogrammeter, erg, joule.

122. Write the dimensional formula for work.

123. A force of 100 dynes acts through a space of 100 cm. Find the work in (*a*) ergs; (*b*) joules; (*c*) gram-centimeters.

124. A force of 100 poundals acts through a space of 100 ft. Find the work in (*a*) foot-poundals; (*b*) foot-pounds.

125. A mass of 10 g moves with a velocity of 10 cm per sec. Find its kinetic energy in (*a*) ergs; (*b*) gram-centimeters.

126. A mass of 10 lb. moves with a velocity of 10 ft. per sec. Find its kinetic energy in (*a*) foot-poundals; (*b*) foot-pounds.

127. Forty-nine joules of work were done by a force acting through a distance of 2 m. Find the force in (*a*) dynes; (*b*) gram-centimeters.

128. A stone having a mass of 20 lb. is carried to the top of a tower 50 ft. in height. (*a*) What potential energy does the stone possess in gravitational units? (*b*) If it be dropped from the top of a tower what kinetic energy in gravitational units will it possess at the instant it strikes the ground?

129. A horse pulls a load weighing 1 ton a distance of 1 mile. If the force exerted upon the traces be 300 lb. what is the work in foot-pounds done by the horse?

130. The lower end of a ladder 30 ft. long stands on the ground at a distance of 6 ft. from the building against which the upper

end rests. How much work is done against gravity in carrying 100 lb. to the top of the ladder?

131. A hammer weighing 8 oz. strikes a nail with a velocity of 10 ft. per sec. driving it 1 in. What average force in pounds is exerted by the hammer upon the nail?

132. A body of mass m moving with a velocity of 5 ft. per sec. possesses kinetic energy equivalent to 25 ft.-lb. Find the mass of the body.

133. A mass of 196 g possesses kinetic energy equivalent to 40 g-cm. Find the velocity.

134. A mass of 10 lb. falls freely from a point 800 ft. above the ground. (*a*) Find its kinetic energy in foot-pounds after it has fallen for 5 sec. (*b*) What is its potential energy at the end of the 5 sec.? (*c*) How far has it fallen?

135. (*a*) What velocity will the body have (problem 134) when it strikes the ground? (*b*) What is its kinetic energy? (*c*) How does this value compare with its potential energy at the instant it starts to fall?

136. If 1 kg be lifted vertically through a space of 78.74 in. how much work is done in (*a*) kilogrammeters; (*b*) gram-centimeters; (*c*) ergs; (*d*) joules?

137. A mass of 200 lb. is hoisted out of a mine. Find in gravitational units the work done when the mass is drawn upward a distance of 200 ft. (*a*) with a uniform velocity of 20 ft. per sec.; (*b*) with an acceleration of 4 ft. per sec. per sec.

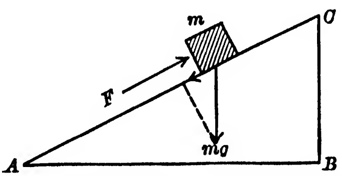

FIG. 22.—Work on inclined plane.

138. Suppose that the body (problem 137) is drawn upward with a uniform velocity of 20 ft. per sec. for 7.5 sec., and then its motion is retarded for 5 sec. at a rate of 4 ft. per sec. per sec. Find in gravitational units the work done over (*a*) the first 150 ft.; (*b*) the remaining 50 ft.

139. Suppose that the mass m (Fig. 22) moves on the incline without friction. The incline AC is 10 ft. in length, and makes an angle of 30° with AB. The mass m is 10 lb. (*a*) If F is 5 lb., how many foot-pounds of work will be done in moving m from A to C, neglecting friction? (*b*) How many foot-pounds will be required to lift a similar mass from B to C? (*c*) In this latter case, what force will be required in pounds; poundals?

140. Suppose that AC (Fig. 22) is 12 ft., angle BAC is 30°, and m is 20 lb. Find (*a*) the force mg in poundals; (*b*) the component of this force down the plane; (*c*) neglecting friction, the work in foot-pounds required to move m from A to C; (*d*) the work in foot-pounds required to lift m vertically from B to C.

141. Define: power, watt, kilowatt, horsepower.

142. Write the dimensional formula for power.

143. A mass of 300 kg is lifted vertically through a distance of 50 m in half a minute. Find the power expended in watts and kilowatts.

144. At what rate (horsepower) is energy expended when a force of 220 lb. is exerted through a distance of 16 ft. in 2 sec.?

145. At what rate (horsepower) is energy expended when a force of 220 poundals is exerted through a distance of 16 ft. in 2 sec.?

146. A mass of 11,190 kg is lifted vertically to a height of 10 m in 10 sec. Find the power expended in (*a*) kilowatts; (*b*) horsepower.

147. Water flows into a mine 500 ft. deep at the rate of 100 cu. ft. per min. Find the horsepower of an engine that will be required to keep the mine dry.

148. In a given steam engine the average pressure exerted on the piston is 180 lb. per sq. in.; the diameter of the piston, 1 ft.; the length of the stroke is 2 ft.; the number of r.p.m. 120. Find the horsepower.

149. Find the horsepower expended in taking a train of 100 tons up an incline of 1 ft. in 200 ft. at the rate of 20 miles per hr., neglecting friction.

150. If a locomotive is rated at 1200 hp., what force in pounds should it be able to exert while running 40 miles an hour?

151. The area of a piston of a force pump is 200 sq. in., and the length of stroke 20 in. The pump is used to force water into the city mains under a pressure of 60 lb. per sq. in. (*a*) How much work is done per stroke of the piston? (*b*) At what rate (horsepower) will the pump be doing work when it supplies 5,000,000 gal. per 10-hr. day, 1 gal. being equivalent to 231 cu. in.?

152. A train having a mass of 120 tons, including the engine, meets an average resistance on the level of 15 lb. per ton. The engine is of 150 hp. Find the "full speed" of the train; that is, the speed at which the engine exerts a force equal to the resistance.

MECHANICAL MOMENTS

37. Moment of a Force.—Consider the force F (Fig. 23) to act on the lever arm of length d, thus tending to produce rotation about the point O. The product of the force F and the lever arm d is called the moment of the force, or the *torque;* that is,

$$\textit{moment of a force} = \textit{force} \times \textit{lever arm} = Fd,$$

in which the lever arm d is a straight line measured from the origin O to the line of direction of the force, and at right angles to it. For example, in Fig. 24 suppose that the force F acts on the lever arm FO in the direction BF, thus tending to produce rotation about O. The lever arm in this case is the line $BO = d$.

38. Moment of a Couple.—Two equal parallel forces acting in opposite senses constitute a *couple.* The *moment of a couple* is the product of one of the forces and the perpendicular distance between them; that is, Fd. This being true, it follows that the moment of a couple is the same, no matter where the origin of rotation be chosen in the plane containing the forces.

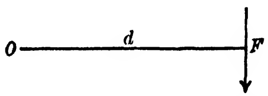

FIG. 23.— Moment of a force, Fd.

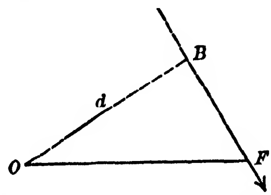

FIG. 24.—Moment of a force, Fd.

Example.—Given the couple represented by two equal, parallel, and oppositely directed forces, $F = F'$ (Fig. 25) to find the moment. *Solution:* Let O be any point on the line AB, and d be the distance between F and F'. The moment tending to produce rotation in a clockwise sense is $+F(d + x)$; the moment tending to produce rotation in a counterclockwise sense is $-F'x = -Fx$. The moment of the couple is the resultant of these two moments; that is, the moment of the couple $= +F(d + x) - Fx = Fd$.

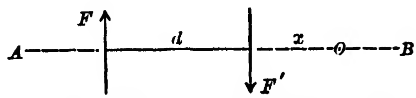

FIG. 25.—Moment of a couple, $Fd = F'd$.

39. Solution of Problems by Moments.—Consider Fig. 26. Suppose that the force F acts downward on the bar AB tending to produce rotation in a clockwise sense about the point B. Suppose also that the bar AB is supported by a string AC, in which there is exerted a force F'. Now the moment due to the force F is Fd; the moment due to the force F' is $F'd'$. The system is in equilibrium, therefore these moments are equal in magnitude and opposite in sense, hence we may write

$$Fd = F'd'.$$

Consider also Fig. 27. Let AO represent a uniform homogeneous bar of length $2l$, the mass of which may be considered to be concentrated at the center of gravity G. The bar is drawn aside so that OA makes an angle θ with OB. The force F represents the weight of the bar and acts downward; F' is a horizontal force required to hold the bar in position. The equation of equilibrium here is $Fd = F'd'$, where $d' = BO$. But $d = l \sin \theta$, and $d' = 2l \cos \theta$, hence we may write

$$Fl \sin \theta = F'2l \cos \theta.$$

Example.—A uniform metal bar AB, of mass 30 lb., length 8 ft., is supported in a horizontal position, as shown in Fig. 26. A weight of 50 lb. is attached to the end A. We wish to find, by the method of moments, the force exerted on the support AC. The length of BC is 10 ft. *Solution:* We shall first find the value of d', in terms of the angle ACB. The tangent of $ACB = 0.8$, and from our tables we find that $\sin ACB = 0.6247$. Then

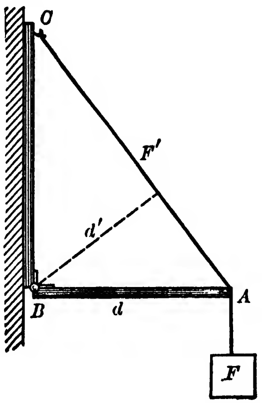

Fig. 26.—Problem of moments.

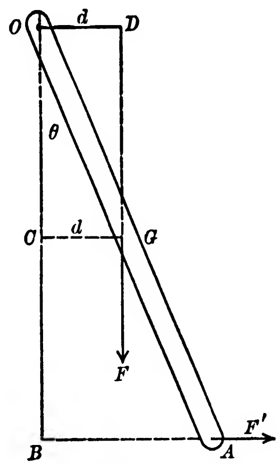

Fig. 27.—Problem of moments.

$d' = 10 \times 0.6247 = 6.247$. The weight of the rod (30 lb.) is considered as if it were concentrated at the center of AB. From the equation of moments ($Fd = F'd'$) we may write $50 \times 8 + 30 \times 4 = F' \times 6.247$, and hence $F' = 83.27$ lb. force.

Example.—Suppose that a rod, of mass 120 lb., and length 12 ft 5,000,000
at O (Fig. 27) is drawn aside, making with BO an angle of 36°. acts at right angles to AO. Find (*a*) the force F'; (*b*) the do in.?
on O; (*c*) the horizontal force on O. *Solution:* (*a*) $Fd = 120$ he engine,
$= 423.22$. Then, from the equation, $FD = F'd'$, 423.22 per ton.
Hence $F' = 35.27$ lb. force. (*b*) The downward force on $O =$ the train;
tical component of $F' = 120 - 35.27 \times \sin 36° = 99.27$ lb.
The horizontal thrust on $O =$ the horizontal component of F' ual to the
$36° = 27.53$ lb. force.

Problems

153. Find the moment of a force of 10 lb. applied to one end of a lever arm 10 ft. in length, when the line of direction of the force makes with the lever arm an angle of (*a*) 90°; (*b*) 30°.

154. A horizontal rod *AB* 100 cm in length is hinged to a vertical support *BC*, as shown in Fig. 26. A string supporting a mass of 2828 g is fastened to the end *A*, and the other end of the string is fastened to the vertical support at the point *C*. Find by the method of moments the force F' exerted on the string *AC*, in gravitational units, when (*a*) *BC* is 100 cm; (*b*) 50 cm.

155. Suppose (problem 154) that the cord *AC* makes an angle of 30° with *AB*, and the weight (2828 g) is suspended from the middle point of *AB*. Find the pull on *AC*.

156. A uniform rod 12 ft. in length and having a mass of 100 lb. is pivoted at one end and hangs in a vertical position (Fig. 27). The lower end of the rod is now drawn aside until it makes an angle of 30° with the vertical. What force applied to the lower end will be required to hold the bar in this position when the force is applied (*a*) at right angles to the rod; (*b*) at right angles to the vertical line *BO*?

157. Find the downward force exerted on the pivot under the conditions (*a*) and (*b*) of problem 156.

158. A uniform steel rod *AB* (Fig. 26) of length 12 ft., mass 200 lb., is pivoted at the end *B* to a vertical support *BC*. The rod *AB* is held in a position at right angles to the support *BC* by means of a rope *AC*, at a point above *B* such that *BC* is 18 ft. Find the force exerted in pounds on the rope.

159. Suppose that the rope *AC* of problem 158 is shortened, thus drawing *A* upward until angle $BAC = 90°$. Make a sketch showing the position of the rod *AB*, with respect to its former position, and find the force (pull) in the rope.

160. Suppose that the rope (problem 158) is slackened until the angle $ABC = 120°$; that is, the angle between the bar and the wall is 60°. Find the force on the rope.

161. A uniform bar 10 ft. in length, mass 5 lb. to the foot, is supported in a horizontal position, as shown in Fig. 26. The angle $ABC = 60°$. Find (*a*) the vertical component at *C*, due to the force along the line *AC*; (*b*) the horizontal component at *C*.

162. A uniform homogeneous bar *OC* (Fig. 28) of length 6 ft. and weight W' 50 lb., supports at the end *C* a cylindrical metal

disc of radius 1 ft. and weight W 200 lb. What force applied in the horizontal direction AB will be required to keep the system in equilibrium, the angle θ being 30°?

163. (*a*) What is the downward pull on the pivot at O when the bar OC hangs in a vertical position? (*b*) What is the vertical (downward) thrust on O when $\theta = 30°$? (*c*) What is the horizontal thrust on the pivot when $\theta = 30°$?

164. A crane lever AC (Fig. 29) carries a suspended weight W in such a position that the angle $\theta = 60°$. The weight $W =$ 1.5 tons, and W', the weight of the bar AC, $=$ 800 lb. (*a*) What is the horizontal pull on the cable CF? (*b*) What is the thrust on the pivot at A in the direction CA?

165. Find (*a*) the horizontal thrust (problem 164) on the pivot; (*b*) the vertical thrust.

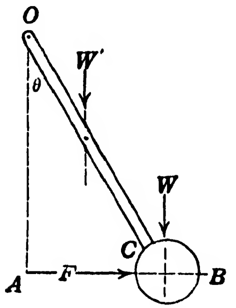

FIG. 28.

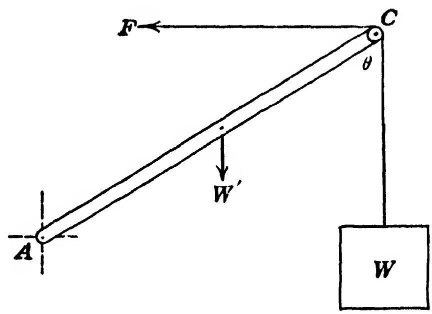

FIG. 29.

166. A telephone pole standing at the corner of a street carries two systems of wires, which act at right angles to each other, and which produce a resultant force of 2000 lb. The pole is supported from the top by means of a guy wire which makes an angle of 60° with the ground. The height of the pole is 30 ft. The resultant force on the pole due to the wires lies in the same plane as the supporting guy wire. Find, by the method of moments, the force in the guy wire, assuming that 40 per cent of the load is carried by the earth around the base of the pole.

40. Center of Mass; One Dimension.—Suppose that we consider two material particles m and m' at a given distance from each other. Let us select a point p, ($pm = x$ and $pm' = x'$), such that $mx = m'x'$; that is, $mx - m'x' = 0$. The point p is called the *center of mass*, or centroid of the system. The products mx and $m'x'$ are the moments of mass. The center

of mass of a body is a point about which the sum of the moments of mass is zero; that is,

$$\Sigma mx = \Sigma my = \Sigma mz = 0.$$

The *center of mass*, or centroid of the system is sometimes called the *center of inertia*.

Center of Gravity.—If we consider the gravitational forces which act on the masses of a body m, m', m'', etc., as parallel, then the center of gravity

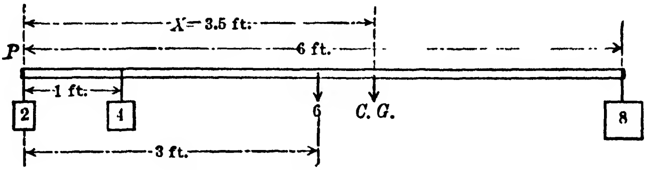

FIG. 30.—Center of gravity.

is identical with the center of mass. *Center of gravity*, then, may be defined as the point through which the total weight of the body, considered as a single vertical force, acts.

For any given plane of reference, it may be shown that the center of mass may be defined by the equation

$$X = \frac{\Sigma mx}{M},$$

in which X = distance from the plane of reference to the center of mass G; m = any material particle of the system, and x = distance of m from the plane of reference; M = mass of the entire system.

Example—Suppose that we have masses m, m', m'', of 2, 4, and 8 lb., respectively, fastened to a uniform homogeneous rod, of length 6 ft. (Fig. 30) and wish to find the center of gravity of the system. Mass m is at one end of the rod; m' is 1 ft. from the end; and m'' is at the other end. The mass of the rod is 6 lb., and since the rod is uniform and homogeneous, we may consider that this mass (6 lb.) is concentrated at the midpoint. We select a plane of reference P at right angles to the line mm'. Now this plane of reference may pass through any point we choose. It is convenient, however, to select one of the end points, as m. *Solution:* According to our equation the center of mass of the system lies at a distance $X = \Sigma mx/M = (2 \times 0 + 4 \times 1 + 6 \times 3 + 8 \times 6)/(2 + 4 + 6 + 8) = 70/20 = 3.5$. The center of gravity G, then, lies at a point on mm', to the right of the plane of reference P, equal to 3.5 ft.

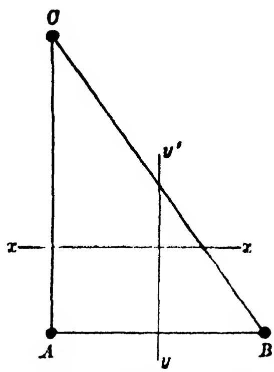

FIG. 31.—Center of gravity.

41. Center of Mass; Two or Three Dimensions.—When a two-dimensional body is considered, as in Fig. 31, we have to deal with the equations $X = \Sigma mx/M$ and $Y = \Sigma my/M$.

Example.—Given three masses of 2, 4, 6, respectively, placed at the vertices of a triangle (Fig. 31) the base of which is 4 ft. and the altitude 6 ft., to find the center of gravity G of the system. In this case we shall choose two planes of reference, one lying in the line AC, and the other in the line AB, and both at right angles to the plane of the paper. *Solution:* (*a*) First let us consider the three bodies with reference to the plane AC. In this case $X = \Sigma mx/M = (4 \times 0 + 2 \times 0 + 6 \times 4)/12 = 2$. This means that the center of mass with reference to AC lies somewhere on the line yy', and at a distance 2.4 ft. from AC. (*b*) We must now determine the center of mass with reference to the plane AB. The equation is $Y = \Sigma my/M = (4 \times 0 + 6 \times 0 + 2 \times 6)/12 = 1$. The center of mass, therefore, lies at the point G, on the intersection of the line xx' and yy', 1 ft. from AB, and 2 ft. from AC.

If we consider a body with reference to its three dimensions it will be necessary to employ, after the manner shown in the last example, all three of the equations,

$$X = \frac{\Sigma mx}{M},\ Y = \frac{\Sigma my}{M},\ Z = \frac{\Sigma mz}{M}.$$

Problems

167. Masses of 10, 20, and 30 g are placed at the following points on the line AB, the length of which is 1 m. The mass 10 is placed at A, the mass 20, 60 cm from A, and the mass 30, at B. Find the center of mass of the system.

168. Suppose that the rod AB (problem 167) has a mass of 40 g. Find the center of inertia (center of mass) of the system.

169. Masses of 10, 20, and 30 g are placed at the points A, B, C, of a right triangle. The base AB is 15 cm and the altitude BC is 12 cm. Find the center of gravity of the system.

170. A bridge having a span of 120 ft. supports a locomotive weighing 60 tons. The center of gravity of the locomotive is 30 ft. from one end of the bridge. The weight of the bridge is 90 tons. Find the force on each pier supporting the bridge.

171. Two parallel forces of 40 and 50 lb., respectively, act in the same direction and sense upon a bar at points 9 ft. apart. Find the magnitude and point of application of the resultant.

172. Find the center of gravity of a system consisting of a shaft 8 ft. long, of a mass of 60 lb., and having a 30-lb. pulley at one end and a 70-lb. pulley at the other end.

173. A uniform metal bar AB, 10 ft. in length, having a mass of 2 lb. to the foot, carries a weight of 6 lb. at the end A, and a weight of 10 lb., 2 ft. from the end B. Where must the bar be supported in order to balance?

174. The centroid of a triangle is at the intersection of its median lines. A right-angled triangular board of uniform thickness has a mass of 2 lb., height 3 ft., base 1 ft. Find the center of gravity of the system when a 2-lb. weight is placed (*a*) at the right-angular vertex; (*b*) at the middle of the longest side.

175. A metal beam of variable cross-section is supported by two pillars, one at each end, the load on the pillars being 100 and 200 lb., respectively. When the pillars are shifted so that each stands 1 ft. from the end of the beam, the loads are 90 and 210 lb., respectively. Find the length of the beam.

176. A uniform homogeneous bar AB 10 ft. in length carries a weight of 36 lb. at the end A and a weight of 20 lb. at the end B. The center of gravity is 4 ft. from the end A. What is the mass of the bar per linear foot?

177. A homogeneous cubical block, 4 ft. on each edge and mass 400 lb., lies on one face on a horizontal plane. The block is turned on one edge so that the horizontal face becomes vertical. (*a*) What is the direction and magnitude of the least force required to start the block? (*b*) In overturning the block, through what vertical height is the center of mass lifted? (*c*) How much work is done?

MACHINES

42. The Law of Machines.—A *machine* is a device for transferring or transforming energy. In accordance with the law of the conservation of

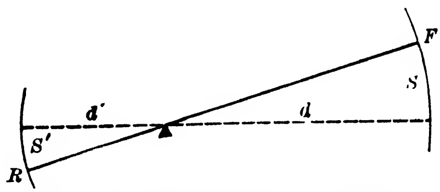

FIG. 32.—Illustrating law of machines.

energy, the work done by a force acting on a machine must equal the work done by the machine. This principle, as applied in mechanics, is expressed by the general law of machines as follows: The force, multiplied by the distance through which it acts, is equal to the resistance overcome, multiplied by the distance through which it acts. This statement of the law takes no account of friction. In terms of Fig. 32, the law may be expressed as

$$Fs = Rs', \text{ or } Fd = Rd',$$

in which F is the force applied, and d its lever arm; R is the resistance overcome, and d' is its lever arm.

43. Mechanical Advantage.—Three advantages may be derived from the use of a machine. (*a*) We may apply a force in the most advantageous direction, as in the lifting of a weight by means of a pulley. (*b*) We may gain in speed at the expense of force, as in the gearing of a bicycle. (*c*) We may use a small force to overcome a large resistance, as in the case of lifting a heavy weight by means of a lever. When we speak of mechanical advantage we usually refer to the third advantage as mentioned above.

Mechanical advantage is the ratio of the resistance overcome to the force or effort applied. In determining the mechanical advantage of a machine two things should be noted: (*a*) friction is not taken into account, and (*b*) the ratio of the resistance overcome to the effort applied may be expressed in terms of certain parts of the machine. Thus

$$\textit{mechanical advantage} = \frac{R}{F} = \frac{s}{s'} = \frac{d}{d'}$$

44. Efficiency.—The *efficiency* of a machine is the ratio of the useful work gotten out to the work put into it; that is,

$$\textit{efficiency} = \frac{\textit{work out}}{\textit{work in}}$$

An ideal machine, that is, a frictionless machine, would have an efficiency of 100 per cent. In the case of a simple lever the efficiency may be nearly 100 per cent, while for the screw it may be as low as 25 per cent. The difference between the work done on a machine and the useful work delivered by it represents the frictional losses, hence

$$\textit{work in} = \textit{work out} + \textit{frictional losses}$$

45. Mechanical Powers.—There are six so-called mechanical powers, or simple machines, as follows: The lever, wheel and axle, inclined plane,

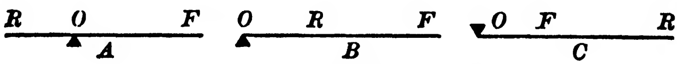

FIG. 33.—Three kinds of levers.

pulley, wedge, screw. All forms of mechanical machines, however complex, may be reduced in principle to one or more of these simple machines. In general, problems relating to machines may be solved by the use of the equations, $Fs = Rs'$, or $Fd = Rd'$.

46. The Lever.—A *lever* is a rigid bar capable of moving about a fixed point called a fulcrum. The three classes of levers are shown in Fig. 33, in which F is the force applied, R the resistance overcome, O the fulcrum, d the force arm, and d' the resistance arm. It should be noted that d is always measured from the fulcrum to the line of direction of the force and at right angles to it; and the resistance arm d' is measured from the fulcrum to the line of direction of the resistance and at right angles to it.

47. The Wheel and Axle.—The *wheel and axle* is an application of the principle of the lever (Fig. 34) in which F may be considered the force applied; R the resistance overcome, OF the force arm d, OR, the resistance arm d'.

48. The Pulley.—The *pulley* is a wheel turning about an axis in a frame or block. A block, or set of blocks, containing one or more pulleys, together with the attached rope is called a *block and tackle*.

The law of machines ($Fs = Rs'$) applies to pulleys. In case A (Fig. 35) when F moves downward 1 ft. R moves upward 1 ft.; that is, $s = s'$. In case B, when F moves 2 ft. R moves 1 ft.; that is, $s = 2$, and $s' = 1$, the mechanical advantage, in this case, being $R/F = 2/1$.

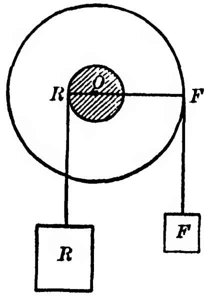

Fig. 34.—Wheel and axle.

In the case of the *differential pulley* (Fig. 39) we have $Fr = \frac{1}{2}W(r - r')$, where r and r' represent the large and small radii of the fixed pulley system.

49. The Inclined Plane.—The *inclined plane* is a device for lifting heavy bodies through a vertical height by sliding or rolling them along an incline. When the force is applied parallel to the incline (Fig. 36), the general equation $Fs = Rs'$ applies, in which case $AC = s$ and $BC = s'$.

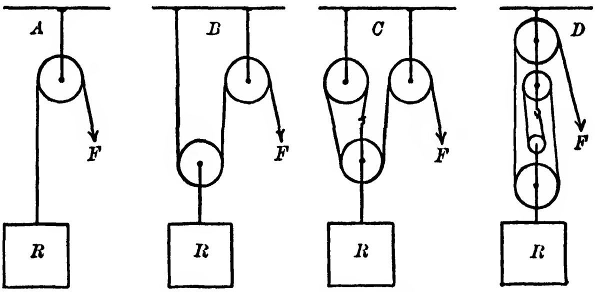

Fig. 35.—Combinations of pulleys.

50. The Screw.—A *screw* is a cylinder having a spiral groove cut around its circumference. The spiral ridge is called the thread, and the distance between two consecutive threads, the pitch. The mechanical advantage of

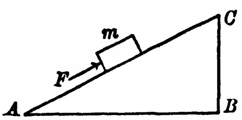

Fig. 36.—Inclined plane.

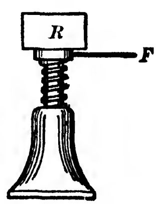

Fig. 37.—Jack screw.

the screw is derived from a combination of the principles of the lever and the inclined plane.

Example.—The lever FR of the jackscrew (Fig. 37) is 1 m in length. If one revolution of the force F causes the screw to move upward 1 cm, what

resistance R will be overcome by a force of 10 kg? *Solution:* Using the equation $Fs = Rs'$ we may write, $2\pi \times 100 \times 10 \times 1000 = R \times 1$. Hence $R = 2{,}000{,}000\pi$ g $= 2000\pi$ kg.

51. Friction.—*Friction* is the resisting force which is encountered when one body slides or rolls over another. This resisting force (friction) depends upon the nature of the surfaces in contact and the force with which they are pressed together. For a given pressure, rolling friction is in general less than sliding friction. The following facts have been experimentally determined with reference to sliding friction: Sliding friction (*a*) is independent of the area of the surfaces in contact, (*b*) is directly proportional to the force with which the surfaces are pressed together, and (*c*) through a considerable range is independent of the speed of the body. The friction between two bodies is greatest at starting (starting friction).

The *coefficient of sliding friction* is the ratio between the force F required to move the body at a constant speed and the normal force N pressing the two surfaces together, or, in equational form, *coefficient of friction* $= F/N = k$, from which

$$F = kN.$$

Example.—An iron weight of mass 25 lb. is drawn at constant speed over a horizontal leather surface, the force required to keep the speed constant being 14 lb. Find the coefficient of friction between iron and leather. *Solution:* Coefficient of friction $k = F/N = {}^{14}\!/_{25} = 0.56$.

Problems

178. Make drawings to illustrate the three classes of levers and explain the mechanical advantage of each.

179. A force of 10 lb. is applied to one end of a lever 10 ft. in length. The resistance is 2 ft. from the fulcrum. Neglecting the weight of the lever, find R when (*a*) the fulcrum is 2 ft. from the end R; (*b*) when the fulcrum is at the end of the lever (Fig. 33, B). (*c*) Compare the mechanical advantages of the two systems.

180. A uniform steel rod 10 ft. in length, having a mass of 100 lb., is used as a lever. The resistance arm d' is 2 ft. Find the force required to overcome a resistance of 1350 lb. when (*a*) the system is used as a lever of the first class; (*b*) second class.

181. Consider Fig. 35. Through what distance must F move in order to lift R 1 ft. in (*a*) case C; (*b*) case D?

182. Find the resistance that may be overcome by a force of 10 lb. in each of the cases illustrated in Fig. 35, neglecting friction and the weight of the pulleys.

183. A force of 10 lb. applied to a system of pulleys as shown in Fig. 38 will support what weight W, neglecting the weight of the pulleys?

184. Neglecting friction, what weight W will be supported when a force of 40 lb. is applied to a differential pulley system (Fig. 39), the radii of the fixed pulley system being $r = 5$ in. when (*a*) $r' = 3$ in.; (*b*) $r' = 4.5$ in.?

185. Assume that the efficiency of a differential pulley system is 75 per cent. The radii r and $r' = 4.5$ and 3 in., respectively. A force of 200 lb. at F (Fig. 39) will lift what weight (including W and the movable pulley)?

186. Given an inclined plane (Fig. 36) the base of which is 8 ft.; the height BC, 6 ft.; the mass m, 20 lb. Consider that we may neglect friction between the mass m and the incline AC. What force, in pounds and poundals, acting parallel to the incline, will be required to slide m up the plane at constant speed?

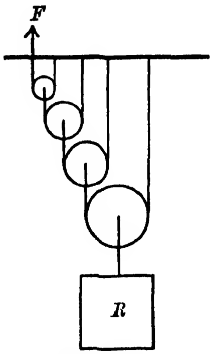

FIG. 38.

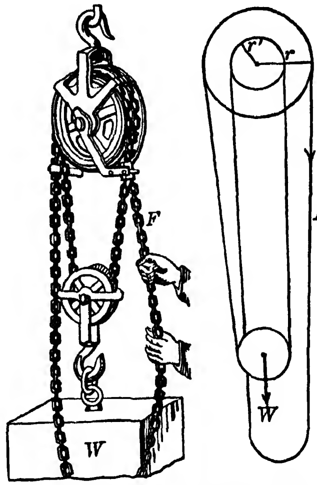

FIG. 39.—The differential pulley.

187. Find the force required to move m up the plane (problem 186) at constant speed, the coefficient of friction between m and the surface of the plane being 0.25.

188. Suppose that the frictional force required to slide a box up an incline, the vertical height of which is 4 ft., is 50 lb. The weight of the box is 600 lb. What must be the length of the incline such that a total force of 250 lb. applied parallel to the incline is required to raise the box to the top of the incline?

189. The lever arm of a jackscrew is 3 ft. in length. The distance between consecutive threads is $\frac{1}{4}$ in. A force of 100 lb. applied to the end of the lever arm will exert what lifting force?

190. A steel tool is pressed with a force of 20 lb. against the face of a grindstone that is rotating at constant speed. The coefficient of friction is 0.6. What backward pull must be exerted on the tool to prevent its moving forward in the direction of rotation of the stone?

MOMENTS OF INERTIA

52. Kinetic Energy of Rotation and Moment of Inertia.—Consider the motion of a single mass particle m of a disc which is rotating at constant speed about an axis through its center and at right angles to its face. The angular velocity of the particle is ω, and its linear velocity $v = \omega r$, where r is the radius of gyration of the particle m. The kinetic energy of the particle is $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\omega^2 mr^2$, in which the quantity mr^2 is the moment of inertia of the mass particle. Now the kinetic energy of rotation for the entire system is $K.E. = \frac{1}{2}\omega^2 \Sigma mr^2$, the quantity Σmr^2 being the general defining term for moment of inertia; that is,

$$\textit{moment of inertia} = \Sigma mr^2 = I$$

The student should note that r in this equation is the radius of gyration of a given mass particle m and not necessarily the radius of the disc or other rotating body. He should note also the similarity of the equations

$$K.E.\ \textit{of translation} = \tfrac{1}{2}Mv^2$$
$$K.E.\ \textit{of rotation} = \tfrac{1}{2}I\omega^2$$

Moment of inertia is expressed numerically in grams centimeter squared (g cm^2) or pounds feet squared (lb. ft.^{2}).

53. Moments of Inertia about the Center of Mass.—The moment of inertia of a body about an axis through its center of mass or centroid is designated by the letter I_o. For moment of inertia about any axis, whether through the center of mass or not, we use the letter I. A few of the more commonly occurring equations for moments of inertia are given below. These all refer to symmetrical bodies rotating about axes through their respective centroids.

1. *Thin Ring.*—The moment of inertia of a thin ring of mass M and radius R, a wire hoop for example, in which all the mass particles are at practically the same distance from the axis of rotation, is

$$I_o = MR^2.$$

2. *Disc or Cylinder.*—The moment of inertia of a disc or cylinder of mass M, radius R, about an axis passing through its center of mass and at right angles to its face, is

$$I_o = \frac{MR^2}{2}.$$

3. *Ring or Hollow Cylinder.*—The moment of inertia of a ring or hollow cylinder of mass M, outer radius R, inner radius R', about an axis through the center of mass and at right angles to the face, is

$$I_o = \frac{M(R^2 - R'^2)}{2}.$$

4. *Sphere.*—The moment of inertia of a sphere of mass M and radius R, about an axis through the center, is

$$I_o = \frac{2MR^2}{5}.$$

5. *Thin Rod.*—The moment of inertia of a thin rod of uniform density, of length l, about an axis through its middle point is

$$I_o = \frac{Ml^2}{12}.$$

6. *Rectangle.*—The moment of inertia of a rectangular body of mass M, length a, width b, about an axis through the center of mass and at right angles to the face, is

$$I_o = \frac{M(a^2 + b^2)}{12}.$$

Example.—A uniform circular disc, having a mass of 200 g and a radius of 10 cm, rotates about an axis through its center and at right angles to its face, making 15 r.p.m. Find (*a*) its moment of inertia; (*b*) its kinetic energy due to rotation. *Solution:* The moment of inertia of a uniform circular disc about an axis passing through its center of mass is (*a*) $I_o = \frac{1}{2}Mr^2 = 10{,}000$ g cm². (*b*) The period of rotation $T = {}^{60}\!/_{15} = 4$ sec. and the angular velocity $\omega = 2\pi/T = \pi/2$ radians per sec. Hence $K.E. = \frac{1}{2}I\omega^2 = 1250\pi^2$ ergs.

Example.—A rectangular plank, mass 12 lb., length 12 ft., width 18 in., rotates about an axis which is at right angles to its face and which passes through its center of gravity, making $1/\pi$ r.p.s. Find (*a*) the moment of inertia; (*b*) the kinetic energy of the rotating system. *Solution:* The moment of inertia of a rectangular body, under the conditions named, is (*a*) $I_o = M(a^2 + b^2)/12 = 12(144 + 2.25)/12 = 146.25$ lb. ft.². Also (*b*), $T = \pi$ sec., and $\omega = 2\pi/T = 2$ radians per sec. Hence $K.E. = \frac{1}{2}I\omega^2 = 292.5$ foot-poundals $= 292.5/32$ ft.-lb.

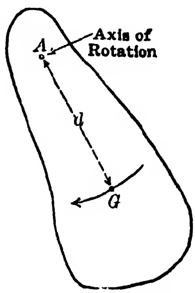

FIG. 40.—Moment of inertia about parallel axes.

54. Moment of Inertia About a Parallel Axis.—Thus far we have considered moments of inertia of systems of symmetrical bodies rotating about axes passing through their centers of mass. It is important, however, to consider the moment of inertia of a body rotating about an axis parallel to the axis through the center of mass. In actual practice we usually have to consider cases of this sort; that is, the moments of inertia of systems rotating about axes other than those passing through their centroids.

It may be shown that the equation for the moment of inertia of such a system is

$$I = I_o + Md^2,$$

in which I = the moment of inertia of the system about the given axis A, (Fig. 40); I_o = moment of inertia about an axis parallel to the given axis and passing through the center of gravity G; M = mass of the body; and d = distance between the two parallel axes.

Example.—A circular disc of mass 20 lb., radius 2 ft., rotates about an axis at right angles to its face and passing through a point 18 in. from the center. Find the moment of inertia of the system. *Solution:* $I = I_o + Md^2$. In this case $I_o = \frac{1}{2}Mr^2 = \frac{1}{2} \times 20 \times 4 = 40$ lb. ft.^{2}; $M = 20$ lb.; $d = 1.5$ ft., hence $d^2 = \frac{9}{4}$. Then $I = 40 + 20 \times \frac{9}{4} = 85$ lb. ft.^{2}.

55. Moment of Inertia and Angular Acceleration.—The relation between the *moment of a force* Fd tending to produce rotation about an axis, the *moment of inertia* I, and the *angular acceleration* α of a rotating system, is represented by the equation

$$Fd = \alpha I,$$

in which F = the force in *absolute units* tending to produce rotation; d = lever arm (Fig. 41); I = moment of inertia about the axis O; and α = the angular acceleration. In dealing with this equation it is important to recall (Art. 27) that for any point in a rotating system where r is the distance of the point to the center of rotation, the linear acceleration $a = \alpha r$.

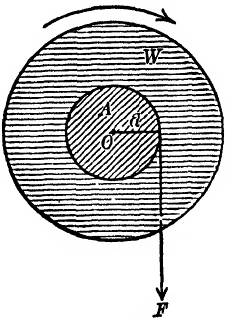

FIG. 41.—Moment of inertia and angular acceleration.

Example.—A string carrying a weight is wrapped around the axle A of a wheel W (Fig. 41). The force F applied to the string is 10 g; the radius of the axle, 2 cm. The weight moves downward with an acceleration of 20 cm per sec. per sec. Find the moment of inertia of the rotating system. *Solution:* Since F in the equation above is expressed in absolute units, the given force must be reduced to dynes; that is, 10 g of force = 9800 dynes. Also, since $a = \alpha r$, then $\alpha = a/r = \frac{20}{2} = 10$ radians per sec. per sec. $Fd = \alpha I$, then $9800 \times 2 = 10I$. Hence $I = 1960$ g cm^2.

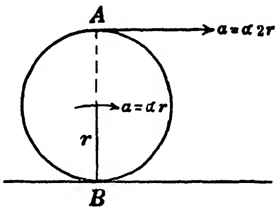

FIG. 42.—Linear and angular accelerations.

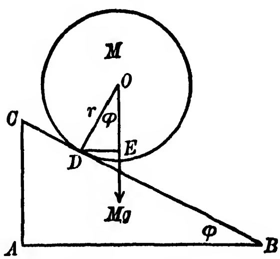

FIG. 43.—Rolling motion on inclined plane.

56. Translation and Rotation.—We wish now to consider the relation which exists between the linear displacement and the angular acceleration of a body having both translatory and rotatory motions. For example, a cylinder, acted upon by a constant force, rolls along a horizontal plane or down an incline (Figs. 42 and 43) with an accelerated motion. Every particle in the circumference, say, has the same instantaneous angular velocity with respect to the center; the instantaneous linear velocities, however, with respect to the plane upon which the disc rolls, vary from zero

at the point of contact B to a maximum at A (Fig. 42). By way of illustration let us assume that a constant force F is applied to a fine string which is wrapped around a cylinder of mass M, causing it to roll with an accelerated motion along a horizontal plane. With respect to the point of contact B, the linear acceleration of the center of the cylinder is $a = \alpha r$, and that of the point A is $a = 2\alpha r$. The moment of inertia about the point B is $I = I_o + Md^2$, where $d = r$, and also $I = Fd/\alpha$. The angular acceleration of the string then is $\alpha = (I_o + Md^2)/Fd$. The instantaneous linear velocity of the center is $v = \omega r$.

In the case of a cylinder rolling down an incline (Fig. 43), the force is Mg and the moment of the force causing rotation is $Fd = Mg \times DE = Mgr \sin \theta$.

Example.—A cylinder of mass 8 lb. and radius 6 in. rolls without sliding or friction down an incline, the direction angle θ of which is 30°. Find (*a*) the moment of inertia about the point of contact with the plane; (*b*) the linear acceleration of the center down the plane; and (*c*) the distance it will travel down the plane in 4 sec. *Solution:* (*a*) $I = I_o + Md^2 = 8 \times \frac{1}{4} \times \frac{1}{2} + 8 \times \frac{1}{4} = 3$ lb. ft.^{2}. (*b*) $Fd = \alpha I$, from which $\alpha = Fd/I = Mgr \sin 30°/I = (8 \times 32 \times \frac{1}{4})/3 = \frac{64}{3}$ radians per sec. per sec. Also, $a = \alpha r = \frac{64}{3} \times \frac{1}{2} = \frac{64}{6} = 10.666$ ft. per sec. per sec. (*c*) $s = \frac{1}{2}at^2 = \frac{1}{2} \times \frac{64}{6} \times 16 = 85.33$ ft.

57. Moment of Inertia and the Period of a Compound Pendulum.—The period (time of one complete vibration) of a compound pendulum is $T = 2\pi\sqrt{l/g}$, where l is the length of an equivalent simple pendulum. To find the period of a compound pendulum it is necessary, first, to find the value of l. This length is $l = I/Mh$ in which I is the moment of inertia of the pendulum about its axis of suspension; M is the mass of the pendulum; and h is the distance from the center of mass to the point of suspension. Our equation then becomes

$$T = 2\pi\sqrt{\frac{I}{Mgh}}.$$

Example.—A small hole is bored through a meter stick 10 cm from one end, and about this point it is suspended so as to vibrate freely. Its mass is 360 g and its length $L = 100$ cm. Find (*a*) the length l of an equivalent simple pendulum, and (*b*) the period of vibration. *Solution:* $l = I/Mh$, where h = distance from center of gravity to point of suspension = 40 cm. We may consider the meter stick as a thin rod in which the width is negligible as compared with the length. $I = I_o + Mh^2 = \frac{ML^2}{12} + Mh^2$, hence $l = \left(\frac{ML^2}{12} + Mh^2\right)/Mh = \frac{L^2}{12h} + h = \frac{10,000}{480} + 40 = 60.8$ cm. Then $T = 2\pi \sqrt{l/g} = 2\pi\sqrt{60.8/32} = 2.76\pi$ sec.

Moments of Inertia

1. Thin circular hoop, axis through center, perpendicular to face, radius R, $I_o = MR^2$.
2. Circular plate or cylinder, axis through center, perpendicular to face, radius R, $I = MR^2/2$.

3. Circular ring, axis through center, perpendicular to face, outer radius R, inner radius R', $I_o = M(R^2 + R'^2)/2$.
4. Sphere, axis through center, radius R, $I_o = MR^2 2/5$.
5. Uniform thin rod, axis through middle, length l, $I_o = Ml^2/12$.
6. Rectangular figure, axis through center, perpendicular to plane, width a, length b, $I_o = M(a^2 + b^2)/12$.
7. Moment of inertia about axis parallel to axis through center of gravity, $I = I_o + Md^2$.
8. Moment of force $Fd = I\alpha = Ia/r$.
9. Period of vibration, $T = 2\pi\sqrt{I/Mgh}$.

Problems

191. Explain each term of the following fundamental equations: (*a*) $v = \omega r$; (*b*) $a = \alpha r$; (*c*) $K.E. = \frac{1}{2}I\omega^2$; (*d*) $Fd = \alpha I$; (*e*) $I = I_o + Md^2$.

192. Write the dimensional formula for moment of inertia.

193. A circular disc, mass 200 lb., radius 2 ft., makes 120 r.p.m. about an axis at right angles to its greatest face and passing through its center. Find (*a*) its angular velocity ω; (*b*) the linear velocity of a point in the circumference; (*c*) the moment of inertia about its center.

194. Find the kinetic energy of the disc (problem 193) in (*a*) absolute units; (*b*) gravitational units.

195. Suppose that the disc of problem 193 is suspended on an axis passing through its face 6 in. from the circumference. Find the moment of inertia about this axis.

196. Given a uniform cylindrical disc having a mass of 20 lb., and a radius of 2 ft. Find its moment of inertia about (*a*) its center; (*b*) a point midway between the center and circumference.

197. Find the kinetic energy of the disc of problem 196, assuming that it makes 5 revolutions every 10 sec.

198. Find the moment of inertia of a meter stick, mass 240 g, about an axis at right angles to its greatest face through (*a*) the middle point; (*b*) one end; (*c*) 20 cm from one end.

199. Given a uniform board 12 ft. in length and 18 in. in width, and having a mass of 24 lb. Find the moment of inertia of the board (*a*) about an axis through its center of gravity; (*b*) about an axis on a median line and 2 ft. from one end.

200. Given a rectangular block, density 10 g per cm^3, length 40 cm, width 20 cm, thickness 10 cm. Find the moment of inertia of this block about its three axes through the center of mass, and at right angles to the three sets of faces.

201. Suppose that a solid metal wheel having a mass of 100 lb. and a radius of 2 ft. rolls along the ground making 15 r.p.m. Find (*a*) the linear velocity of the center of the wheel with respect to the ground; (*b*) the velocity of a particle at its highest point with respect to the ground; (*c*) the velocity of the particle in contact with the ground, with respect to the ground.

202. The wheel of problem 201 possesses kinetic energy due to its linear velocity ($K.E. = \frac{1}{2}Mv^2$) and also due to its rotation ($K.E. = \frac{1}{2}I_o\omega^2$). Find the total kinetic energy due to these two factors.

203. The total kinetic energy of the wheel may be found in another way; that is, by considering its motion with reference to a point in contact with the ground. In this case we use the equation $K.E. = \frac{1}{2}I\omega^2$, in which $I_o = I + Md^2$. Find the kinetic energy of the wheel by this method. How does the result obtained compare with that of problem 202?

204. A given circular disc rolls along a level track with a uniform velocity. At a given instant the linear velocity of a particle at the highest point of the disc is 20 ft. per sec. with respect to the ground. Find (*a*) the velocity of this particle with respect to the center of the disc; (*b*) the linear velocity of the center of the disc; (*c*) the distance the disc will roll in 10 sec.; (*d*) the angular velocity of the disc; (*e*) its period of revolution.

205. Find the moment of inertia of a disc of mass 1 kg, radius 15 cm, rotating about an axis at right angles to the plane of the disc and at a distance of 5 cm from its circumference.

206. Find the kinetic energy of the disc of problem 205 when its period of rotation is π sec.

207. The metal rim of a wheel has a mass of 100 lb. Its moment of inertia is 312.5. lb. ft.^{2}. Its outer radius is 2 ft. Find the inner radius of the rim.

208. A cylindrical projectile of mass 20 lb., radius 6 in., is shot through the air end-on with a linear velocity of 100 ft. per sec. and a rotational velocity equivalent to $20/\pi$ r.p.s. Find its total kinetic energy in foot-pounds.

209. A solid spherical body of radius 4 cm and mass 2 kg has a linear velocity of 20 cm per sec. and an angular velocity of 20 radians per sec. Find its total kinetic energy in joules.

210. A spherical ball having a mass of 200 g, radius 2 cm, has a linear velocity of 10 cm per sec. and an angular velocity of 10

radians per sec. Find its total kinetic energy in (*a*) ergs; (*b*) gram-centimeters.

211. A grindstone weighing (having a mass of) 90 lb., and having a radius of 8 in. is supported upon ball bearings so that it turns without appreciable friction. A string is wrapped several times around the stone and a force of 6 lb. is applied to the string, causing the stone to rotate about its axis. Find the acceleration of the string.

212. If the grindstone (problem 211) is placed at rest upon the floor and a force of 6 lb. is applied to the string (Fig. 42), the stone will roll along the floor. Consider the motion of the disc about the point B, in terms of $Fd = \alpha I$. Find (*a*) the angular acceleration of the string; (*b*) the linear acceleration of the center of the rolling stone.

213. Find (*a*) the linear velocity of the center of the stone, and (*b*) its angular velocity 10 sec. after it starts to roll.

214. A cylindrical disc (Fig. 43), mass 200 lb., radius 18 in., rolls, due to the force of gravity, down an inclined plane CB which makes an angle of 30° with its base AB. Find (*a*) the force in poundals acting on the center of the disc; (*b*) the moment of the force ($Mg \times DE$) tending to rotate the disc about its point of contact D with the incline CB.

215. (*a*) Find the moment of inertia of the disc about its point of contact D with the incline (Fig. 43). (*b*) Find the angular acceleration of the disc as it rolls down the plane.

216. Find (*a*) the linear acceleration of the disc (problem 214) as it rolls down the plane; (*b*) the velocity at the end of 4 sec.; (*c*) the distance it will roll in 4 sec.

217. Let a be the acceleration of a body sliding without friction down an inclined plane, and a' the acceleration of a solid cylinder rolling down the same incline, the motion of both bodies being due to gravity. Suppose that both bodies start from rest and move for the time t. Show (*a*) that $a' = \frac{2}{3}a$; and (*b*) $v' = \frac{2}{3}v$.

218. Suppose that the two bodies of problem 217 move to the bottom of the plane, a distance s. Let v be the velocity of the sliding body, and v' the linear velocity of the rolling body. Show that at the bottom of the plane $v'^2 = \frac{2}{3}v^2$.

219. A sphere and a cylinder, both having radii equal to r, start from rest and roll down the same incline for t sec. Let

s be the space traversed by the sphere in the time t, and s' the space traversed by the cylinder. Show that $s' = {}^{14}\!/_{15}s$.

220. An inclined plane, length 5 m, makes an angle of 30° with the base. A cylinder of mass 2000 g, radius 10 cm, rolls from the top to the bottom of the plane. Find (*a*) the angular acceleration of the cylinder; (*b*) the linear acceleration; (*c*) the time required to reach the bottom.

221. Find the total K. E. of the cylinder (problem 220) at the instant it reaches the bottom of the plane.

222. A metal disc is suspended so as to serve as a torsional pendulum (Fig. 21). A metal ring of mass 1000 g, outer radius 12 cm, inner radius 8 cm, is placed symmetrically upon the disc with respect to its center. How much will the moment of inertia of the system be increased by the addition of the ring?

223. A circular wire hoop 2 ft. in diameter is suspended from a nail so as to vibrate freely in its own plane. (*a*) Find the value of l. (*b*) How does l in this case compare in length with the diameter of the hoop? (*c*) What is its period of vibration?

224. A thin metal disc 2 ft. in diameter is suspended so as to vibrate freely in its own plane about a stiff wire axis which passes through the disc practically at its circumference. (*a*) Find l for the disc pendulum. (*b*) How does l compare in length with the diameter of the disc? (*c*) What is the period of vibration of the disc?

225. A thin rod of length L is suspended so as to vibrate freely about an axis through one end. Show that $l = {}^2\!/_3L$.

226. Given a rectangular piece of board, length AB 30 in., width BC 10 in. Find its period of vibration when suspended from a point in one end of the board midway between B and C.

CHAPTER IV

MECHANICS OF FLUIDS

FLUIDS AT REST

58. Properties of Fluids.—A fluid may be either a liquid or a gas. A liquid conforms to the shape of the containing vessel and always has a definite surface; a gas, on the other hand, has no definite surface and always tends to fill completely the containing vessel. Liquids are practically incompressible; gases are highly compressible.

The general properties of fluids at rest are: (*a*) the pressure which a fluid exerts against the walls of a container is always normal to the surface; (*b*) pressure at a given point is equal in all directions; (*c*) fluids possess perfect elasticity.

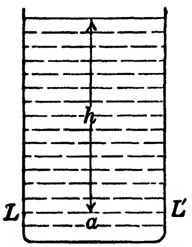

FIG. 44.—Pressure on a, h, d.

59. Pressure in Liquids.—*Pressure* is force per unit area; that is, $P = F/A$. Pressure on a unit of area a in the line LL' (Fig. 44) is directly proportional to the depth and density of the liquid. Thus, for unit area, we may write

$$P = hd,$$

where h is the vertical distance from the surface plane of the liquid to the center of area of the unit area of surface pressed upon, and d is the density of the liquid, that is, the mass per unit volume.

In general, pressure is measured in grams or dynes per square centimeter, or in pounds or poundals per square inch or square foot.

60. Force and Pressure on Plane Surfaces.—The force exerted upon any immersed surface, due to the pressure of a liquid, is

$$F = PA;$$

that is,

$$F = AHD,$$

gravitational units, or

$$F = AHDg,$$

absolute units.

In the above equation, A is the area pressed upon; H is the vertical distance from the surface plane of the fluid to the center of area (centroid) of the surface pressed upon; D is the density of the fluid in grams or pounds per unit of volume; and g is the acceleration of gravity.

The *center of area* is the same as the center of mass or center of gravity of a body of surface thickness only. The following table gives the centers of area of some common surface areas:

Figure	Center of Area
Parallelogram....	intersection of diagonals
Triangle......	intersection of median lines
Circle.....	geometrical center of figure
Spherical shell	center of sphere
Hemispherical bowl	$\frac{1}{2}$ radius, normal to plane surface
Right cone (hollow)	$\frac{2}{3}$ distance from vertex to base

The student should bear in mind that the data given above refer to *surface areas.* For example, while the center of area of a hollow hemisphere is $\frac{1}{2}r$, the center of gravity of a homogeneous solid hemisphere is $\frac{3}{8}r$, measured from the plane surface; and likewise, while the center of area of a right hollow cone is $\frac{2}{3}h$ from the vertex, the center of gravity of a homogeneous solid right cone is $\frac{3}{4}h$ from the vertex. It is also important to note that the equation $F = AHD$ applies to plane surfaces, that is, to parallelogram, triangle, and circle areas, as given in the above table. A discussion of force and pressure as applying to curved surfaces will follow.

61. Discussion of the Equation $F = AHD$.—This equation is perfectly general in its application to plane surfaces, no matter in what direction they may lie, that is, horizontal, vertical, or slantwise. It tells us that the normal force exerted upon any submerged plane surface depends on three factors only, namely, A the area, H the vertical distance from the surface line to the center of area, and D the density. Let us, as an example, take the following case. Suppose that a thin rectangular piece of metal having a given area and a plane surface is placed successively in three different positions, I, II, and III (Fig. 45), the center of area in each case being kept in the line LL'. Imagine that we are looking at the rectangle side on. Now since the factors A, H, and D are the same for all three positions, it follows that the force is the same in all three cases.

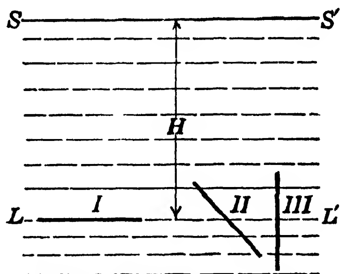

FIG. 45.—Pressure = HD.

The student should make careful note of the fact that by force we mean a force *normal* (that is, at right angles) to the surface pressed upon. For example, suppose that the F is the force acting upon the plate in the three positions shown in Fig. 45. In I, the direction of F is vertical; in II, oblique; and in III, horizontal. In case II, the force acting vertically downward is $F \cos \theta$, where θ is the angle between the oblique line and the horizontal line LL'; the horizontal component is $F \sin \theta$.

Example.—As an example illustrating the forces exerted on the bottom, sides, and end consider the following: In Fig. 46 we have an end view of a vessel in the shape of a right-angled prism filled with water. We wish to find the force exerted on (*a*) the bottom, (*b*) the vertical side BC, (*c*) the sloping side AC, and (*d*) the triangular end ABC. The base AB = 8 ft., the vertical height BC = 6 ft., the sloping side AC = 10 ft., and the length of the prism, that is, the distance away from the observer = 10 ft. *Solu-*

tion: (*a*) In this case, the area A of the bottom $8 \times 10 = 80$ sq. ft., and H is the depth of the liquid $= 6$ ft. Then $F = AHD = 80 \times 6 \times 62.4 = 29{,}952$ lb. (*b*) $A = 6 \times 10 = 60$ sq. ft., and H = half the depth of the liquid $= 3$ ft. Then $F = AHD = 60 \times 3 \times 62.4 = 11{,}232$ lb. (*c*) The *horizontal* force on side AC is the same as that on BC. The *normal* force on AC, however, is $F = AHD = 10 \times 10 \times 3 \times 62.4 = 18{,}720$ lb. The horizontal component of this normal force $= 18{,}720 \times \sin\theta = 18{,}720 \times 0.6 = 11{,}232$ lb. (*d*) The area A of the triangular end $ABC = \frac{1}{2} \times 8 \times 6 = 24$ sq. ft. The center of area of a triangle is at the point of intersection of its median lines, which point is on the median line CC', two-thirds of the distance from the vertex C. The vertical distance from the surface line SC to the center of area of the triangle, therefore, is $H = \frac{2}{3} \times 6 = 4$ ft. Then $F = AHD = 24 \times 4 \times 62.4 = 5990$ lb.

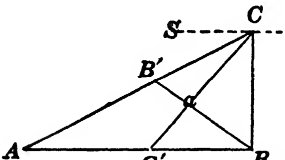

FIG. 46.

62. Force on Curved Surfaces.—All the cases considered thus far have involved the finding the resultant F of a number of parallel forces, in which the vector or geometric and arithmetic sums are identical. Now the arithmetical sum of all the forces on the elements of a surface due to fluid pressure is sometimes called the *whole pressure*, a term frequently occurring in the literature of hydrostatics. When the surface under consideration is plane, the *whole pressure* and the *resultant force* are the same. It should be noted, however, that when the surface is not plane, the whole pressure and the resultant force are not necessarily alike.

By way of illustration, let Fig. 47 represent the cross-section of a cylinder or sphere. The center of area is at the center of the circle, and consequently $H = r$. Now all the pairs of elementary forces acting normal to the curved surface are equal in magnitude and oppositely directed, and therefore the resultant for each pair is zero. For example, the whole pressure on the circle, according to the formula $F = AHD = 2\pi r \times r \times D = 2\pi r^2 D$. But the resultant of all the elements of force around the entire circle is zero, and therefore the term "whole pressure" has, in a case of this sort, no physical meaning. This does not imply, however, that vessels having curved surfaces, such as pipes, cylinders, and spheres, when filled with a liquid, are not subjected to tangential forces, called tensile stresses, which tend to burst or tear them apart. The question of tensile stresses in pipes and cylinders is an important one in hydraulic engineering practice.

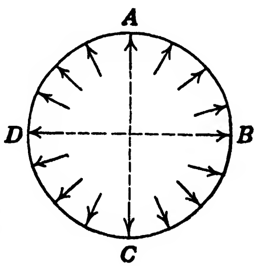

FIG. 47.

63. Uniform Pressure on Cylindrical Surfaces.—Figure 47 may represent the cross-section of a cylinder standing in a vertical position and subjected to a uniform internal hydrostatic pressure. Suppose, for example, that the cylinder in question has a diameter of 2 ft. and a vertical height of 10 ft. It is filled with water. We wish to find what force the water exerts

in tending to force the two vertical halves of the cylinder apart along the line AC (Fig. 47). Any other diameter might be chosen. The force in question acts at right angles to the line AC and, let us say, to the right. In this case the effective area is that of a rectangle having an area A equal to $2 \times 10 = 20$ sq. ft. The value of H is equal to half the depth of the water, and therefore $H = 5$ ft. Then $F = AHD = 20 \times 5 \times 62.4 = 6240$ lb. Now since this force is supported by the two edges, A and C, of the half cylinder, the force on one edge, at A say, is $\frac{1}{2} \times 6240 = 3120$ lb. This force per edge per linear inch is what engineers call the *tensile stress;* in other words, it is the force per unit of length that a pipe or cylinder must be capable of exerting in order to overcome the bursting force of the liquid.

64. Uniform Pressure on Spherical Surfaces.—Suppose that we have two hemispherical shells placed face to face, thus forming a hollow sphere. Assume that the sphere is filled with water. Then the hydrostatic force tending to force the hemispherical shells apart will be equal to the hydrostatic force acting over area A equal to that formed by a great circle of the sphere, that is, $A = \pi r^2$. Let the radius of the sphere be 2 ft. The center of area of a sphere is at the center, and hence $H = r$. Then $F = AHD = \pi 4 \times 2 \times 62.4 = 499.2\pi$ lb.

Problems

227. A rectangular vessel 20 cm long, 10 cm wide, and 10 cm deep is filled with water. Find the force due to the weight of the water on (*a*) the bottom; (*b*) one side; (*c*) one end.

228. The vessel of problem 227 is filled with mercury. Find the force exerted on (*a*) the bottom; (*b*) one side; (*c*) one end.

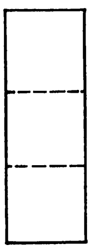

FIG. 48.

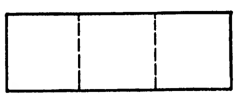

FIG. 49.

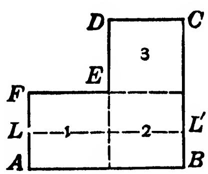

FIG. 50.

229. A tank 20 ft. long, 10 ft. wide, and 10 ft. deep is filled with water. Find (*a*) the force due to the weight of the water on (*a*) the bottom; (*b*) one side; (*c*) one end.

230. Find tion total force exerted on the side, end, and bottom of the vesir radjoblem 227) when it is filled with brine, having a specifi vertical of 1.4.

231. aced on el shown in Fig. 48 has a cross-sectional area of $10 \times$ ssel; (*b*) tl a length of 30 cm. It is filled with water,

closed and placed on end. (*a*) Find the force on the bottom due to the weight of the liquid; (*b*) the force on one side.

232. The vessel (problem 231) is placed in a horizontal position (Fig. 49). Find the force on (*a*) the bottom; (*b*) side; (*c*) end.

233. In Fig. 50 there is shown a vessel having the same volume as that of Figs. 48 and 49. Cross-sectional areas of end *AF* and *CD* are each 10 × 10 cm. Length of base *AB* is 20 cm; height *BC*, 20 cm. The line to *LL′* has reference to a later problem. Find the force on (*a*) the bottom; (*b*) the end *BC*; (*c*) the end *AF*.

234. What is the force in the face *ABCDEF* (problem 233)?

235. Three vessels I, II, III, (Fig. 51) having bottoms of the same area (20 cm on each edge) and sides of the same height (20 cm) are filled with water. The sides of each box are vertical. The ends of I are vertical. One end of II is vertical, and the other end slopes outward making an angle of 30° with the vertical. One end of III is vertical, and the other end slopes inward making an angle of 30° with the vertical. (*a*) Find the weight of water in each vessel. (*b*) Find the force exerted on the bottom of each vessel by the water. (*c*) How does the weight of water in each compare with the force exerted on the bottom? (*d*) Under what conditions is the force on the bottom of a vessel equal to the weight of the liquid?

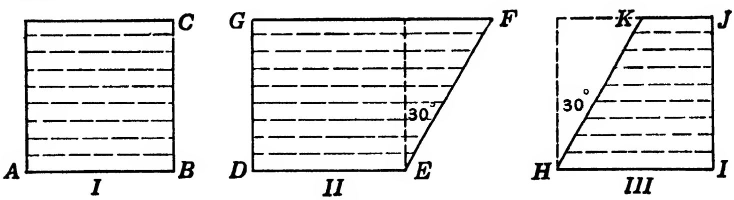

Fig. 51.

236. (*a*) Find the force on the end *BC*, *DG*, and *IJ*, respectively. (*b*) What is the normal force on *EF* of vessel II? (*c*) What is the horizontal thrust on this end? (*c*) How does the horizontal thrust on the sloping end *EF* compare with the force on *DG*?

237. Find the force on the face (side tov the observer) of vessel II.

238. A thin piece of metal, 20 by 30 c ravity position placed at the bottom of a rectangular tank, 1 m in de vess, for ex manner that the 20-cm edge of the metal rests um and vertical heig the the water exe

tank, and the 30-cm edge is vertical. Find the normal force exerted on one side of the metal.

239. Suppose that the 30-cm edge of the metal (problem 238) makes an angle of 30° with the bottom of the tank. Find the normal force exerted on the metal.

240. Suppose that the piece of metal (problem 238) is placed at the bottom of the tank in such a manner that one corner touches the bottom and the diagonal is vertical. Find the force exerted on one side.

241. What are the vertical and horizontal components of the force acting on the metal sheet of problem 239?

242. A vessel having the shape of a right triangular prism, 1 m in height, length, and width, is placed in such a position that one rectangular face is horizontal and one vertical and is filled with water. Find the normal force in grams exerted on each face of the vessel.

243. The radius of the base of a cylindrical vessel is 10 cm, and the height 30 cm. The vessel is filled with water. Find (*a*) the weight of the water in the vessel; (*b*) the force exerted on the bottom; (*c*) the force tending to push the vertical halves of the cylinder apart.

244. A cylindrical tank has a radius of 10 ft. and a height of 30 ft. The vessel is filled with water. Find (*a*) the weight of the water in the tank; (*b*) the force exerted on the bottom; (*c*) the horizontal bursting force.

245. (*a*) What is the bursting force per linear inch along a vertical seam in the tank? In engineering practice the tensile stress (bursting force per linear inch) is given by the formula $\frac{1.3}{6} \times h \times d$, where h is the hydrostatic head (distance from the surface to the center of area), and d is the diameter in inches. (*b*) How does your result obtained in (*a*) compare with the result obtained by the use of this formula?

246. A spherical vessel, 2 ft. in diameter, is filled with water. Let Fig. 47 represent a great circle of the sphere in question. What is the horizontal force tending to push apart the two hemispherical sections that are shown in outline by *ABC* and *CDA*?

247. The radius of the base of a right conical vessel is 10 cm, and the vertical height 30 cm. The vessel is filled with water and placed on its base. Find (*a*) the weight of the water in the vessel; (*b*) the force exerted on the bottom.

248. A steel railroad water tank consists of a cylinder of radius 10 ft., height 30 ft., and a hemispherical base. Find (*a*) the weight of water in the tank when it is full; (*b*) the force acting downward on the hemispherical bottom of the tank.

65. Center of Pressure.—Thus far in our study of fluid pressure we have been concerned in calculating the *magnitude* of the resultant force, $F = AHD$. We have now to consider the point in the given area at which this resultant force acts, a point called the *center of pressure.* The center of hydrostatic pressure on any immersed surface is the point of application of the resultant of all the elementary hydrostatic pressures against the elements of the surface. If the area pressed upon is a plane surface all the elementary pressures are parallel, and the problem consists in finding the resultant of a system of parallel forces.

Center of pressure may be defined by means of the equation $X = \Sigma fx/\Sigma f$ in which X is the distance from the surface of the liquid, considered as a plane of reference, to the center of pressure of the area acted upon; f is the force acting on an element of area; fx is the moment due to this force.

66. Center of Pressure of a Parallelogram.—The center of pressure of a rectangle having one edge in the surface of the liquid is on a median line at a distance equal to two-thirds of the depth of the lowest edge below the surface. This holds whether the rectangle is vertical or inclined.

67. Center of Pressure of a Triangle.—(*a*) The center of pressure of a triangle whose vertex lies in the surface and whose base is horizontal is on a median line at a distance equal to three-fourths of the depth of the base from the surface. (*b*) The center of pressure of a triangle whose base lies in the surface is on a median line at a distance equal to one-half the depth of the vertex from the surface.

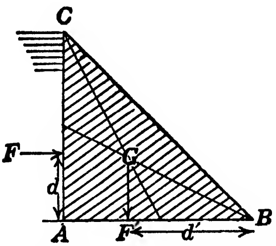

FIG. 52.—Center of pressure and center of gravity.

Example.—A dam, whose cross-section is a right-angled triangle ABC (Fig. 52) 3 m high, is built of a material whose density is 4 g per cm³. If the water reaches to the top on the vertical side, what must be the breadth of the base in order that the moment of the force due to gravity about the point B is just equal to the moment of the force due to the water about the point A? *Solution:* Consider unit width of the dam. (*a*) The area A pressed upon is 300 cm². $F = AHD = 300 \times 150 \times 1 = 45{,}000$ g of force. The point of application of this force, the center of pressure, $= \frac{2}{3}AC = 200$ cm from surface, that is, 100 cm from A. The moment of the force about A is, then, $Fd = 45{,}000 \times 100 = 4{,}500{,}000$. (*b*) We must now find the moment of the force about the point B, due to the weight of the dam. Let F' be the weight of the section of the dam under consideration; and let X be the width of the base AB. Then $F' = (300 \times X \times 4)/2 = 600X$ g of force. Since the center of gravity G of the section is at the point of intersection of the medians of the triangle ABC, the line of direction of the force F' cuts the base AB at a point equal to $\frac{2}{3}X$, measured from $B = d'$. The moment of

the force due to the weight of the dam is, therefore, $F'd' = 600 \times \frac{2}{3}X^2 = 400X^2$. According to the conditions of the problem, the two moments Fd and $F'd'$ are equal in magnitude and opposite in sense, therefore $4{,}500{,}000 = 600X \times \frac{2}{3}X = 400X^2$, and hence $X = 106$ cm.

Problems

249. A right triangular prism, 9 ft. on the base, 6 ft. in height, 10 ft. in length, supports a column of water 6 ft. deep against its vertical face. The density of the prism is 500 lb. per cu. ft. Find (*a*) the moment of the force due to the water tending to overturn the prism; (*b*) the moment of the force due to the weight of the prism tending to hold it in place.

250. Assume that the water presses against the sloping face of the prism (problem 249). Find (*a*) the weight of the water acting vertically downward upon the sloping face; (*b*) the force exerted by the water normal to the sloping face; (*c*) the moment of the force with reference to the point B (Fig. 52) acting horizontal to the base; (*d*) the moment of the force with reference to B, acting at right angles to the base.

251. A cubical box 4 ft. on each edge is filled with water. One face of the box is hinged at the lower edge, and is held vertically in place by a clasp at the middle point at the top. (*a*) What is the moment of the force tending to turn the face outward about the lower edge? (*b*) What is the force exerted on the clasp?

252. The head gate of a flume is 9 ft. in height and 10 ft. in width. It supports, in a vertical position, a column of water 9 ft. in depth. (*a*) Where must a single support be placed so that the head gate will be retained in position? (*b*) What is the moment of the force acting upon the support, tending to turn the head gate about its lowest point?

253. A vessel in the form of a rectangular pyramid 12 ft. in height rests on its base, the area of which is 4×4 ft. The vessel is filled with water. Find the moment of the force tending to turn a given face outward.

68. Pascal's Law.—Pascal's law states that pressure applied to any given area of an enclosed fluid is transmitted undiminished to every like area of the containing vessel. The work done on the small piston (Fig. 53) is equal to that done by the large piston, that is, $W = Fs = F's'$, in which s and s' are the spaces through which the respective pistons move. Then $F/F' = s'/s$. We may therefore say: (*a*) The forces exerted by the pistons are directly proportional to their cross-sectional areas, and (*b*) the spaces through which the pistons move are inversely proportional to their cross-

sectional areas. The practical application of this principle is exemplified in the use of the hydraulic press, the hydraulic elevator, and apparatus of similar design.

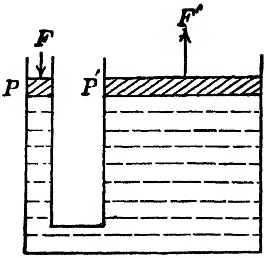

FIG. 53.—Hydraulic press.

Example.—The area A of the small piston of a hydraulic press is 4 sq. in.; the radius of the large piston is 1 ft. A force of 10 lb. is applied to the small piston. Find the upward force exerted by the large piston. *Solution:* Let A be the area of the small piston; A' the area of the large piston. Then $A = 4$ sq. in.; and $A' = \pi r^2 = 144\pi$ sq. in. Let F be the force exerted on the small piston, and F' that on the large piston. The forces on the two pistons are proportional to the areas; that is $F:F' = A:A'$. Hence $F' = FA'/A = 360\pi$ lb.

Problems

254. A rectangular vessel 10 × 10 × 10 cm is filled with water and covered on top. Through an opening in the top there is inserted a piston-like plug of cross-sectional area 1 cm², to which there is applied a force of 10 g. What is the force on (*a*) the bottom, (*b*) side, (*c*) top due to the water and plug pressures?

255. A rectangular tank 10 ft. on each edge is filled with water and tightly covered. A pressure of 10 lb. per sq. in. is applied to the interior of the tank by means of a plug. What is the total force due to the weight of the water and its pressure of the plug combined on (*a*) the bottom; (*b*) side; (*c*) top?

256. Three tanks, each 4 × 4 ft. at the base, stand in vertical positions and are filled with water. Tank I is uniform in cross-sectional area and is 12 ft. in height. Number II is a tight cubical box, 4 × 4 × 4 ft., into the top of which there is fitted an 8-ft. tube having an inside cross-sectional area of 1 sq. ft. Number III is like II, except that the cross-sectional area of the tube is 1 sq. in. Numbers II and III are filled with water to the same depth as I, that is, 12 ft. (*a*) Find the weight of water in vessels I and II. (*b*) How does the force exerted on the bottom of I compare with that of II?

257. Find the horizontal force exerted on the sides of II and III, respectively.

258. What is the upward force in the top of (*a*) tank II; (*b*) tank III?

259. A cylindrical tank 10 ft. in diameter and 10 ft. in height has inserted into the top a vertical tube 20 ft. in length. Both

tank and tube are filled with water. What is (*a*) the force exerted on the bottom of the tank; (*b*) the pressure in pounds per square inch?

260. Compare the force exerted on the bottom of a cylindrical tank filled with water, cross-sectional area 78.54 sq. ft., height 20 ft., with that exerted on the bottom of another cylindrical tank, of radius 5 ft. and height 5 ft., into the top of which is fixed a pipe of cross-sectional area of 1 sq. in., height 15 ft., also filled with water.

261. The radii of the pistons of a hydraulic press are 1 in. and 1 ft., respectively. A force of 100 lb. is applied to the smaller piston. (*a*) Find the force exerted by the larger piston. (*b*) When the larger piston moves 1 in., through what space does the smaller piston move?

262. A weight of 1.8 tons is placed upon the large piston P' (Fig. 53). What weight will have to be placed on P in order to keep the system in equilibrium, the diameter of P' being 2 ft. and the cross-sectional area of P' 4 sq. in.?

263. Suppose that the small piston P (Fig. 53) is free to move in a rather long vertical cylinder. The diameter of P' is 2 ft., and that of P 2 in. When the large piston P' has a velocity downward of 2 ft. per sec., what is the upward velocity of the small piston?

264. A closed cylindrical steel laboratory tank, 2 ft. in diameter and resting upon one end is filled with water. If the tank were connected directly with the city water mains, what would be the bursting force on the top, assuming that the water pressure is 40 lb. per sq. in.?

265. In manufacturing plants, hydraulic presses are frequently operated by connecting them directly with the city water system, which exerts, let us say, a pressure of 60 lb. per sq. in. What must be the diameter of the large piston in order that a force of 12π tons may be exerted?

BUOYANCY OF FLUIDS

69. Archimedes' Principle.—Archimedes' principle, sometimes called the principle of buoyancy, states that a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced. A body that sinks in a fluid displaces its own volume; a body that floats displaces its own weight. In both cases the body is buoyed up by a force equal to the weight of the fluid displaced.

70. Stability of Floating Bodies.—Let Fig. 54 represent the cross-section of a boat. The point G is the center of gravity of the boat, and B is the center of buoyancy of the water displaced. The *center of buoyancy* of a floating body is the center of volume (centroid) of the space occupied by the liquid displaced. The *metacenter* M is the point of intersection of a vertical line through B with the middle line ab (Fig. 54). At the point G, we may consider that there is a force equal to the weight of the floating body acting *downward;* at B, a force equal to the weight of the water displaced acting *upward.* So long as the metacenter M lies above the center of gravity G, the action of the two forces is to *right the boat.* When, however, M lies below G, the action of the two forces is to *tip the boat over* (Fig. 55).

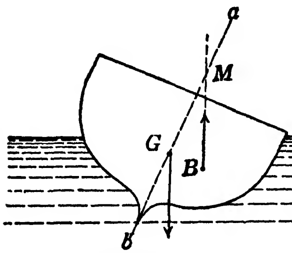

FIG. 54.—Center of gravity below metacenter. Forces G and B tend to right boat.

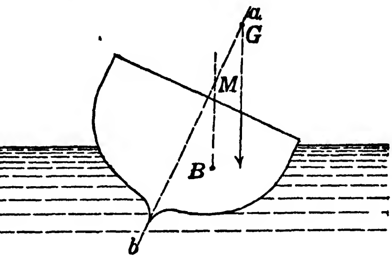

FIG. 55.—Center of gravity above metacenter. Forces G and B tend to overturn boat.

Problems

266. A piece of metal having a mass of 100 g, and a volume of 10 cm³ will displace (*a*) what volume of water; (*b*) how many grams of water? (*c*) It will be buoyed up by what force; (*d*) will have what weight in water?

267. A cubic foot of lead of mass 700 lb. is immersed in water. (*a*) What volume of water is displaced? (*b*) What weight of water is displaced? (*c*) What buoyant force acts upon it? (*d*) What is its weight in water?

268. A cubic foot of a certain substance weighing 60 lb. is thrown into water. (*a*) Will it sink or float? (*b*) How many pounds of water will it displace? (*c*) What is the buoyant force acting upon it?

269. A cubical block 1 ft. on each edge is immersed in water. Find the buoyant force exerted by the water upon the block (*a*) when the block is just submerged, that is, when its upper face lies in the surface of the water; (*b*) when the upper face is 1 ft. below the surface of the water; (*c*) 10 ft. below the surface.

270. A cubical block of wood, mass 250 lb., 2 ft. on each edge, is submerged in a tank of water, so that the upper face of the

block lies in and parallel with the surface. The tank is 10 ft. deep. How much work in foot-pounds will be required to force the block to the bottom?

271. Suppose that the block of problem 270 were allowed to float on the surface. What part would be submerged?

272. A balloon having a capacity of 60,000 cu. ft. is filled with hydrogen. (*a*) Find the buoyant force acting on the balloon due to the air displaced. (*b*) Assuming that the weight of the balloon when collapsed is W lb., find the net buoyant force.

273. The British dirigible R-34, which was the first lighter-than-air ship to cross the Atlantic, July 6, 1919, had a capacity of 2,000,000 cu ft. This balloon was filled with hydrogen. What loss in net buoyancy would have occurred if it had been filled with helium?

274. The capacity of the Graf Zeppelin, with which Count Eckener of Germany circumnavigated the globe, was 3,708,043 cu. ft. The total lift of this balloon was 20 tons. What was its gross weight?

71. Density and Specific Gravity.—*Density* D is mass per unit volume; that is, $D = M/V$. *Specific gravity* is the ratio of the density of a given body to that of another substance taken as a standard. Specific gravity may also be defined as the weight of a given volume of a substance divided by the weight of an equal volume of the standard. For solids and liquids, distilled water at 4°C. is the standard; for gases, air or hydrogen is chosen as the standard.

Example 1.—The mass of a cubic centimeter of a given metal is 8 g. Find (*a*) the density; (*b*) the specific gravity. *Solution:* (*a*) $D = M/V = 8/1 = 8$ g per cm³. (*b*) Sp. gr. = weight of body/weight of equal volume of water. One cubic centimeter of water weighs 1 g. Hence sp. gr. = $8/1 = 8$.

NOTE.—In metric units, density and specific gravity are numerically equal to each other.

Example 2.—A cubic foot of a given metal has a mass of 500 lb. Find (*a*) its density; (*b*) specific gravity. *Solution:* (*a*) $D = M/V = 500/1 = 500$ lb per cu. ft. (*b*) Sp. gr. = weight of body/weight of equal volume of water = 500/62.5 = 8.

NOTE.—In English units, density is not numerically equal to specific gravity. Densities, however, are now almost universally expressed in terms of metric units, and as such are numerically equal to specific gravities.

72. Methods of Determining Densities.—Some of the more important methods of finding densities are considered briefly in the following paragraphs:

a. Solids Having Regular Outlines.—Determine mass and volume by direct measurement; find the density by the use of the equation $D = M/V$. See problem 275.

b. Solids of Irregular Outline.—Since densities are now almost universally expressed in terms of metric units (grams per cubic centimeter), and also since densities expressed in metric units are numerically equal to specific gravities, we may write D = *sp. gr.* = *weight of body/weight of equal volume of standard.* And since a body immersed in water is buoyed up by the weight of the fluid displaced, that is, by the loss of weight in water, we may also write D = *weight of body/loss of weight in water.* See problem 276.

c. Solids Lighter than Water.—In determining the density of a solid of irregular outline, insoluble in but lighter than water, we attach to the body a sinker sufficiently heavy to sink it, and then proceed according to the equation D = *sp. gr.* = *weight of body/loss of weight in water.* See problem 277.

d. Density of Solids Soluble in Water.—In this case we must determine the specific gravity of the solid in some substance in which it is not soluble, and then compute its density in terms of water. See problem 278.

e. Density of Liquids.—The usual method of determining density in the laboratory is that known as the "specific gravity bottle" method. See problem 279.

A second method is to weigh a sinker in water and then in the given liquid. D = *sp. gr.* = *loss of weight in the given liquid/loss of weight in water.* See problem 280.

A practical method of determining densities of acids, syrups, milk, and other liquids used in large quantities is by use of the hydrometer, the fundamental principle in the employment of which is illustrated in problem 281.

For density values, see Table VI, Appendix.

Problems

275. A rectangular block, 20 cm in length, 10 cm in width, and 5 cm in height has a mass of 5 kg. Find its density.

Ans. 5 g/cm^3.

276. Find the density and the specific gravity of a body which weighs 10 lb. in air and 8 lb. in water.

Ans. Specific gravity, 5; density, 5 g/cm^3.

277. A sinker which weighs 33 g in air and 30 g in water is attached to a piece of wood which weighs 10 g in air. The wood and sinker weigh 20 g in water. Find the density of the wood.

Ans. 0.5 g/cm^3.

278. A piece of rock candy weighs 20 g in air and 10 g in kerosene, in which it is insoluble. The density of the kerosene is 0.8 g per cm^3. Find (*a*) the specific gravity of the candy relative to the kerosene, and (*b*) its density with reference to water.

Ans. (*a*) 2; (*b*) 1.6 g/cm^3.

279. Find the density of glycerine from the following data: Weight of bottle, 15 g; weight of bottle filled with water, 65 g; weight of bottle filled with glycerine, 78 g.

Ans. 1.26 g/cm^3.

280. Density of a liquid by the sinker method. A sinker weighs 20 g in air, 18 g in water, and 18.36 g in alcohol. Find the density of the alcohol.

Ans. 0.82 g/cm^3.

281. A given hydrometer consisting of a cylindrical glass rod, closed at both ends, is weighted at one end with mercury, so as to float upright in water. The length of the rod is 60 cm; its cross-sectional area is 1 cm^2. When placed in pure water 54 cm of the rod were submerged. When placed in sulphuric acid 30 cm were submerged. (*a*) What volume of water was displaced by the hydrometer? (*b*) How many grams of water were displaced? (*c*) What was the mass of the hydrometer? (*d*) How many grams of sulphuric acid were displaced? (*e*) What was the density of the acid?

Ans. (*e*) 1.8 g/cm^3.

282. A given piece of metal weighs 10 g in air and 8 g in water. Find (*a*) the volume of the metal; (*b*) its density; (*c*) specific gravity.

283. Substitute pounds for grams in problem 282 and find (*a*) the volume of the metal in cubic feet; (*b*) its density in pounds per cubic foot; (*c*) its specific gravity; (*d*) its density in grams per cm^3.

284. A cubic foot of a given metal weighs 600 lb. in air. (*a*) What is its weight in water? (*b*) What is its specific gravity; (*c*) density?

285. A cubical block 10 cm on each edge has a density of 8 g per cm^3. Find (*a*) its weight in air; (*b*) in water; (*c*) in alcohol; (*d*) mercury.

286. A piece of wax having a mass of 10 g, is attached to a sinker which weighs 20 g in air and 18 g in water. The wax and sinker combined weigh 8 g in water. Find the density of the wax.

287. A piece of wax, density 0.8 g per cm^3, is attached to a cubical sinker 2 cm on each edge, specific gravity 8. The wax and sinker combined weigh 36 g in water. Find the volume of the wax.

288. A piece of metal, density 10 g per cm^3 contains within its interior a certain cavity. The metal weighs 800 g in air, and 700 g in water. Find the volume of the cavity.

289. A piece of metal, mass 100 g, density 10 g per cm^3 is suspended by means of a thread in a beaker containing 500 g of water. Find (*a*) the force in grams on the string due to the weight of the metal and to the buoyancy of the water combined; (*b*) the force on the bottom of the beaker due to the metal and the water.

290. A cylindrical wooden rod 1 m in length, density 0.8 g per cm^3 has fastened to one end a cylinder of brass of the same size as the wood. The density of the brass is 8 g per cm^3. The rod is placed in water and allowed to float in a vertical position. Find the length of the brass cylinder such that 10 cm of the wooden rod will float above the surface.

291. A uniform wooden rod is weighted at one end so that when placed in water it floats in a vertical position. The weight of the rod is 100 g. In water it sinks to a given mark. An additional weight of 20 g has to be added to the rod in order to cause it to sink to the same mark in a given liquid. Find the density of the given liquid.

292. The density of aluminum is 2.6 g per cm^3; the density of silver, 10.6 g per cm^3. Compare the weight of a cylinder of aluminum, diameter 1 cm, height 1 cm, with that of a sphere of silver of diameter 1 cm.

293. An irregular piece of metal, having a specific gravity of 8, has a cavity within it. The weight of the metal in air is 1000 g. When immersed in water it displaces 150 cc. Find the volume of the cavity.

294. A bottle weighs 2 oz. Filled with water, it weighs 6 oz. Filled with oil, 5.5 oz. Find the density of the oil, and state the denomination of the result.

295. A glass stopper weighs 40 g in air, 24 g in water, and 12 g in sulphuric acid. Find (*a*) the density of the glass; (*b*) the specific gravity of the acid.

296. An iron casting weighs 160 lb. in air. There are reasons to suspect that there are blowholes in it. It is, therefore, weighed in water and found to weigh 138.576 lb. Find the volume of the blowholes, assuming that a cubic foot of the iron weighs 480 lb.

297. A balloon when collapsed weighs 100 kg; when filled with hydrogen of density 0.89 g per l, it will lift an additional weight of 90 kg. The density of air is 1.293 g per l. Find the volume of the balloon when filled.

73. Boyle's Law.—Liquids are practically incompressible; gases, on the other hand, are very compressible. When a gas is compressed, its volume decreases and its density and pressure increase. The relation of volume to pressure is expressed by Boyle's law which states that for constant temperature the product of pressure and volume equals a constant; that is,

$$pv = c = p'v',$$

where p and p' = the pressures exerted upon the gas; v and v' = the corresponding volumes; and c = a constant. For example, if the volume of a given mass of gas, under a pressure equivalent to 76 cm, be 100 cm^3, then $pv = c = 7600$. If now the pressure be reduced to 38 cm, the volume will become 200, and as before $p'v' = c = 7600$.

It should be noted that the equation $pv = c$, for constant temperature is the equation for a rectangular hyperbola, and the curve (Fig. 56) represents an *isothermal line*, the characteristics of which are that $pv = p'v' =$ *constant*, which means that the area pv (Fig. 56) is equal to the area $p'v'$.

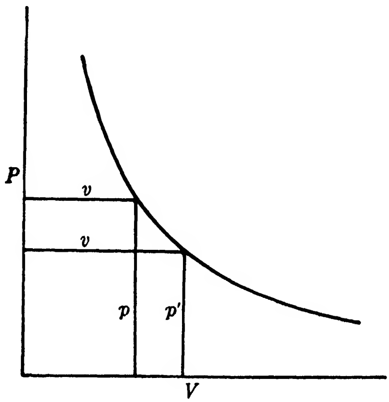

FIG. 56.

Example.—If for constant temperature conditions a given mass of gas, under a pressure of 80 cm, is 300 cm^3, what will be the volume under a pressure of 60 cm? *Solution:* Since $pv = p'v'$, $80 \times 300 = 60 \times v'$, therefore $v' = 400$ cm^3.

The application of Boyle's law is very useful in the solution of physical problems, within certain limitations.

74. Van der Waals' Law.—A perfect gas may be defined as one which obeys Boyle's law exactly. It is assumed, however, that no such gas exists in nature, since it has been observed that gases deviate more or less from Boyle's law, especially when the compression is relatively great. Van der Waals proposed a modification of Boyle's equation as follows

$$\left(p + \frac{a}{v^2}\right)(v - b) = \textit{constant}.$$

The quantity a/v^2 is introduced to make allowance for the effect of mutual attraction between the molecules and $v - b$ to make a corresponding allowance for the effect of the size of the molecules. The factors a and b depend upon the amount and nature of the gas.

Problems

298. A gas having a volume of 25 cm^3 is under a pressure of 1 atmosphere. What will be its volume when the pressure is decreased to 72 cm?

299. A cylinder 20 in. in length and closed at one end is filled with air under a pressure of 1 atmosphere (30 in. of mercury). A tightly fitting piston is inserted into the open end and pushed down to within 5 in. of the end. What is the pressure of the enclosed air when the temperature has become constant? Give your result in pounds per square inch.

300. Two hollow spherical vessels, inside radii 8 and 10 cm, respectively, contain equal masses of a given gas. Find (*a*) the ratio of the densities of the two gases; (*b*) the ratio of their pressures.

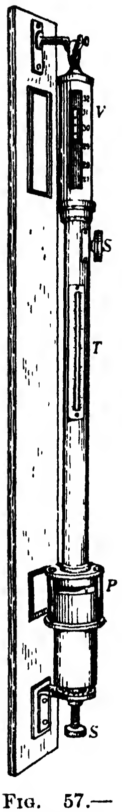

FIG. 57.—Standard barometer.

75. Atmospheric Pressure.—Under standard conditions, that is, at sea level and at 0°C., the density of air is 0.001,29 g per cm³ = 0.08 lb. per cu. ft. Since air has weight, the atmosphere exerts pressure. For standard conditions, a *pressure of* 1 *atmosphere = pressure due to* 76 *cm of mercury* = 30 *in. mercury* = 1033.3 *g per cm*² = 1,012,632 *dynes per cm*² = 14.7 *lb. per sq. in.* Atmospheric pressure is measured by means of a barometer, a standard mercurial barometer being shown in Fig. 57.

In the mercury barometer, the vernier scale V is adjusted by means of the screw S. Before a reading is taken the mercury in the well at the lower end of the instrument is adjusted by means of the screw S' until the surface of the liquid just touches the tip of the pointer P. The vernier is then adjusted so that readings may be taken to a fraction of the least unit of the main scale. Temperature readings are taken by means of the thermometer T. In accurate work, corrections have to be made for expansions due to changes of temperature. The corrected height H_o of the mercurial column is $H_o = H\left\{1 - \frac{(\beta - \alpha)t}{(1 + \beta t)}\right\}$, in which H_o is the corrected height for 0°C.; H and t the observed height and temperature; β = 0.000,181,8, the coefficient expansion of mercury; and α = 0.000,018,4, the coefficient of linear expansion of the brass scale.

An *aneroid barometer* is an instrument which indicates atmospheric pressure changes by the movement of a pointer over a scale. It contains no mercury or other liquid. The principal working part consists of a small air-tight cylindrical metallic box with a thin cover which is somewhat curved. Change of pressure causes the curvature of the cover to change slightly, which motion is communicated to the pointer by means of a system of delicately adjusted levers.

The scale readings of an aneroid barometer are determined by comparing it with a mercury barometer. It is not so accurate as the mercury barometer; it is, however, very much more convenient for certain purposes, since because of its shape and size it may readily be carried about from

place to place. It is very sensitive, a good aneroid barometer registering differences of atmospheric pressure between the top of a table and the floor.

An *altimeter* is an instrument, constructed on the aneroid principle, for measuring altitudes; it is used by mining, civil, and aeronautical engineers, and in practical aviation. A Paulin system altimeter, field type, is shown in Fig. 58. This instrument has a range, as recorded on the double scale, of from −900 to +9700 ft.

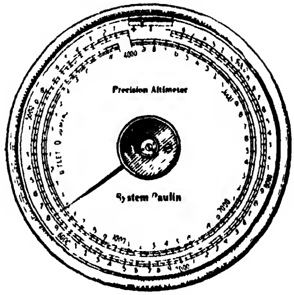

FIG. 58.—Altimeter.

At places near sea level a vertical ascent of about 12 m causes a fall of barometric pressure of 0.1 cm; an ascent of 90 ft., a fall of 0.1 in.

Problems

301. An auditorium is 80 ft. wide, 200 ft. long, and 20 ft. high. Assuming that the density of air on at a given time is 0.001,23 g per cm^3, find the weight of the air in the auditorium.

302. On a day when the barometric reading is 76 cm, an aneroid barometer is carried from the ground floor of an office building to the top. At the top the barometric reading is 29.1 in. What is the height of the building in feet?

303. Under standard conditions the density of air at the surface of the earth is 0.08 lb. per cu. ft., and the pressure which it exerts is 14.7 lb. per sq. in. If the atmosphere were homogeneous (of uniform density) what would be its height in miles?

304. The halves of a set of laboratory Magdeburg hemispheres, 6 inches in diameter, are fitted closely together and the air thoroughly exhausted. What force in pounds will be required to pull the hemispheres apart?

305. Towns *A* and *B* are located on a railroad which runs in a north-south direction. There is a 2 per cent grade (rise of 2 ft. for every 100 ft. of incline) between *A* and *B*. An aneroid barometer carried from *A* to *B* registers a barometric fall of 0.3 in. What is the distance from *A* to *B*, measured along the track?

FLUIDS IN MOTION

76. The Siphon.—The siphon is a device for transferring liquids from a given level to a lower level, over an intervening elevation. It depends for its operation on atmospheric pressure. Figure 59 shows a siphon in opera-

tion. The short arm *ab* of the siphon is measured from the point of application of atmospheric pressure in vessel *A* to the highest point of the siphon; the long arm *ce*, is measured from the highest point to the point of application of atmospheric pressure, as at *e*.

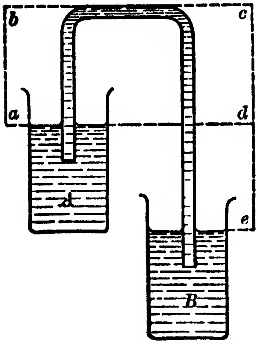

FIG. 59.—Principle of the siphon.

Action of the Siphon.—Let W be the pressure of the atmosphere; w the downward pressure due to the weight of the water in arm ab; w' the downward pressure due to the weight of the water in arm ce. The upward pressure on both arms is that due to the atmosphere, W. The *effective* upward pressure on the short arm is $W - w$; the effective upward pressure on the long arm is $W - w'$. Since $w' > w$ it follows that $W - w > W - w'$. That is, the greater effective upward force acts on the short arm, and hence the liquid flows toward the long arm.

Since the pressures (forces per unit area), w and w', are due to the weight of the liquid in the respective arms, $w = hd$ and $w' = h'd$, in which d is the density of the liquid, $h = ab$, and $h' = ce$. Now if we let P be the effective upward pressure on the short arm, and P' the effective upward pressure on the long arm, then the *net* pressure producing the flow is

$$P - P' = (W - hd) - (W - h'd).$$

Problems

306. Will a siphon work in a vacuum? Why?

307. Explain (*a*) why the flow ceases when the two arms of a siphon are of the same length; (*b*) why the liquid flows back into the vessel when the outer arm is shorter than the inner arm; (*c*) why increasing the outer arm increases the rate of flow.

308. What is the height of a water barometer when the mercurial barometer reads 76 cm, the density of mercury being given as 13.6? Give results in (*a*) meters; (*b*) feet.

309. The specific gravity of kerosene is 0.8; that of sulphuric acid 1.84; mercury, 13.6. Over what height can each of the liquids be siphoned, measured in feet, when the barometric pressure is 76 cm?

310. Over what height in feet can we siphon brine, the density of which is 1.35, when the barometer reads 72 cm?

311. Consider Fig. 59. Suppose that the siphon tube is of uniform bore and has a cross-section of 1 cm². Length of tube *ab*, 20 cm; length of tube *ce* 35 cm; density of liquid 1.5. Find the net effective pressure ($P - P'$) in dynes causing the liquid to flow from the arm *ce*.

77. Bernoulli's Theorem.—This theorem is a formulation of the law of conservation as applied to incompressible fluids, on the assumption that friction effects may be neglected. Bernoulli's principle states that the total energy of a definite mass of liquid moving along a given stream line remains constant. By way of illustration, assume that water is flowing from a tank into a tube at O (Fig. 60). The whole body of liquid is in motion, each particle moving toward the orifice along a definite path called a *str am line.* The lines 1, 2, 3, 4, 5 are stream lines. Consider that mass m of unit volume at a is moving downward along stream line 1. Since density = mass per unit volume and since the volume = 1, density in this case is numerically equal to mass, that is, $d = m$. According to Bernoulli's principle the total energy of the moving particle at a is equal to its energy at b and c, and so on for all points in the stream line. Let the surface of the liquid and the line CO be chosen as levels of reference. The height h = the distance of the mass at a above the datum line CO; v = the velocity of the moving mass; and h'=its distance from the surface. Now the energy of the moving particle at any given instant is made up of three parts as follows: First, the potential energy of *position* with reference to $CO = dgh$; second, the kinetic energy due to its velocity $= \frac{1}{2}mv^2 = \frac{1}{2}dv^2$; and third, the *pressure* potential energy $= h'dg = p$. Equating the total energy received and handed on per unit volume we have, according to Bernoulli's theorem,

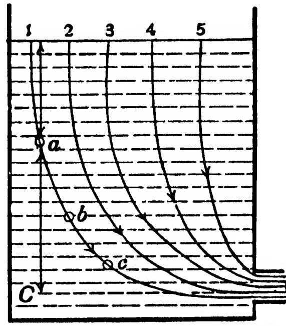

FIG. 60.—Stream lines of flow.

$$dgh + \frac{dv^2}{2} + p = constant = E.$$

This is one form of Bernoulli's equation. Dividing by dg we obtain the equation that is usually used by engineers

$$h + \frac{v^2}{2g} + \frac{p}{dg} = constant = H,$$

in which each term represents a height or length. In the case of water flow engineers call h the *head*, $v^2/2g$ the *velocity head*, p/dg the *pressure head*, and H the *total head.* The important point to be noted about this equation is that when one of the factors, h say, decreases one or both of the other factors must increase, since the sum of the three factors is a constant.

Bernoulli's theorem is of fundamental importance in the science of hydrodynamics.

78. Velocity of Efflux. Torricelli's Theorem.—This proposition deals with the velocity with which a liquid will flow through an orifice in a vessel when the pressure causing the flow is due to the weight of the liquid alone. If we consider a tank to be filled up to a constant level the case becomes one of steady flow, and therefore Bernoulli's principle may be applied. It may be shown mathematically that the theoretical velocity with which a liquid issues from an orifice in the side of a vessel is

$$v = \sqrt{2gh},$$

in which v = theoretical velocity of efflux, h = vertical distance from the surface of the liquid to the orifice, and g = acceleration of gravity. Since at a depth h, $p = hdg$, we may write $h = p/dg$. Substituting this value of h in the equation $v = \sqrt{2gh}$ we have as an expression for the velocity of efflux

$$v = \sqrt{\frac{2p}{d}}.$$

This equation applies not only to liquids but to fluids in general, and therefore the velocity of flow of a gas from a vessel in which the pressure is in excess of the outside atmospheric pressure by an amount equal to p may be computed from the same formula.

Experiment shows that the actual velocity of efflux depends upon the shape of the orifice, being 97 per cent of the theoretical value in the case of an orifice having a sharp edge.

The volume of liquid which will flow out of the orifice is

$$V = Avt,$$

in which V = volume of liquid; A = cross-sectional area of the orifice; v = velocity of efflux; and t = time in seconds. In actual practice this value is never reached. If the opening be a simple orifice without a mouthpiece of any sort, the actual volume of liquid discharged is only about 62 per cent of the theoretical volume. By using a mouthpiece of proper shape and dimensions, the flow may be made about 80 per cent of the theoretical value.

Problems

312. A steel tank is filled with water to a depth of 16 ft. and maintained at this level. A hole is drilled through the side of the tank, at the bottom, and a short cylindrical tube is fitted to it. The hole in the tube has a cross-sectional area of 1 sq. in. How many cubic feet of water will flow through the tube per minute assuming that the actual discharge is 80 per cent of the theoretical value?

313. The water in a tank is maintained at a constant level 20 ft. from the bottom. Find the total quantity of water that will flow out per hour from two openings of equal cross-sectional area A, one of which is at the bottom of the vessel and the other one halfway down the side, the ratio of the actual volume delivered to the theoretical volume being k.

314. A certain tank is filled with water to a depth of 65 ft. A hole is drilled in the side of this tank 16 ft. from the bottom, which rests on the ground. How far from the base of the tank would the water issuing from this opening strike the ground, provided v have its theoretical value and we neglect air friction?

315. The density of hydrogen is to that of oxygen as 1:16. Compare the velocities with which these two gases will effuse through the walls of a porous vessel, the pressure difference being the same for both.

316. If oxygen effuses through a porous vessel at the rate of 500 cc per min. when the pressure difference between the interior and exterior of the vessel is 1 atmosphere, find the rate of effusion of hydrogen when the difference of pressure is 4 atmospheres.

79. Flow of Liquids in Pipes of Variable Section.—Consider water flowing in a pipe of variable cross-section, as shown in Fig. 61. It may be shown mathematically, as well as experimentally, that as the velocity approaches a maximum in the constricted portion of the tube the pressure approaches a minimum; that is,

$$p < p', \text{ and } p < p''.$$

This means that when a fluid in motion is constrained at any point to increase its velocity, the lateral pressure at that point in the fluid is diminished.

The mathematical demonstration of the principle of diminished pressure is based upon a special case of Bernoulli's equation, $dgh + \frac{1}{2}dv^2 + p =$

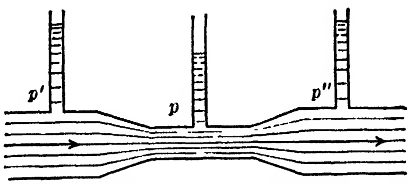

FIG. 61.—Principle of the Venturi water meter.

constant. If a pipe of variable cross-section is placed in a horizontal position, the factor h may be considered as constant and the equation becomes

$$\tfrac{1}{2}dv^2 + p = constant.$$

When the moving fluid reaches the constricted part of the pipe its velocity v increases. But since $\frac{1}{2}dv^2 + p$ is equal to a constant, it follows that if v increases p must decrease. The greater the increase in velocity the less the pressure. Indeed the pressure in the constricted part of the pipe may be reduced to or even below that of 1 atmosphere.

This principle has an important application in the operation of the Venturi water meter, a modification of which is employed in measuring the water passing through the great Croton aqueduct, which supplies New York City. The principle may also be used to explain the operation of the jet pump, the atomizer or spraying machine, the "card and tube" experiment, the curving of a pitched baseball, and the lift of an airplane.

80. The Airplane.—An airplane in flight is supported by the reaction of the air against the wings. In Fig. 62 there is shown a section of the wing of an airplane in level flight. The direction of motion is from D to T.

The angle which the lower face of the wing makes with the longitudinal axis of the plane is called the angle of incidence or *angle of attack.* The supporting reaction is represented by the resultant force R, which is due to two primary causes. The first is an upward force due to the compression of the air on the under side of the wing, and the second is due to the diminished pressure on the upper side of the wing caused by the increased speed of the air stream over the upper curved surface. To understand the reason for this diminished pressure on the upper side of the wing, let us consider two particles of air, M and N, at the leading edge of the wing. Suppose that particle M travels along the upper curved side of the wing in a curved path, and N along the under side. Now, other things being equal, the particle M on the upper side will travel with a greater speed than N, because it has farther to go in a given time. But we have learned (Art. 79) that when the speed of a fluid stream is increased, the lateral pressure is diminished. The

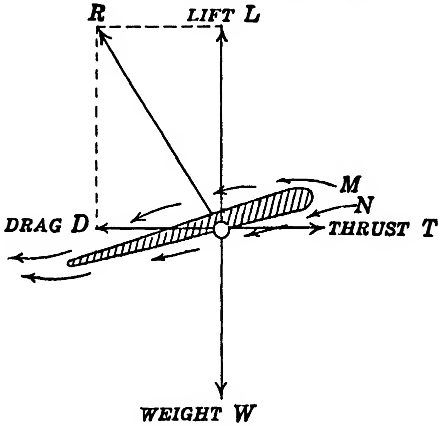

FIG. 62.

pressure on the upper side of the wing is thus much less than on the lower side. As a result of the motion of the plane we therefore have an increase of pressure on the under side of the wing and a diminution of pressure on the upper side. It is estimated that about *one-fourth* the sustaining force comes from the compression of the air below the wing and about *three-fourths* from the reduced pressure above.

The problem of airplane flight involves a study of the various components (forces) which act upon the plane. These are (*a*) the weight of the plane, OW (Fig. 63); (*b*) the thrust OT, a force due to the rotation of the propeller; (*c*) the drag OD due to the resistance of the plane against the air; and (*d*) the lift L, which is a component of the reaction R. The lift L is always considered as acting at right angles to the direction of motion, that is, at right angles to DT. The force W of course always acts vertically downward.

The fundamental equations for lift and drag may be written as follows:

$$\textit{lift } L = K_l \times A \times v^2,$$
$$\textit{drag } D = K_d \times A \times v^2,$$

where A = wing area, v = speed, and K_l and K_d = coefficients of lift and drag, respectively.

The conditions for level flight are that the lift shall be equal to the weight, that is $L = W$, and also that the thrust must be equal to the drag, $T = D$. When the thrust is equal to the drag the plane moves forward with a uniform motion, in virtue of its inertia.

The conditions of weight, thrust, lift, and drag, in order that a plane may rise in a direction OT (Fig. 63) are that

$$\textit{lift } L = \textit{weight component } W_1,$$

and

$$\textit{thrust } T = \textit{drag } D + \textit{weight component } W_2.$$

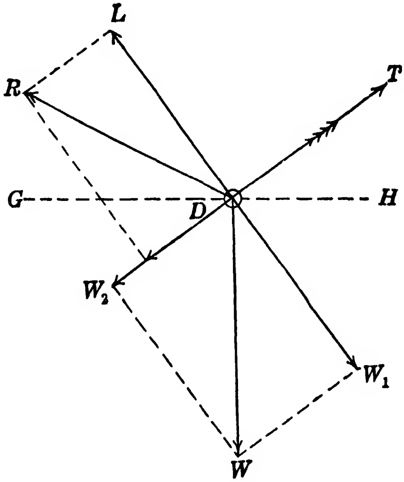

FIG. 63.

Conditions for descent are that

$$\textit{lift } L = \textit{weight component } W_1,$$

and

$$\textit{thrust } T = \textit{drag } D - \textit{weight component } W_2.$$

It will be noted that the fundamental difference between the conditions for ascent and descent is that for ascent $T = D + W_2$, while for descent $T = D - W_2$.

When an airplane with power shut off descends without acceleration or rotation, it is said to *glide*. The angle of glide must be such that the lift L is equal to the weight component W_1, and the drag D must be equal to the weight component W_2. These conditions for gliding may be expressed as follows:

$$\textit{thrust } T = 0,$$
$$\textit{lift } L = \textit{weight component } W_1,$$

and

$$\textit{drag } D = \textit{weight component } W_2.$$

Problems

317. If air be blown through a tube having a flaring lip *ab*, and a card *C* be placed against the lower opening, as shown in Fig. 64, it will be found that the card cannot be blown from the tube, even if the tube be held in a vertical position. Explain this phenomenon in terms of the principle of diminished pressure as illustrated in the case of the flow of water in a constricted pipe (Fig. 61).

318. Consider Fig. 65. Here we have illustrated the various motions of a "pitched" ball. The line *AB* represents the general direction and sense in which the ball is moving at a given instant. *R* is the rotary motion in a clockwise sense; *BC* is the direction and sense in which the ball "curves." We may

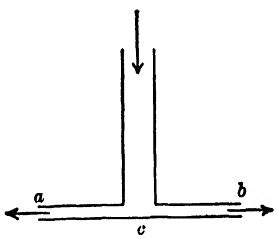

Fig. 64.—Card-and-tube experiment.

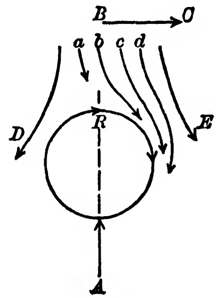

Fig. 65.—Curving of a pitched ball.

consider the ball as moving against the air, or, on the other hand, we may consider the air as moving against the ball with a velocity represented by the lines *D* and *E*. As the ball rotates it carries air around with it as shown by the lines *a*, *b*, *c*, *d*. Explain why the ball curves to the right; that is, in the sense *BC*.

319. Consider an airplane in ascending flight (Fig. 63). Let the angle *ROL* be 20°, and angle WOW_1, 30°. The reactive force exerted along the line *OR* by the compression of the air on the under side of the wing is 1000 lb. Find (*a*) the total reactive force *R*; (*b*) the lift *L*; (*c*) the drag *D*.

320. If the weight component W_1 (problem 319) is 3759 lb., what is (*a*) the weight of the plane; (*b*) the component W_2; (*c*) the thrust *T*?

81. Circulatory Motion of the Air.—Few fluid motions are of greater importance to us than those of the air. Especially is this true of what is

known to the meteorologist as "cyclones," those immense whirls of air, many miles in diameter, which give rise to the varying conditions of the "weather." The air being heated at a given point gives rise to a diminished barometric pressure, known as a *low* region (Fig. 66). The cold heavy air rushing into this low region, together with the motion imparted due to the rotation of the earth on its axis, gives rise to the cyclonic storms which pass

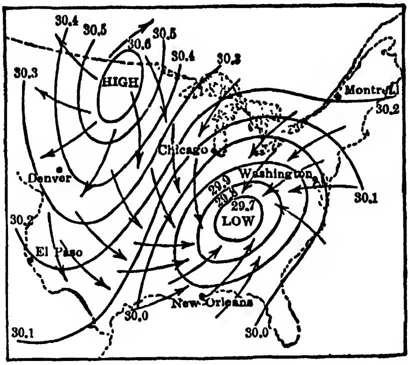

FIG. 66.—Weather map showing rotary motion of the air.

periodically over the country from west to east. In the Northern Hemisphere these wind storms always rotate in a counterclockwise sense.

An explanation of this counterclockwise motion is involved in the solution of the problems which follow.

Problems

321. Consider the three points *a*, *t*, *c*, on a meridian of the hemisphere (Fig. 67) which rotates about a vertical axis through *P* in the sense indicated by the arrow. Suppose that *t* is a target, and that guns situated at *a* and *c* are aimed directly at its center. Considering the motion of the sphere, we wish to find where the balls fired from the guns will strike the target, with reference to its center. All three points move in the same direction and sense, but with different velocities. (*a*) How does the velocity of *c* compare with that of *t*. (*b*) How does the velocity of *t* compare with that of *a*? (*c*) Will the ball fired from *c* strike the target to the right or left of the center, looking toward *P*? (*d*) Where will the ball from *a* strike the target, with reference to its center? (*e*) If the target were free to move about its center,

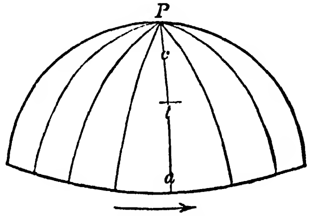

FIG. 67.—Explanation of cyclonic motion of air.

in what sense would it tend to rotate due to the action of the two balls?

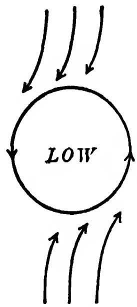

FIG. 68.—Circular motion of the air in "low" area.

322. Explain why the air rushing into the low area (Fig. 68) tends to set up a circular (cyclonic) motion in a counterclockwise sense.

323. A person stands with his back to the wind (Fig. 66). On which hand (right or left) is the storm center? Why?

324. Suppose that a gun situated at the north pole of the earth is aimed along a meridian at the center of a target distant 1200 ft. from the pole. The linear velocity of the target, due to the rotation of the earth is approximately 1 in. per sec.; the velocity of a ball fired from the gun is 1500 ft. per sec. Where will the ball strike the target with reference to its center?

325. If the position of the gun and target (problem 324) be exchanged, where will the ball strike the target?

CHAPTER V

MOLECULAR MECHANICS

SURFACE PHENOMENA IN LIQUIDS

82. Surface Tension.—Surface tension is the force exerted in the surface of a liquid, per unit of length of its boundary line; that is,

$$\textit{surface tension} = T = \frac{F}{l},$$

where T = dynes per unit length (1 cm) of a single surface of the liquid, F = total force exerted, and l = the surface boundary considered. In the case of a capillary tube, l = the circumference of the capillary bore = $2\pi r$, where r is the radius of the bore.

The force F always acts at right angles to the boundary of the liquid and in the plane of the surface at the point of contact.

SURFACE TENSIONS IN DYNES PER CM

Mercury	547
Water	75
Olive oil	32
Turpentine	27
Alcohol	23

83. Surface Tension and Surface Energy.—Suppose that a frame ABC (Fig. 69) is lowered into a given liquid S, and then drawn upward so that a

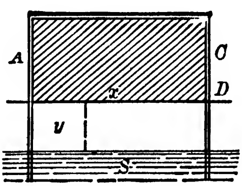

FIG. 69.—Energy in a stretched film.

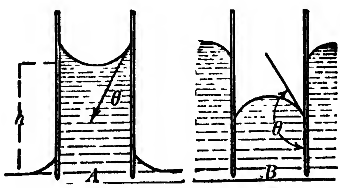

FIG. 70.—Angles of contact in capillary tubes.

film is stretched across its surface. Now let a light rod D be laid across the surface of the film, and the film below the rod broken. The upper section of the film will contract drawing the rod upward through a distance y. It must be noted that in the case of a free film we have two surfaces to take into account. The surface tension is therefore $2T$. The entire diminution in surface is $2xy$. If E be the surface energy of the film per *unit area*, the potential energy for both surfaces has been decreased by an amount equal

to $2Exy$. Now if T be the surface tension per *unit width* of the film, the total surface tension lifting the rod is $2Tx$. The distance moved is y. The work done against gravity is $2Txy$, which is equal to the loss of potential energy, or

$$2Txy = 2Exy,$$

which means that the surface tension T per unit length of the film is numerically equal to the surface energy E per unit of area.

84. Capillary Action.—When a capillary tube is thrust into a liquid, the liquid ascends or is depressed in the tube according as its surface is concave or convex (Fig. 70), A and B, the elevation or depression being

$$h = \frac{2T}{rdg} \cos \theta$$

where h = elevation or depression of the liquid in centimeters; T = surface tension in dynes per unit length; θ = angle of contact; r = radius of the tube in centimeters; d = density of the liquid in grams per cm³; and g = acceleration of gravity.

In the case of capillary action between two plates

$$h = \frac{2T}{udg} \cos \theta$$

where u is the distance between the plates.

The angle of contact between water and clean glass is so nearly equal to 0° that it is usually assumed for these substances that $\cos \theta = 1$.

85. Normal Pressure on a Curved Film.—A stretched curved film always exhibits a pressure normal to its surface and directed toward the concave side. It may be shown that for a *cylindrical* film of radius R, the pressure $P = T/R$.

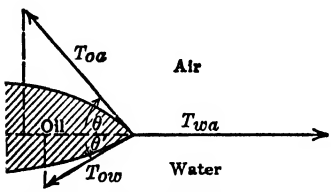

Fig. 71.—Form of drop of oil on water.

In the case of a *spherical* globule enclosed in a film having *one surface*, as a drop of water, for example, $P = 2T/R$.

In the case of a hollow spherical film (soap bubble) in which there are *two surfaces* of practically equal radii, the pressure is $P = 4T/R$.

Example.—The surface tension of water against air is 75 dynes per cm. A spherical drop of water has a radius of 2 mm. Find (*a*) the pressure normal to the drop; (*b*) the total force acting on the surface of the drop, due to surface tension; (*c*) the potential energy in the surface. *Solution:* (*a*) $P = \frac{2T}{R} = 2 \times \frac{75}{0.2} = 750$ dynes per cm². (*b*) Area of drop $= 4\pi r^2 = 0.16\pi$ cm². $F = PA = 750 \times 0.16\pi = 120\pi$ dynes. (*c*) The potential energy E per unit area is numerically equal to the surface tension T; that is, $E = 75$ ergs per cm². The total energy in the entire surface of the drop is, therefore, $EA = 75 \times 0.16\pi = 12\pi$ ergs.

86. Oil on Water.—When a drop of oil is placed on the surface of water, the shape which the drop assumes (Fig. 71) depends upon the relative magnitudes of the following surface tensions: T_{oa}, the surface tension between oil and air; T_{ow}, the surface tension between oil and water; T_{wa}, the surface

tension between water and air. The drop will be flattened by the tension T_{wa} until the projections of the vectors T_{oa} and T_{ow} are equal to T_{wa}; that is,

$$T_{wa} = T_{oa} \cos \theta + T_{ow} \cos \theta'.$$

Problems

326. In Fig. 69 we have represented a rectangular wire support upon which there is stretched a soap film, of length 5 cm, width 5 cm, the surface tension of which is 36 dynes per cm. (*a*) What is the force exerted upon the rod *D*, due to the surface tension of the two surfaces of the film? (*b*) What potential energy does the film possess?

327. What is the height to which water will rise in a capillary tube 0.06 cm in diameter, assuming the angle of contact to be negligible?

328. How much will the surface of mercury be depressed in a glass tube of radius 0.02 cm, the angle of contact being 135°?

329. Compute the diameter of a glass tube in which pure water rises to a height of 12 cm, the angle of contact being negligible.

330. A soap bubble of radius 4 cm is made from a solution having a surface tension of 25 dynes per cm. Find (*a*) the pressure normal to the surface of the bubble; (*b*) the force exerted upon the air within the bubble.

331. Two spherical globules of mercury having radii of 1 mm and 2 mm, respectively, unite to form one drop. (*a*) How does the surface of the resulting drop compare with the sum of the surfaces of the two drops? (*b*) How does the surface energy of the resulting globule compare with the surface energies of the original drops?

332. Find the angle of contact of turpentine with glass from the following data: Radius of capillary, 0.1 mm; density of turpentine, 0.86 g per cm^3; surface tension, 28.8; height to which liquid rises in tube, 6.52 cm.

333. The horizontal component of the surface tension of olive oil against air is 36.9 dynes per cm, and that against water, 20.7 (Fig. 71). The surface tension of water is 75. (*a*) How does the sum of the components of the surface tensions named above compare with the pull (surface tension) of the water? (*b*) What will happen to the drop of oil if it is placed upon water?

334. The surface tension of grease is greater than that of gasoline. In cleaning a grease spot from cloth, in what direction (inward or outward) will the grease move when the gasoline is

placed (*a*) in the center of the spot (*b*) in a ring around the margin of the spot? (*c*) How should gasoline be applied to the spot so as to prevent the grease from spreading into the cloth?

DIFFUSION

87. Diffusion of Liquids.—If two liquids which are miscible are introduced into a vessel so that the lighter lies above the denser, diffusion will take place, some of the lighter liquid passing downward, and some of the denser liquid passing upward. The essential facts relating to diffusion of liquids are: (*a*) The rate of diffusion is very slow. (*b*) In general the rate of diffusion increases with an increase of temperature. (*c*) The rate of diffusion is proportional to the ***concentration gradient*** c, where c is the change of concentration per unit of length considered. Concentration of a solution is measured in terms of the number of gram-molecules of the substance (solute) dissolved in one liter of the solvent. For example, the molecular weight of sodium chloride (NaCl) $= 23 + 35.5 = 58.5$. A normal solution of sodium chloride in water, then, is 1 gram-molecule (58.5 g) dissolved in 1 l. A tenth normal ($n/10$) solution $=$ 5.85 g of the solute (NaCl) in 1 l of the solvent (water). (*d*) The rate of diffusion is proportional, also, to the ***diffusion constant*** k, where k is the number of grams of the solute which diffuses through unit area (1 cm^2), per unit concentration gradient, per unit time (1 day). The mass m which will diffuse through any given area in the time t is

$$m = k \times c \times a \times t,$$

where m = mass in grams; k = diffusion constant; c = concentration gradient; a = surface area lying between the two solutions; t = time in days.

By way of illustration, the diffusion constants of a few substances are given in the following, where n = the concentration (gram-molecules per liter), $t°$ = temperature, and k = diffusion constant.

Diffusion Constants

Substance	n	t°C.	k
Hydrochloric acid....	1 0	5	1 74
Hydrochloric acid.	1 0	12	2 09
Sodium chloride ...	1 0	5	0 76
Sodium chloride	1 0	10	0 91
Sugar........	1 0	9	0 31
Albumen	1 0	13	0 06

Problems

335. A cylindrical vessel having a radius of 4 cm contains a salt solution, above which there is a quantity of pure water. The vertical concentration gradient of the salt solution is 2. The

diffusion constant of the salt solution is 0.76. Find the number of grams of salt which will diffuse into the water in 6 hr.

Ans. 6.08π g.

336. What time in days will be required for 10.44π g of hydrochloric acid, possessing unit concentration gradient, to diffuse through a circular area having a diameter of 4 cm, the temperature being 5°C.?

337. From the following data compute the diffusion constant for cane sugar. It was found that 1.4 g of sugar from a solution, the concentration gradient of which was unity, passed through 2 cm^2 of surface in 48 hr. Find k.

338. It was shown experimentally that 9.1 g of sodium chloride at 10°C. passed through a surface of 4 cm^2 in 2 days. Find the concentration gradient.

88. Diffusion through Membranes. Osmosis.—Liquids diffuse readily through certain membranes. The diffusion of substances through such

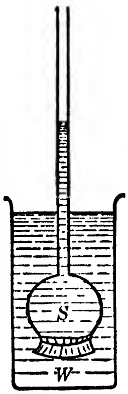

Fig. 72.—Osmotic-pressure apparatus.

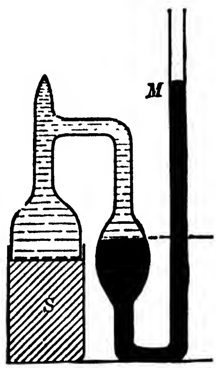

Fig. 73.— Pfeffer's osmotic-pressure apparatus.

septa is called *osmosis.* The separation of crystalloids from colloids by means of an intervening membrane is called *dialysis.* If an aqueous solution of a salt or of sugar be separated from water by means of a membrane (Fig. 72) which allows osmosis to take place from W to S more readily than from S to W, the solution rises in the tube giving rise to a pressure equivalent to hdg, which measures the osmotic pressure of the solution in S.

Pfeffer's Experiment.—Pfeffer performed a series of classic experiments in which he measured the osmotic pressure of sugar solutions, using an apparatus as shown in Fig. 73. The osmotic pressure of the solution in S was measured by means of the mercury manometer M. He found that a concentration of 20.16 g of sugar per l of water gave an osmotic pressure equivalent

to 101.6 cm of mercury. In another experiment, at the same temperature, he found that 61.19 g of sugar dissolved in 1 l of water gave a pressure of 307.5 cm.

The three laws of osmotic action, which may be illustrated by data taken from Pfeffer's experiments, are as follows:

1. Temperature remaining constant, osmotic pressure is proportional to the concentration. This is equivalent to Boyle's law, as applied to solutions; that is, $pv = c$, in which p = osmotic pressure, v = the volume of the solution containing 1 gram-molecule of the solute, and c = a constant.

2. For a given temperature, gram-molecules of different substances dissolved in equal volumes of the solvent exert equal osmotic pressures. For example, the molecular weight of cane sugar is 342 and that of glucose 180. Now if 342 g of cane sugar be dissolved in 1000 cc of water and 180 g of glucose be dissolved in an equal volume of water, the osmotic pressure exerted by the two solutions will be equal.

3. For a given concentration, osmotic pressure is proportional to the absolute temperature.

Example 1.—Referring to Pfeffer's data, find how the ratio of the osmotic pressures observed compares with the ratio of the concentrations of the two sugar solutions. *Solution:* $P/P' = 101.6/307.5 = 0.330$, and $C/C' = 20.16/61.19 = 0.329$. That is to say, the osmotic pressures were practically proportional to the concentrations in accordance with the first law.

Example 2.—Find the volume of solution in each case containing 1 gram-molecule of sugar, the molecular weight of sugar being 342. *Solution:* Since there are 20.16 g in 1000 cc of solution, 342 g will require $(342/20.16) \times 1000 = 16{,}964$ cc; and $(342/61.19) \times 1000 = 5589$ cc. That is, $V = 16{,}964$ and $V' = 5589$.

Example 3.—Find the product of pressure and volume in each case. *Solution:* $pv = 101.6 \times 16{,}964 = 1{,}723{,}500$, and $p'v' = 307.5 \times 5589 = 1{,}718{,}600$. Since $pv = p'v'$ = a constant (nearly) it follows that within the limits of experimental error Boyle's law applied to the solutions used in Pfeffer's experiment.

Example 4.—*Van't Hoff's application of Pfeffer's data.* The gas law states that the product of pressure and volume is equal to the absolute temperature times a constant R; that is, $pv = RT$ in which R is the gas constant $= 8.3 \times 10^7$ ergs per degree Centigrade, and T = absolute temperature. Van't Hoff showed by means of Pfeffer's data that a dilute solution of sugar in water obeys the same general law as a gas. Selecting the first set of data given in the example above, find the value of R, the temperature being 15°C. *Solution:* The pressure $p = 102 \times 13.6 \times 980 = 1359 \times 10^3$ dynes per cm². The volume v containing a gram molecule of the solute $= 16{,}964$ cc. The absolute temperature $T = 15 + 273 = 288$. Then $R = 1359 \times 10^3 \times \frac{16{,}964}{288} = 8 \times 10^7$, a close approximation to the theoretical value.

Problems

339. Pfeffer found that a certain concentration of sugar gave a pressure of 53.5 cm of mercury, the product of pv expressed in

cm and cc being 1,822,000. Find the concentration in grams per liter. *Ans.* 10.04 g/l.

340. At a given temperature, a 1 per cent solution of sugar in water (10 g per l) gave an osmotic pressure equivalent to 50.5 cm of mercury. The computed value for R was 82,202,000. What was the temperature at the time of the experiment?

Ans. 280° Abs.

341. Sugar does not dissociate in water; that is, 100 molecules of sugar will go into solution forming, let us say, 100 active particles which take part in exerting osmotic pressure. Salts, acids, and bases, on the other hand, dissociate into ions. It is estimated, for example, that a thousandth normal ($n/1000$) solution of KCl is completely dissociated, as follows: KCl = K + Cl; 100 molecules of KCl will therefore give 200 active particles in solution. What concentration of potassium chloride in water will be required to give the same osmotic pressure as that exerted by the sugar solution of problem 340?

Ans. 5 g/l.

ELASTICITY

89. Stress and Strain.—When a body is stretched, twisted, bent, or compressed it is said to be distorted. Within the limits of elasticity of a given substance the distortion is proportional to the force applied.

Stress.—When a force is applied to a medium producing a distortion, a stretch for example, there is set up in the medium a reaction called a *stress*. When the system is in equilibrium, the stress is equal to the force per unit area producing the distortion; that is,

$$stress = \frac{F}{a}.$$

Strain.—When a body is distorted the relative change of configuration of the system is called a *strain*. In the case of the distortion due to a stretching force, the strain is the change of length per unit of length; that is,

$$strain = \frac{l}{L}.$$

90. Hooke's Law.—*Hooke's law* states that stress is proportional to strain; that is, $stress = e \times strain$, where e is a proportionality factor known as the *coefficient of elasticity*.

91. Coefficients of Elasticity.—From Hooke's law we have

$$coefficient\ of\ elasticity = e = \frac{stress}{strain}$$

Volume Elasticity.—In the case of a change of volume due to a distorting force e is the *coefficient of volume elasticity*, which is

$$C.V.E. = \frac{stress}{strain} = \frac{F/a}{v/V} = \frac{FV}{av},$$

where F = distorting force; a = cross-sectional area upon which the force acts; v change of volume; and V original volume.

Young's Modulus.—When a rod or wire is stretched, the coefficient of elasticity expressed by the ratio of the stress to the strain is called *Young's modulus*, whence

$$Y.M. = \frac{stress}{strain} = \frac{F/a}{l/L} = \frac{FL}{al},$$

where F = stretching force; a = cross-sectional area of rod or wire; l = change in length; and L = original length.

Coefficient of Simple Rigidity.—In a distortion due to twisting, as in the case of a rod fastened at one end and twisted, as shown in Fig. 74, the ratio of the shearing stress to the shearing strain is called the *coefficient of simple rigidity*, or *rigidity coefficient n*, which may be written

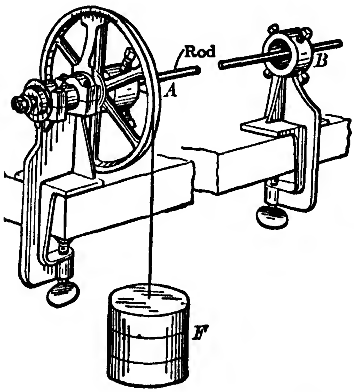

FIG. 74.—Coefficient of rigidity apparatus.

$$n = \frac{2L\mathfrak{T}}{\pi\theta r^4},$$

where n = coefficient of simple rigidity; L = length of the rod; $\mathfrak{T}$ = torque, or moment of a force ($F \times$ radius of wheel) producing the distortion; θ = angle of distortion, *measured in radians;* and r = radius of the rod.

Example.—Given a steel rod AB, 6 ft. in length, radius 0.2 in., clamped at one end, as shown in Fig. 74. The end A is twisted through an angle of 2° by a force of 5 lb., acting on a circumference of the wheel, the radius of which is 3 in. Find the coefficient of simple rigidity n in pounds per inch squared. *Solution:* The length of the rod is 72 in.; the torque $\mathfrak{T}$, in pound inches, is $5 \times 3 = 15$; $\theta = \frac{2 \times 2\pi}{360} = \frac{\pi}{90}$ radians; $r^4 = 0.0016$ in. Then $n = \frac{2L\mathfrak{T}}{\pi\theta r^4} = 2 \times 72 \times 15 \times 90 \times \frac{10{,}000}{\pi^2 \times 16} = \frac{1215 \times 10^5}{\pi^2} = 12.3 \times 10^6$ lb./in.^{2}.

For Elastic Constants, see Table IX, Appendix.

92. Determination of Coefficient of Rigidity by Torsional Pendulum.—The torsional pendulum furnishes a method of determining the coefficient of rigidity of a given metal when the latter is in the form of a wire or very thin rod. If a cylinder (Fig. 75) be suspended by means of a wire, it may be made to oscillate in a horizontal plane with the period T, such that

$$T = 2\pi\sqrt{\frac{2LI}{\pi n r^4}},$$

where L = length of the supporting wire; I = moment of inertia of the vibrating cylinder; n = coefficient of simple rigidity, expressed in *absolute units;* and r = radius of the wire.

Example.—Suppose that we wish to determine the coefficient of simple rigidity of a piece of metal from which a given sample of wire is made. A

cylindrical disc, having a mass of 10 lb., and a radius of 4 in., is suspended from the middle point of one face by means of a piece of the wire whose length is 6 ft., and radius, 0.05 in. The disc makes 26 vibrations per min. Find the coefficient of simple rigidity in pounds per square inch. *Solution:* $T = {}^{60}\!/_{26} = 2.3$ sec. The moment of inertia I of the disc is $\frac{1}{2}Mr^2 = 80$; $L = 72$ in.; $r^4 = 625/10^8$. Now from equation $T = 2\pi\sqrt{\dfrac{2LI}{\pi n r^4}}$ we may write

$$n = \frac{8\pi IL}{r^4T^2} = 8\pi \times 80 \times 72 \times \frac{10^8}{625 \times 5.29} = 4378 \times 10^6 \text{ poundals/in.}^2.$$

Now to reduce this value to gravitational units we divide by $g = 32 \times 12$ in./sec./sec. That is $4378 \times \dfrac{10^6}{32 \times 12} = 11.4 \times 10^6$ lb./in.^{2}.

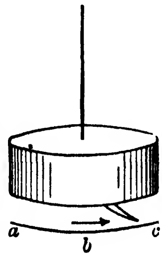

Fig. 75.—Coefficient of rigidity pendulum.

93. Energy Stored in a Strained Body.—The work required to stretch a body within the limits of its elasticity is stored in the body as potential energy W. This energy is

$$W = \tfrac{1}{2} \times AL \times \textit{stress} \times \textit{strain} = \tfrac{1}{2}Fl,$$

where A = cross-sectional area of the rod; L = length of the rod; F = stretching force; and l = elongation.

Example.—An iron rod, length 5 m, cross-sectional area 0.02 cm^2, is stretched 1 mm by a force of 7 kg. Find the energy in absolute units stored in the rod. *Solution:* $L = 500$ cm; $A = 0.02$ cm^2; stress = force in dynes/cm^2 $= 7000 \times 980 \times {}^{100}\!/_{2}$; strain $= l/L = {}^{1}\!/_{5000}$. Then $W = (\frac{1}{2} \times {}^{2}\!/_{100} \times 500 \times 7000 \times 980 \times 50 \times 1)/5000 = 333{,}000$ ergs. Also, $W = \frac{1}{2}Fl = \frac{1}{2} \times 7000 \times 980 \times {}^{1}\!/_{10} = 333{,}000$ ergs.

94. Impact.—The phenomena of impact vary with the masses, the velocities and the elasticities of the colliding bodies. Consider two elastic bodies (two metal balls, say) in which,

m and m' = the masses of the balls,
v and v' = their velocities before impact,

and

u and u' = their velocities after impact.

Newton showed that $u - u' = k(v' - v)$, where k is called the *coefficient of restitution,* a factor depending upon the elasticity of the bodies in collision. Also, since according to the third law of motion "action is equal to reaction," that is, the change of momentum of one body is equal to change of momentum of the other, we may write $m(v - u) = -m'(v' - u')$.

Now solving for u and u' we have

$$u = \frac{(mv + m'v')}{(m + m')} - \frac{km'(v - v')}{(m + m')},$$

$$u' = \frac{(mv + m'v')}{(m + m')} + \frac{km(v - v')}{(m + m')}.$$

In connection with the above equations it should be noted that the vector quantities v and v' must be taken with their proper signs; that is, if velocity

in one sense (to the right, for instance), is considered positive, then velocity in the opposite sense (to the left) will be negative.

We consider that bodies are perfectly elastic when $k = 1$; and perfectly inelastic when $k = 0$.

Example.—The coefficient of restitution k between two balls A and B is 0.5. The mass of A is 100 g, and its velocity v is 20 cm per sec. The mass of B is 50 g, and its velocity 10 cm per sec. Find the velocity of each ball after impact when (*a*) the balls are moving in opposite senses; (*b*) in the same sense. *Solution:* When the balls are moving in opposite senses, v will be positive, and v' negative. Mass $m = 100$, and $m' = 50$. In case (*a*) $v = +20$, and $v' = -10$, from which $v - v' = 20 - (-10) = 20 + 10 = 30$. Then (*a*) $u = [100 \times 20 + 50 \times (-10)]/150 - [0.5 \times 50 \times 30]/150 = (2000 - 500)/150 - (25 \times 30)/150 = (1500 - 750)/150 = +5$, and $u' = (1500 + 750)/150 = +15$. In case (*b*) $v = +20$ and $v' = +10$. Then $u = (100 \times 20 + 50 \times 10)/150 - (0.5 \times 50 \times 10)/150 = (2500 - 250)/150 = +15$, and $u' = (2500 + 250)/150 = +18.3$.

Rebound.—If a small body m be allowed to fall, striking a large body of mass m' at rest, the velocity of m' will not be appreciably changed, and consequently v' and $u' = 0$. In this case the height of the fall of $m = H$, and $v^2 = 2gH$ downward. The height of the rebound $= h$, and $u^2 = 2gh$ upward. It follows that $k = -v/u = h/H$.

95. Loss of Kinetic Energy in Impact.—When two bodies collide a certain amount of kinetic energy is lost in the form of heat. The loss of kinetic energy due to impact is

$$\frac{(1 - k^2)(mm')(v - v')^2}{2(m + m')}.$$

Problems

342. A volume of 10 cc of gas under a pressure of 1 atmosphere is enclosed in the short arm of a J-tube by means of mercury. Additional mercury is now poured into the open end of the tube until the volume of the enclosed air is 5 cc. The cross-section of the tube is 1 cm^2. (*a*) The diminution of volume from 10 to 5 cc was due to what pressure? (*b*) Find the coefficient of volume elasticity of the air. (*c*) How does e in this case compare with the total pressure upon the gas?

343. A mass of gas having a volume of 1000 cc is under a pressure of 2000 g. An additional pressure of 500 g is applied to the gas, causing the volume to become 800 cc. Find the coefficient of volume elasticity, and compare this value with the total pressure upon the gas.

344. A cylinder having a cross-sectional area of 10 cm^2, and a length of 100 cm, is filled with gas, which is under a pressure of 300 g per cm^2. Into one end of the cylinder there is fitted

a piston, which may be considered to move without friction. An additional force of 1 kg is applied to the piston, causing it to move into the cylinder a distance of 25 cm. Find the coefficient of elasticity in absolute units (dynes per cm^2).

345. A column of water 1 m in length is enclosed in a rigid tube, having a cross-sectional area of 4 cm^2. The coefficient of elasticity of water is 20×10^9 dynes per cm^2. A force of 50 kg applied to the end of the tube will reduce the column of water by what length?

346. A column of water, length 4 m, cross-sectional area 2 cm^2 is reduced in volume by 1.96 cm^3, by a force of 100 kg. Find the coefficient of elasticity of the water in (*a*) gravitational units; (*b*) absolute units.

347. A rod 5 m in length, and 4 cm^2 in cross-sectional area is stretched 0.05 mm by a force of 10 kg. Find (*a*) the stress, in both absolute and gravitational units; (*b*) the strain.

348. A steel wire 4 m in length, cross-sectional area 0.01 cm^2, is stretched 0.3 mm by a force of 2 kg. Find Young's modulus for this wire (*a*) in grams per cm^2; (*b*) dynes per cm^2.

Ans. (*a*) 26.6×10^8 g/cm^2; (*b*) 26.1×10^{11} dynes/cm^2.

349. A metal rod 12 ft. long, radius 0.1 in., is stretched 0.3 in. by a weight of 300π lb. applied to one end. Find Young's modulus for this rod in pounds per square inch.

350. Young's modulus for a given sample of copper wire is 12×10^{11} dynes per cm^2. The length of the wire is 5 m; its diameter, as determined by a micrometer gage, is 0.6 mm. What force in kilograms will be required to stretch the wire 0.2 cm?

351. A rod of Bessemer steel 10 ft. long, cross-sectional area 0.05 sq. in. is stretched 0.02 in. by a force of 270 lb. Find Young's modulus in (*a*) gravitational units; (*b*) absolute units.

352. Find the energy in foot-pounds stored in the rod of problem 351 while under strain.

353. A brass wire 500 cm long and 1 mm^2 in cross-section, has a 10-kg mass suspended upon it. How much will the wire be stretched?

354. What would be the stretch of the wire in problem 353 if it were of steel?

355. A wire drawn from Bessemer steel, 4 m in length, 0.01 cm^2 in cross-section, is stretched by a force of 3 kg. Find the energy in absolute units stored in the wire due to the strain.

356. A brass rod 1.5 m in length, radius 0.4 cm, is clamped at one end. To the other end there is attached a circular disc of radius 5 cm (Fig. 74). A force of $130\pi^2$ g causes the disc to rotate through an angle of 4°. Find the coefficient of simple rigidity n for this rod in (*a*) gravitational units; (*b*) absolute units.

Ans. (*a*) 3.42×10^8 g/cm^2; (*b*) 3.35×10^{11} dynes/cm^2.

357. A metal rod, length 4 ft., radius 0.5 in., is clamped at one end and a circular disc attached to the other end. The radius of the disc is 6 in. A force of 100 lb. applied to the rim of the disc causes a twist of 3°. Find the coefficient of rigidity in pounds per square inch.

358. A steel rod 60 in. in length and 0.5 in. in diameter is clamped at one end and has a circular disc of radius 4 in. attached to the other. If the rigidity modulus of this rod (coefficient of rigidity) be 12×10^6 lb. per sq. in., through what angle, in radians, will the disc be turned by a force of 20 lb. applied to the rim of disc?

359. A rod having a length of 10 ft. and a radius of 0.2 in. is clamped at one end, and to the other end is attached a circular disc having a radius of 5 in. To the rim of this disc there is applied a force of 10 lb. If the coefficient of rigidity be 10×10^6 lb per in.^{2}, through what angle in degrees will the disc be turned?

360. The rod of problem 359 is clamped at one end, and suspended in a vertical position. To the lower end there is attached a metal cylinder having a diameter of 1 ft. and a mass of 30 lb. Find the period of torsional vibration of this system.

Ans. $T = 0.5$ sec.

361. A metal cylinder, of radius 10 cm and mass 1 kg, is suspended from a steel wire, of length 1 m and radius 0.07 cm. The period of vibration of the system is 4 sec. Find the coefficient of simple rigidity.

362. Given two perfectly elastic balls ($k = 1$), A and B. The mass of A is 100 g and it is moving in a positive sense (from left to right) with a velocity of 20 cm per sec. Find the velocity of the balls after impact under the following conditions: (*a*) when B is at rest at the instant of impact; (*b*) when B is moving in the opposite sense to A with a velocity of 50 cm per sec.; (*c*) when B is moving in the same sense as A with a velocity of 50 cm per sec.

Ans. (*a*) $u = +13.3$ cm/sec., $u' = +20$ cm/sec.; (*b*) $u = -3.3$ cm/sec., $u' = +20$ cm/sec.; (*c*) $u = +30$ cm/sec., $u' = +20$ cm/sec.

363. Solve problem 362, assuming that both balls are perfectly inelastic, that is $k = 0$.

364. Solve problem 362, assuming that the coefficient of restitution between the two balls is 0.5.

365. An ivory ball falls through a height of 1 m, striking a marble slab. The coefficient of restitution between ivory and marble is 0.8. (*a*) To what height will the ball rebound, neglecting air friction? (*b*) A second ball dropped from the same height (1 m) upon the slab rebounds to a height of 70.6 cm. Find the coefficient of restitution between the ball and the slab.

CHAPTER VI

HEAT

TEMPERATURE MEASUREMENTS

96. Heat and Temperature.—*Heat* is a form of energy which is capable of affecting the physical condition of a body and which is accompanied in general by a change of state or temperature. Heat may be measured in thermal units (calories or B.t.u.), or in units of work (ergs, joules, gram-centimeters, foot-pounds).

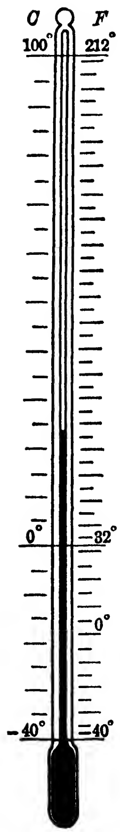

Fig. 76.—Centigrade and Fahrenheit scales.

Temperature is the condition of a body which affects our sensations of warmth and cold. Temperature changes are accompanied by certain physical changes, as for example changes in pressure, volume, electrical resistance, electromotive force. Any one of these changes may be chosen as a basis for temperature measurements. We shall consider, first, temperature measurement based on volume changes, as illustrated by the liquid-in-glass thermometer.

97. Thermometric Substances. *Mercury.*—The limitations of a thermometric substance are in general fixed by its freezing point and its boiling point. Mercury freezes at −38.8°C.; boils at +357°C. under a pressure of 1 atmosphere. It is possible to increase the boiling point of a mercury-in-glass thermometer to 500°C. by increasing the pressure of the gas enclosed within the instrument.

Alcohol.—For measuring temperatures below −38.8°C. alcohol is often used as the thermometric substance. This liquid is usually colored red or blue to render it visible against the glass. The freezing point of alcohol is −130°C.; its boiling point +78°C.

Toluene.—The freezing point of toluene is −80°C.; its boiling point +110°C.

98. Thermometric Scales.—On the *Centigrade scale* the interval between the freezing point and the boiling point is divided into 100 grades or degrees. On the *Fahrenheit scale* the interval between the freezing point and the boiling point is divided into 180°. The zero of this scale is 32° below the freezing point, thus making the interval from the zero of the scale to the boiling point 212°. The relation of the two scales is shown in Fig. 76. Since 100°C. is equivalent to 180°F., we may write,

$$1\text{C. } unit = \tfrac{9}{5}\text{F. } units,$$

and

$$1\text{F. } unit = \tfrac{5}{9}\text{C. } unit.$$

Then

$$\tfrac{9}{5}\text{C.} + 32 = \text{F.},$$

and

$$\tfrac{5}{9}(\text{F.} - 32) = \text{C.}$$

Example 1.—A reading of $-15°$C. is equivalent to what reading on the F. scale? *Solution:* $\frac{9}{5}(-15) + 32 = -27 + 32 = +5°$F.

Example 2.—A reading of $+23°$F. is equivalent to what reading on the C. scale? *Solution:* $\frac{5}{9}(+23 - 32) = \frac{5}{9}(-9) = -5°$C.

Example 3.—A reading of $-13°$F. is equivalent to what reading on the C. scale? *Solution:* $\frac{5}{9}(-13 - 32) = \frac{5}{9}(-45) = -25°$C.

99. Scale Corrections.—Because of expansion or contraction of a thermometer after it has been calibrated it is sometimes necessary to make corrections of the scale readings in order to get the true temperature readings, as illustrated by the following example:

Example.—It was found on test that the freezing-point mark of a given C. thermometer is $-0.2°$, and the boiling point $+100.8°$. What is the true temperature when this thermometer registers 50°, assuming the tube to be uniform in bore? *Solution:* From -0.2 to $+100.8$ as registered on this thermometer = 101 scale parts. Since 101 scale parts = 100°, then 1 scale part = $(100/101)°$. Now a reading of 50° on this thermometer = 50.2 scale parts above the true freezing point. Hence the true reading = $50.2 \times (100/101) = 49.7°$C.

Problems

366. Give the following C. readings their equivalents on the F. scale: (*a*) $+10°$; (*b*) $-10°$; (*c*) $+30°$; (*d*) $-30°$; (*e*) $-40°$.

367. Give the following F. readings their equivalents on the C. scale: (*a*) $+32°$; (*b*) $+77°$; (*c*) $+5°$; (*d*) $-4°$; (*e*) $-40°$.

368. (*a*) Change the following F. readings to C.: $+95°$, $+14°$, $-49°$. (*b*) Change the following C. readings to F.: $+230°$, $-15°$, $-273°$.

369. Change the following C. readings to F.: (*a*) 20° above freezing; (*b*) 20° below freezing.

370. Change the following F. readings to C.: (*a*) 20° above freezing; (*b*) 20° below freezing.

371. Twenty degrees below zero on the F. scale is what reading on the C. scale?

372. A thermometer tube of uniform bore has 18 F. divisions to the centimeter. How many divisions per centimeter would there be if it were graduated to give C. readings?

373. At what temperature is the C. reading twice the F. reading?

374. At what temperature is the F. reading twice the C. reading?

375. At what temperature does the F. thermometer read the same as the C. thermometer?

376. In testing a C. thermometer it was found that the scale readings for the boiling point was $+101°$, and for the freezing point $+0.5°$. What is the true reading when the mercury stands at the 60° mark on this scale?

EXPANSION

100. Coefficients of Expansion.—*Coefficient of linear expansion* is increase in length, per unit length, per degree. That is, $\alpha = l/Lt$ in which α is the coefficient of linear expansion; l is the *change* in length; L is the original length of the body; and t is the *change* in temperature. The relation between the length at 0°C. (L_o) and the length at t degrees is

$$L_t = L_o(1 + \alpha t)$$

The *coefficient of volume expansion* (β) is the increase in volume, per unit volume, per degree. We may write

$$V_t = V_o(1 + \beta t)$$

It may be shown that, as a first approximation, $\beta = 3\alpha$.

NOTES.—1. In cubical, as in linear expansion, there exists no strict proportionality between increase of volume and increase of temperature. This explains why the reading of thermometers filled with different liquids, such as mercury and alcohol, do not exactly agree.

2. If an empty flask be heated, it expands as if it were solid throughout. See problem 383.

3. The coefficient of volume expansion β increases considerably with increase of temperature, and becomes quite large near the boiling point. The equation $V_t = V_o(1 + \beta t)$ should therefore be considered only an approximation, and in the case of liquids, when temperatures are taken near the boiling point, it is better to write $V_t = V_o(1 + \beta t + \beta' t^2)$. In connection with this equation, note problems 384 and 385. For Coefficients of Expansion, see Tables X, XI, XII, Appendix.

Problems

377. A brass wire 5.1 ft. in length at 0°C., elongates 0.1 in. when heated to 90°C. Find the coefficient of linear expansion of brass.

378. A given steel rod increased 1 in. in length when heated from 0 to 200°C. Find the length of the rod at 0°C. See Table X, Appendix.

379. If steel rails, 30 ft. in length, are laid at a temperature of 59°F., how large a gap, in inches, must be left between the ends,

if the highest temperature allowed for be 113°F., assuming the coefficient of linear expansion of steel to be 0.000,011,5 per degree C.

380. A certain distance measured with a steel tape was found to be 1025 ft. The measurement was made when the temperature was 30°C. The tape was correct at 16°C. What was the actual distance measured, the coefficient of expansion of the tape being 0.000,012?

381. A steel bridge of 200 ft. span will change in length by how many inches when the temperature rises from −20°C. to +20°C., the coefficient of linear expansion being 0.000,012?

382. Two similar wires *A* and *B*, at 0°C., length of each 10 m, cross-sectional area 0.01 cm^2, Young's modulus 20×10^8 g per cm^2, coefficient of expansion 12×10^{-6} are caused to elongate, one by a stretching force, and the other by heating. The wire *A* is stretched by a weight of 1 kg. To what temperature must *B* be raised to have an equal length?

383. A flask made of glass having a linear coefficient of expansion of 0.000,008 is calibrated to hold 1000 cc at 0°C. How much will it hold at 100°C.?

384. At 0°C. the volume of a given mass of alcohol is 1000 cc. What will be its volume at 70°C.? See Note 3, Art. 100; also Table XII, Appendix.

385. A given mass of ether has a volume of 250 cc at 0°C. Find its volume at 5° below its boiling point.

386. A certain mass of mercury has a volume of 120 cc at 0°C., and a volume of 121.32 cc at 60°C. Find the coefficient of volume expansion of mercury.

387. A solid displaces 500 cc when immersed in water at 0°C., and displaces 503 cc when immersed in water at 30°C. Find (*a*) the coefficient of cubical expansion of the solid; (*b*) its coefficient of linear expansion.

388. At 0°C. a given sample of iron has a density of 7.5 g per cm^3. Its coefficient of linear expansion is 0.000,012. What will be the density of the iron when heated from 0° to 200°C.?

CHANGE OF VOLUME AND PRESSURE IN GASES

101. Pressure and Volume Coefficients of Gases.—According to the laws of Charles and Gay-Lussac as applying to perfect gases (*a*) for conditions of constant volume, the pressure coefficient = $1/273$ = 0.003,66, and (*b*) for constant pressure the volume coefficient = $1/273$ = 0.003,66; that is,

$$\alpha_p = \alpha_v = 1/273 = 0.003{,}66.$$

102. Absolute Zero.—If the volume of a given mass of gas be kept constant, we may write

$$P_t = P_0(1 + \alpha t),$$

in which P_t = pressure exerted by the gas at a given temperature; P_o = pressure at 0°C.; and α = a coefficient which is called the *coefficient of pressure* for constant volume, and which is numerically equal to the coefficient of expansion for constant pressure. If now we select a temperature t such that the pressure is zero, then $P_t = 0$, and from the equation $P_t = P_o(1 + \alpha t)$ we have $t = -273$°C.; that is

$$\textit{absolute zero} = -273\text{°C.}$$

103. To Change Centigrade and Fahrenheit Readings to Absolute.—Since absolute zero = T = −273°C., and since F. = $\frac{9}{5}$C. + 32, we may write

$$\textit{absolute } T \textit{ on } \text{C. } \textit{scale} = \text{C.} + 273,$$

and

$$\textit{absolute } T \textit{ on } \text{F. } \textit{scale} = \text{F.} + 459.4.$$

Example 1.—What is the reading on the Abs. scale of (*a*) +20°C.? (*b*) −20°C.? *Solution:* (*a*) +20 + 273 = 293° Abs; (*b*) −20 + 273 = 253° Abs. C.

Example 2.—Freezing point on the F. scale is what reading on the Abs. scale? *Solution:* Freezing point F. = +32. Then +32 + 459.4 = 491.4 Abs. F.

104. Relation of Pressure and Volume to Absolute Temperature.—As a deduction from the laws of Charles and Gay-Lussac, we may state that for a perfect gas the product of pressure and volume is proportional to the absolute temperature; that is,

$$\frac{pv}{p'v'} = \frac{T}{T'}.$$

Example.—Under a pressure equivalent to 1 atmosphere (76 cm of mercury) and a temperature of −3°C. the volume of a given mass of gas is 1 liter. Find its volume when the pressure is 72 cm and the temperature is 37°. *Solution:* $T = 270$ and $T' = 310$. Then, since $pv/p'v' = T/T'$ we have $(76 \times 1000)/(72 \times v') = {}^{270}\!/_{310}$ from which $v' = 1211.9$ cc.

105. The General Gas Equation.—Combining Boyle's law with that of Gay-Lussac (law of Charles) and using absolute temperature, we may write

$$pv = nRT,$$

in which R is called the gas constant. The factor R, however, is a constant only under the conditions that p, v, and n are expressed in certain specific units.

R in Metric Units, Centigrade.—R is a constant for any gas when p = pressure in dynes per cm², v = volume in cc, n = number of gram-molecules, and T = absolute temperature C. Under the above conditions

$$R = 83{,}150{,}000 \textit{ ergs/degree Centigrade.}$$

NOTES.—1. R = 83,150,000 = a constant for *any* gas, under the conditions given above.

2. The factor n in the equation $pv = nRT$ is the number of gram-molecules of the gas under pressure. A gram-molecule of a substance is its

molecular weight in grams. For example, a gram-molecule of oxygen = 32 g, and a gram-molecule of nitrogen = 28 g.

3. The total mass of the gas under pressure = $n \times$ *molecular weight.*

4. Under standard conditions of temperature (0°C.) and pressure (1 atmosphere = $76 \times 13.6 \times 980$ = 1,012,900 dynes per cm²), the volume of a gram-molecule of any gas = 22,412 cc.

Example.—The molecular weight of oxygen is 32. A given mass of O enclosed in a vessel having a volume of 50 l exerts a pressure of 75 cm at 27°C. Find (*a*) the number of gram-molecules, and (*b*) the mass of the gas. *Solution:* (*a*) A pressure of 75 cm = $75 \times 13.6 \times 980$ = 999,600 dynes per cm². Then from the equation $pv = nRT$ we have $999{,}600 \times 50 \times 1000 = n \times 83{,}150{,}000 \times 300$, and $n = 2$ gram-molecules; (*b*) mass of O = $2 \times 32 = 64$ g.

R in English Units, Fahrenheit.—In engineering practice the value of R has been determined for a large number of gases in terms of pressure in pounds per square foot, volume in cubic feet, and the F. scale. For example, for air under standard conditions

R = 53.37 *foot-pounds/degree Fahrenheit*

In engineering texts the gas equation is usually written $pv = wRT$, in which p = pressure in pounds per square foot, v = volume in cubic feet, w = weight (mass) of the gas in pounds, and T = absolute temperature F. It should be noted that the value of R given above (R = 53.37) is for air, values for other gases being given in engineering tables.

Example.—A tank contains 10 lb. of air at a temperature of −9.4°F. under a pressure of 200 lb. per sq. in. Find the volume of the air. *Solution:* From $pv = wRT$ we have $200 \times 144 \times v = 10 \times 53.37 \times 450$. Hence v = 8.34 cu. ft.

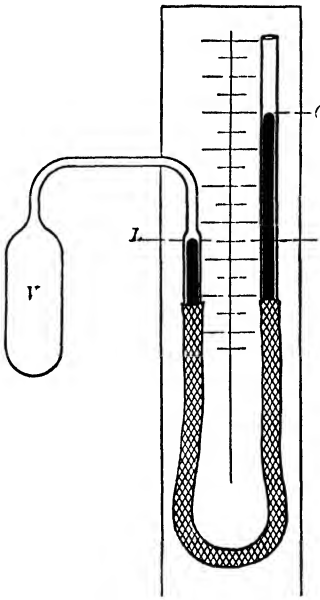

FIG. 77.—Gas thermometer.

106. The Constant Volume Gas Thermometer.—Figure 77 shows in outline the essential parts of a standard hydrogen thermometer. The volume of the gas in V is kept constant by raising or lowering the open tube C, thus keeping the mercury at L at a constant level. With this instrument the temperature is measured in terms of the pressure exerted. Starting with the equation $P = P_0(1 + \alpha\theta)$ we may write

$$\theta = \frac{100(P - P_0)}{(P_{100} - P_0)},$$

where θ = temperature in degrees on the pressure-temperature scale; P = pressure exerted by the gas in V at the temperature θ; P_0 = pressure corresponding to the zero of the scale; that is, when V is placed in melting ice; P_{100} = pressure corresponding to the boiling point of water, under 1

atmosphere's pressure. When P_0 and P_{100} are once fixed for a given instrument, the temperature may be determined by finding the value of P.

Example.—When the bulb of an air thermometer was placed in ice, the mercury column stood at a point 20 mm above the line L (Fig. 77); when placed in boiling water, the pressure was equivalent to 305.6 mm. When the bulb was placed in a given bath, the pressure was 105.6 mm. The barometric reading was 76 cm throughout the entire experiment. Find the temperature θ of the bath. *Solution:* Since the pressure of the atmosphere during the experiment = 760 mm, $P_0 = 20 + 760 = 780$ mm; $P_{100} = 305.6 + 760 = 1065.6$ mm; and $P = 865.6$ mm. Then $\theta = 100(865.6 - 780)/(1065.6 - 780) = 30°$.

Problems

389. Change the following C. readings to Absolute Centigrade: (*a*) +10°; (*b*) −10°; (*c*) freezing point; (*d*) boiling point.

390. Change the following F. readings to Absolute on the F. scale: (*a*) +68; (*b*) −10; (*c*) freezing point; (*d*) boiling point.

391. (*a*) How many C. degrees are there from freezing point to Abs. zero? (*b*) How many F. degrees from freezing point to Abs. zero? (*c*) How many F. degrees from zero F. to Abs. zero?

392. Change 491.4 F. degrees Abs. to (*a*) Fahrenheit; (*b*) Centigrade.

393. Change 373 C. degrees Abs. to (*a*) Centigrade; (*b*) Fahrenheit.

394. Change the following Abs. C. readings to Centigrade: (*a*) 293; (*b*) 233; (*c*) 273.

395. Change the following Abs. F. readings to Fahrenheit: (*a*) 671.4; (*b*) 491.4; (*c*) 459.4.

396. A given mass of gas at a temperature of 0°C., under a pressure of 76°C. occupies a volume of 1 l. Find the volume when the pressure is 72 cm and the temperature 100°C.

397. A mass of gas has a volume of 100 cc at a temperature of −3°C. and pressure of 74 cm. Find the pressure if the volume be kept constant and the temperature be changed to +27°C.

398. When the barometric pressure is 30 in. and the temperature is 32°F., the volume of a given mass of gas is 10 cu. ft. Find the volume when the barometric pressure is 28 in. and the temperature (*a*) −10°F.; (*b*) 10°F. above freezing point.

399. The capacity of a metal tank is 8 cu. ft. The tank is filled with air under standard conditions (p = 14.7 lb./sq. in. and t = 32°F.). Find the pressure in pounds per square inch when the temperature is increased to 212°F.

400. A liter of gas at 100°C. and 76 cm pressure, will exert what pressure if the temperature be raised to 200°C. without changing the volume?

401. A liter of air at 23°C. and 50 cm pressure will have what volume under standard conditions?

402. If the density of air is 1.293 g per l under standard conditions, what density will it have at 80°C. and 60 cm pressure?

403. Find the volume of 5 gram-molecules of a gas at a temperature of 270°C. and under a pressure of 2 atmospheres.

404. The molecular weight of oxygen is 32. If 64 g of oxygen be enclosed in a vessel having a volume of 1000 cc at a temperature of 27°C., what pressure will it exert in (*a*) dynes; (*b*) grams?

405. If 10 lb. of air in a tank at 68°F. exert a pressure of 100 lb. per sq. in., what is the volume of the tank?

406. If 10 lb. of air under a pressure of 100 lb. per sq. in. occupy a volume of 20 cu. ft. what is the temperature?

407. A vessel containing nitrogen (molecular weight 28) has a volume of 50 l. When the temperature is 27°C. the pressure is 70 cm. Find the number of grams of N in the vessel.

408. A volume of 50 cc of hydrogen is collected in a tube over mercury. The mercury in the tube stands 20 cm above that in the bath. The barometer reads 74 cm and the temperature is 27°C. Hydrogen weighs 0.0896 g per l under standard conditions. Find the weight of hydrogen in the tube.

409. A liter of air under standard conditions of pressure and temperature has a mass of 1.293 g. What mass of air will a liter flask contain at −50°C. and a pressure equivalent to 160 cm of mercury?

410. A hydrogen thermometer is used to measure the temperature of the water in a certain tank. Let L (Fig. 77) represent the level of the mercury in the closed tube. Let C be the level of the mercury in the open branch. The volume of air in the bulb V is kept constant. When the bulb V is in melting ice C is 10 cm above L. When V is in steam above boiling water at a pressure of 1 atmosphere, C is 41.5 cm above L. When V is in the water whose temperature is to be found, C is 18.5 cm above L. Find the temperature of the water, the barometric pressure being 76 cm. throughout the experiment.

411. Consider the hydrogen thermometer of problem 410. What will be the height of the mercury column above L when the

bulb V is placed in a vessel containing liquid the temperature of which is (a) $+30°$; (b) $-30°$?

HEAT MEASUREMENTS

107. Heat Units.—The idea of *quantity of heat* involves three factors namely, *mass*, *specific heat*, and *change of temperature*. Quantity of heat may be measured in terms of calories, or in terms of British thermal units (B.t.u.).

A *calorie* is the heat required to raise the temperature of 1 g of water 1°C. When very accurate determinations are required, the 1° is understood to mean from 15° to 16°C.

A *B.t.u.* is the heat required to raise 1 lb. of water 1°F.

108. Specific Heat and Thermal Capacity.—The *specific heat* of a substance is numerically equal to the number of heat units (calories or B.t.u.) required to raise unit mass (gram or pound) 1° (Centigrade or Fahrenheit). The sp. h. of water = 1. The sp. h. of copper, for example, = 0.09; this means that it requires 0.09 cal. to raise 1 g of copper 1°C., or 0.09 B.t.u. to raise 1 lb. of copper 1°F.

The *thermal capacity* of a body is the heat (calories or B.t.u.) required to raise its temperature 1°. The thermal capacity is therefore a quantity which is equal to the product of the mass of the body multiplied by its specific heat, or

$$\textit{thermal capacity} = m \times s$$

where m = mass in grams or pounds, and s = specific heat.

Example.—A copper vessel has a mass of 1050 grams, which is equivalent to 2.315 lb. The sp. h. of copper is 0.09. Find the thermal capacity of this vessel in (a) calories per degree C.; (b) B.t.u. per degree F. *Solution:* $T.C. = m \times s =$ (a) $1050 \times 0.09 = 94.5$ cal./°C.; (b) $2.315 \times 0.09 = 0.208{,}35$ B.t.u./°F.

For specific heats, see Tables XIII and XIV, Appendix.

109. Specific Heat by Method of Mixtures.—Specific heat is usually determined by the so-called method of mixtures. A hot body is dropped into water (or other liquid) contained in a calorimeter. The temperature of the hot body A falls; the temperature of the water B and the calorimeter C rises. We assume that the *heat lost by* A = *heat gained by* $B + C$; that is,

$$mst = m's't' + m''s''t'',$$

where m = mass of the body, s = its specific heat, t = its change of temperature; m', s', and t' = the mass, specific heat, and change of temperature of the water; and m'', s'', t'' = mass, specific heat, and change of temperature of the calorimeter.

Note that t, t', and t'' represent *changes* of temperature.

Example.—Five hundred grams of lead shot at a temperature of 98°C. are poured into 350 g of water contained in an iron vessel having a mass of 300 g. The initial temperature of the water in the calorimeter was 20°C.; its temperature after the addition of the lead shot is 23°C. Find the sp. h. of the lead. *Solution:* From specific heat Table XIII, Appendix, we find that

the sp. h. of iron $s'' = 0.116$. Then $500 \times (98 - 23) \times s = 350 \times (23 - 20) \times 1 + 300(23 - 20) \times 0.116$, from which $s = 0.03$.

110. Heat of Fusion.—The *heat of fusion* of a substance is the heat (calories or B.t.u.) required to change unit mass from a solid to a liquid, without change of temperature.

The *heat of fusion of ice* at the melting point and for 1 atmosphere's pressure = 80 cal./g = 144 B.t.u./lb.

111. Heat of Vaporization.—The *heat of vaporization* of a substance is the heat (calories or B.t.u.) required to change unit mass of the substance at a given temperature from a liquid to a vapor without change of temperature.

The *heat of vaporization of water* at the boiling point for 1 atmosphere's pressure = 538 cal./g = 970 B.t.u./lb.

112. Heat of Combustion.—The *heat of combustion* is the heat (calories or B.t.u.) liberated when unit mass of the substance is burned. For example the H.C. of anthracite coal is about 8000 cal./g = 14,000 B.t.u./lb.

For heats of combustion, see Tables XX and XXI, Appendix.

HEAT ENGINES

113. Mechanical Equivalent of Heat.—The *mechanical equivalent of heat* (Joule's equivalent), usually designated by the letter J, expresses the relation between heat and mechanical work as follows:

$$1 \ cal. = 4.186 \times 10^7 \ ergs,$$
$$1 \ B.t.u. = 778 \ ft.\text{-}lb.$$

Example.—How much energy is expended in changing 20 g of ice at −5°C. to steam at 120°C., the sp. h. of ice being 0.5, and the sp. h. of steam for the given condition being 0.46. Express the result in (*a*) calories; (*b*) ergs. *Solution:* The heat required to bring about the various changes from ice at −5°, to steam at 120° may be summarized as follows:

		Calories
Change from -5 to $0 = 20 \times 5 \times 0.5$	=	50
Change from ice to water at 0° = 20×80	=	1,600
Change from 0° to B. P. = 20×100	=	2,000
Change from water to steam at 100° = 20×538	=	10,760
Change from 100° to 120° = $20 \times 20 \times 0.46$	=	184
Total heat required	=	14,594

This is equivalent to $14{,}594 \times 4.186 \times 10^7$ ergs = 61,090 joules.

114. Efficiency of Heat Engines.—The efficiency of any piece of heat apparatus, whether it be a steam engine, a furnace, or an ordinary gas burner under a kettle, is the ratio of the useful energy gotten out to the total energy put in; that is,

$$efficiency = \frac{useful\ energy\ out}{heat\ energy\ in} = \frac{output}{input}$$

Example.—One cubic foot of gas is burned under a kettle containing a gallon (8 lb.) of water. The temperature of the water is changed from 68 to 110°F. Find the efficiency of the burner and kettle. *Solution:* One cubic

foot of gas gives 600 B.t.u. of heat = input. To change 8 lb. of water from 68°F. to 110°F. (110 − 68 = 42) requires 8 × 42 = 336 B.t.u. = output. Therefore the efficiency = $^{336}/_{600}$ = 0.56 = 56 per cent.

The term "efficiency" as applied to heat engines may be used in several different senses, as, for example, boiler efficiency, engine efficiency, the thermal or fuel-engine efficiency. (*a*) *Boiler efficiency* is the ratio of the heat energy in the steam generated in the boiler to the energy contained in the fuel used. In general, boiler efficiency is from 60 to 80 per cent. (*b*) *Engine efficiency* is the ratio of the mechanical work done on the flywheel shaft to the heat energy delivered by the steam. (*c*) *Fuel-engine* efficiency includes the efficiency reckoned from fuel to drive shaft. In the case of the steam engine, it takes account of both boiler and engine efficiency. For example, suppose that the boiler efficiency of a certain boiler-engine system is 60 per cent, and that the engine efficiency is 18 per cent, then the fuel-engine efficiency is 0.18 × 0.80 = 0.108 = 10.8 per cent. (*d*) Efficiency is also sometimes expressed in pounds of coal required for each horsepower-hour.

115. Carnot's Ideal Engine.—An *ideal engine* is one in which all the processes may be conceived of as being carried on without loss of any sort,

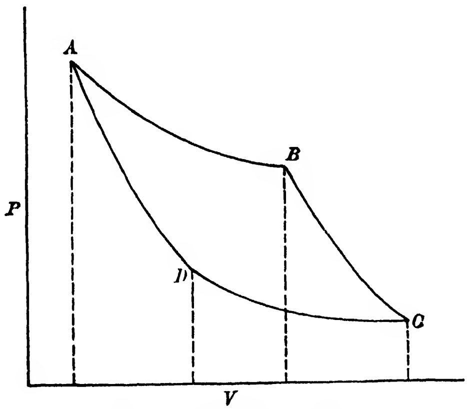

FIG. 78.—Carnot's cycle.

as by radiation, conduction, or any other means whatsoever. No such engine actually exists because, of course, there is always bound to be some heat loss. This ideal engine is called a *Carnot's engine*, from the name of the man (Sadi Carnot, a French physicist and engineer) who first used the idea as the basis for the development of certain important mathematical equations expressing the relationship between heat and work. The cycle shown in Fig. 78 is known as *Carnot's cycle.* This cycle represents an ideal cycle for a perfect engine using a perfect gas. The curves *AB* and *CD* represent *isothermal changes* in pressure and volume; curves *BC* and *DA* represent *adiabatic changes.*

The equation for an isothermal curve, representing Boyle's law, is pv = c = *constant;* the equation for an adiabatic curve is pv c, where γ = 1.41.

During the isothermal expansion AB, H units of heat are absorbed; during the isothermal compression, H' units of heat are given out. During the whole cycle, external work is done equal to $W = ABCD$. This work has been obtained by the transformation of $H - H'$ units of heat; that is, $W = H - H'$.

Carnot showed that for an ideal engine

$$\textit{efficiency} = \frac{T - T'}{T},$$

in which T is the temperature in *absolute* units of the medium (steam, say) received by the engine and T' is the absolute temperature of the medium discharged. No engine can have an efficiency greater than that represented by the fraction $(T - T')/T$.

Example.—A steam engine receives superheated steam from the boiler at a temperature of 200°C. and delivers the spent steam to a condenser at 40°C. Find the efficiency of an ideal engine working between these temperature limits. *Solution:* 200°C. = 473° Abs. and 40°C. = 313° Abs. Then for these temperature limits, *efficiency* $= \frac{473 - 313}{473} = 0.338 = 33.8$ per cent.

In this connection the student should bear in mind two things: the figures just given (33.8 per cent) refer (*a*) to *engine efficiency*, no account being taken of boiler losses, and (*b*) to *ideal* engine efficiency. In actual practice engine efficiencies between the temperature limits given are much lower than 33.8 per cent.

The fuel-engine efficiency of gas and oil engines is relatively high, because the fuel is burned directly in the engine, boiler and transmission losses being thus eliminated.

In the following table are given the approximate thermal (fuel-engine) efficiencies of different types of heat engines.

Thermal Efficiencies

	Per cent		Per cent
Non-condensing engine	9	Steam turbine	26
Condensing engine	17	Gas engine	28
Quadruple expansion engine	24	Oil engine, Diesel	36

116. Conduction.—Heat is transmitted through a body by conduction. The rate at which conduction occurs is represented by

$$H = \frac{kA\tau(t - t')}{l}.$$

When c.g.s. units are employed, H = calories; k = coefficient of conductivity ("conducting power") of the given material; A = area considered, in cm²; $(t - t')$ = difference in temperature (Centigrade) between the surfaces through which conduction takes place; τ = time in seconds; l = thickness of the conducting material in centimeters.

When English units are used, H = B.t.u.; k = conducting power of the medium; A = area in square feet; $(t - t')$ = difference in temperature Fahrenheit; τ = time in hours; l = thickness in inches.

For thermal conductivities, see Tables XXIII and XXIV, Appendix.

Example.—How much heat will be conducted through a plate-glass window, 3 by 4 ft., ¼ in. in thickness, from 7 a.m. to 10 p.m., the temperature inside the room being 20°C. and outside 5°C.? *Solution:* The value of k for glass (Table XXIV, Appendix) is 7; the difference of temperature = 27°F.; and the time = 15 hr. The heat transmitted through the glass, then, is $H = (7 \times 3 \times 4 \times 27 \times 15)/0.25 = 136{,}080$ B.t.u.

Problems

412. A piece of copper, mass 120 g, temperature 100°C., and sp. h. 0.09, is dropped into 240 g of water at 20°C. contained in a metal cup of mass 300 g. The resulting temperature is 23°C. Find (*a*) the specific heat of the cup; (*b*) its thermal capacity.

413. A copper calorimeter weighing 100 g contains 200 g of water at 10°C. 300 g of copper at 100°C. are dropped into the water. Find the final temperature of the water.

414. A piece of copper, mass 100 g, temperature 96°C., is dropped into 240 g of water at 20°C., contained in a zinc vessel of mass 300 g. (*a*) Find the final temperature of the water; (*b*) the thermal capacity of the zinc calorimeter.

415. A piece of lead, mass 340 g, is heated to 90°C. and is then dropped into 300 g of water contained in an aluminum calorimeter. The initial temperature of the water and the calorimeter is 24°C., the final temperature 26°C. Find the mass of the calorimeter. Note.—For specific heat of lead and aluminum see Table XIII, Appendix.

416. Three hundred grams of copper, temperature 100°C., are dropped into 400 g of alcohol, temperature 20°C., contained in an aluminum vessel of mass 100 g. Find the rise in temperature of the alcohol.

417. Find the specific heat of a crystal that is soluble in water but insoluble in turpentine, from the following data: The mass of the crystal is 45 g. It is heated to 80°C. and is then dropped into a copper calorimeter containing turpentine at a temperature of 22°C. Mass of calorimeter, 30 g; mass of turpentine, 131 g; sp. h. of turpentine, 0.43; final temperature of the turpentine, 30°C.

418. Ten grams of ice at −10°C. are changed to steam at 100°C. Assuming that the specific heat of ice is 0.5, how much

heat will be required (*a*) to change the temperature of the ice from -10 to 0°C.; (*b*) to melt the ice; (*c*) to change the temperature of the 10 g of ice water from 0° to 100°C.; (*d*) to change the 10 g of water at 100° to steam at 100°C.?

419. How many B.t.u. will be required to change 10 lb. of ice at 0°F. to steam at 212°F.?

420. Ten pounds of steam at 300°F. are condensed to water, the temperature of which falls to 62°F. Assuming that the specific heat of the steam is 0.46 (Table XIV, Appendix), find the number of B.t.u. liberated.

421. Find the specific heat of a given metal from the following data: 300 g of the metal at 99°C. are dropped into a hole in a block of ice, the temperature of which is 0°C. The hole is immediately covered with another block of dry ice. A total mass of 33.5 g of ice is melted.

422. A mass of 1000 g of copper at 100°C. is placed in a cavity in a block of ice. It remains in the cavity until it comes to the temperature of the ice. How many grams of ice are melted?

423. A piece of metal having a specific heat of 0.1, and a temperature of 90°C. is placed in a cavity in a block of ice at 0°C. If 100 g of ice are melted, what is the mass of the metal?

424. Find the temperature after mixing 10 lb. of water at 100°F., 10 lb. of alcohol (sp. h. 0.6) at 70°F., and 10 lb. of mercury (sp. h. 0.033) at 20°F., neglecting the thermal capacity of the containing vessel.

425. Ten pounds of steam under a pressure of 1 atmosphere and at 212°F. are mixed with 100 lb. of water at 72°F. and 3 lb. of ice at 22°F. Find the resulting temperature of the mixture.

426. How many calories of heat will be required to change 100 g of water at 20°C. to steam at 150°C., the specific heat of steam for the given temperature and pressure being 0.46?

427. How many B.t.u. are required to changed 20 lb. of ice at -8°F. to steam at 300°F., the specific heat of steam for the given conditions being 0.5?

428. Assume the specific heat of ice to be 0.5 and that of steam at constant pressure to be 0.48. Find the result of putting in contact with each other 500 g of ice at -15°C., 400 g of water at 50°C. and 100 g of steam at 120°C. at atmospheric pressure.

429. Find the result of mixing 40 g of ice at 0°C. with 40 g of water at 35°C.

430. A liter of water is heated from 20°C. to the boiling point. Find (*a*) the number of calories consumed; (*b*) the energy required in ergs, (*c*) the energy in joules.

431. If we heat a quart of water (2 lb.) from 52°F. to the B. P., (*a*) how many B.t.u. are consumed? (*b*) How many foot-pounds of energy are put into the water?

432. Find the energy in joules necessary to warm 50 kg of copper through a temperature range of 60°C.

433. Find the energy in foot-pounds necessary to melt 100 lb. of ice and warm the resulting water to 88°F.

434. What horsepower would be required to change ice at 32°F. to water at 212°F. at the rate of 5 lb. per min.?

435. A train of mass 300 tons has a speed of 60 ft. per sec. How much heat in B.t.u. is developed at the brakes when it is stopped?

436. One kilogram of steam at 100°C. in a radiator condenses to water at 100°. How much will the air in a room 5 by 4 by 3 m be warmed, considering the density of air to be 0.001,29 g per cm^3 and its specific heat to be 0.235?

437. The water of Niagara Falls descends about 50 m. Assuming that none of the heat is dissipated, find the increase of temperature per gram due to the fall.

438. Consider the average temperature of a lake to be 12°C. If the temperature falls to 10°C., how much heat would be given up (*a*) in calories per cubic meter; (*b*) in B.t.u. per cubic foot?

439. If a tight vessel which neither absorbs nor transmits heat could be made, what would happen if 300 g of ice at 0° and 50 g of steam at 100° were put into it at the same time?

440. A lead bullet, mass 10 g, strikes a target with a velocity of 500 m per sec. Assuming that 20 per cent of the energy heats the bullet, what will be its rise in temperature?

441. The earth moves in its orbit nearly 19 miles per sec. or about 3,000,000 cm per sec. What is the heat equivalent of the energy of each gram of the earth's mass due to this motion?

442. A given gas on combustion yields 560 B.t.u. per cu. ft. and costs $1 per 1000 cu. ft. Find the cost of heating a gallon of water (8 lb.) from 72°F. to the boiling point, assuming that the burner and kettle have a combined efficiency of 40 per cent.

443. (*a*) How many cubic feet of illuminating gas (Table XXI, Appendix) will have to be burned in order to furnish the same amount of heat (B.t.u.) as that given by 1 ton of Scranton coal?

(*b*) Compare the cost of the two, basing your estimate on the present rates in your town.

444. Analysis of a given grade of coal shows the following composition: C, 80 per cent.; H, 5 per cent.; O, 2 per cent. Find the value in B.t.u. (Table XXI, Appendix) of a ton of this coal, assuming that all the carbon is burned to CO_2 and 95 per cent of the hydrogen is burned to H_2O.

445. If the efficiency of a gas engine is 20 per cent, how many horsepower-hours of work can be developed from the combustion of 1 lb. of gasoline, the heat of combustion being 20,000 B.t.u. per lb.?

446. The explosion temperature in a certain internal combustion engine is 1200°C. and the temperature of exhaust is 700°C. What is the maximum theoretical efficiency of this engine?

447. If the "conducting power" of brickwork be 4.8, how much heat will be conducted through a solid brick wall, 10 by 10 ft., 8 in. thick, in 8 hr., the difference in temperature on the two sides of the wall being 60°F.?

448. Compare the quantity of heat conducted through the walls of a boiler on a winter's day when the temperature is 20°F. and in summer when the temperature is 110°F., assuming that the temperature on the inside of the boiler is the same in both cases.

CHAPTER VII

SOUND

WAVE MOTION

117. Characteristics of Wave Motion.—Energy may be transferred through a medium in two ways, (*a*) by a bodily transfer of matter as in the case of the flowing of a stream of water, and (*b*) by wave motion. Wave motion is the periodic handing on of a disturbance from one point in a medium to another, without transfer of the medium. A *disturbance* is a condition of unstable equilibrium.

The conditions for wave motion are, first, a source of disturbance and, second, a medium capable of handing the disturbance on from point to point. This transfer of energy involves two types of motion, one the forward motion of the wave itself, and the other the periodic motion of the individual particles of the medium.

118. Transverse and Longitudinal Waves.—In general waves are of two kinds, transverse and longitudinal. *Transverse waves* are those in which the motion of the particles is at right angles to the direction of propagation,

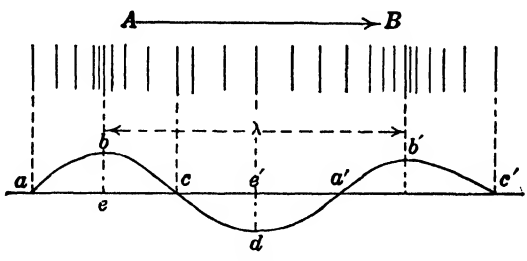

FIG. 79.

lower part of Fig. 79. *Longitudinal waves* consist of condensations and rarefactions, upper part of Fig. 79.

119. Wave Length, Amplitude, and Frequency.—*Wave length* λ is the distance measured from a given point in a wave to the corresponding point in the next wave of the same system, as from a to a', or from b to b' (Fig. 79). *Amplitude* of vibration is measured by one-half the path swept out by the vibrating particle, as be or de' (Fig. 79). *Frequency* n is the number of waves (vibrations) that run by a given point per second. *Period* T is the time of a single vibration, that is, $T = 1/n$.

That portion of a wave which includes a complete vibration, as aa' or cc', represents a *cycle*. Any point in a cycle is spoken of as having a certain *phase*. Phase may be measured in terms of the period T (Art. 31) or in

degrees. For example, a (Fig. 79) is at the zero phase, b is at the 90-deg. phase, c the 180-deg. phase, and so on to a', when the cycle starts over again.

The fundamental equation for wave motion is

$$v = \frac{\lambda}{T} = n\lambda$$

where v = velocity of the wave, λ = wave length, T = period, and n = frequency.

120. Water Waves.—Waves on the surface of water are transverse. Energy may also be transmitted through water (below the surface) by longitudinal waves, as explained in Art. 122. We shall consider here surface water waves. A section of a series of water waves moving in the direction AB is shown in Fig. 80. Such a wave system involves both potential and kinetic energy conditions. For example, the vertical displacements at the crests C and C' and the troughs T and T' constitute the potential energy effects. In front of each crest, at such points as b and b', the water is rising, while behind the crests, at such points as a and a', the water is falling. Also, at crest and trough there is a certain amount of horizontal motion. The motions (transverse and horizontal) at these points constitute the kinetic energy effects. The disturbance in this case consists in the elevation

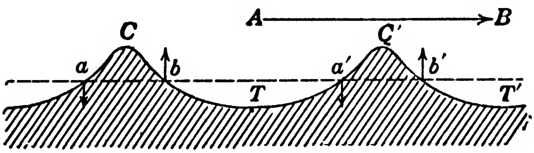

FIG. 80.—Water waves.

of a portion of the surface water above the normal level, and the depression of certain other portions. Gravity tends to restore equilibrium, hence surface water waves are sometimes called *gravitational* waves.

Now there are two forces which tend to restore a surface water wave to its condition of equilibrium, namely, *gravity* and *surface tension*. It may be shown that the velocity of a wave on the surface of water, in which the depth is greater than the wave length, is

$$v = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda D}},$$

where v = velocity of the surface water wave, g = acceleration of gravity, λ = wave length, T = surface tension, and D = density.

An inspection of this equation reveals the fact that if the wave length λ is great, the fraction $2\pi T/\lambda D$ is small compared with $g\lambda/2n$, and hence may be neglected. In this case gravity is the important factor and the equation becomes

$$v = \sqrt{\frac{g\lambda}{2\pi}}.$$

On the other hand, if λ is small, then $2\pi T/\lambda D$ is great compared with $g\lambda/2\pi$, and consequently $g\lambda/2\pi$ may be neglected. This means that surface

tension plays the important part in the propagation of the wave. Short waves of this sort are called *ripples.* When λ is greater than 4 in. or 10 cm the equation $v = \sqrt{g\lambda/2\pi}$ applies, and the wave is called a gravitational wave; when λ is less than 4 in. or 10 cm, $v = \sqrt{2\pi T/\lambda D}$ applies and the wave is a ripple. For wave lengths between these two limits we have to take into account the effect both of gravity and of surface tension. Also, since the velocity due to gravity alone increases as λ *increases,* and that due to surface tension alone increases as λ *decreases,* it follows that there is a certain wave length for which the velocity is a minimum. For water this minimum velocity = 9 in./sec. = 23 cm/sec.

Example.—Taking the density of water as 1 g per cm^3 and its surface tension as 75 dynes per cm, find the speed of a water wave when the wave length is (*a*) π *m*; (*b*) π *cm*; (*c*) π *mm*. *Solution:* (*a*, Since λ is relatively great the factor $2\pi T/\lambda D$ drops out, and we have $v = \sqrt{g\lambda/2\pi} = \sqrt{980 \times \pi \times 100/2\pi} =$ 221.4 cm/sec. (*b*) In this wave-length range the whole equation applies, hence $v = \sqrt{(980 \times \pi/2\pi) + (2\pi \times 75/\pi)} = 25.3$ cm/sec. (*c*) In this case $\lambda = 0.1 \times \pi$ cm, and hence $v = \sqrt{2\pi T/\lambda D} = \sqrt{2\pi \times 75/0.1 \times \pi} =$ 38.7 cm.

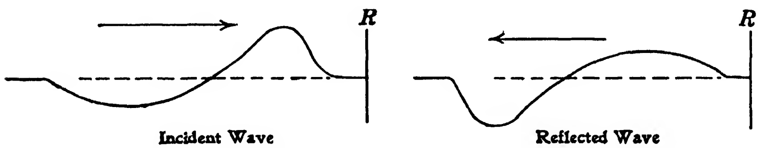

FIG. 81.—Incident and reflected transverse waves.

121. Transverse Waves in a Stretched Cord.—When a disturbance is set up in a flexible stretched cord or string it moves forward as a transverse wave, the particles in front of the wave rising and those behind falling. When the wave is reflected, the crest becomes a trough and the trough a crest (Fig. 81). Note that upon reflection the wave shape is retained, but the wave is turned end for end.

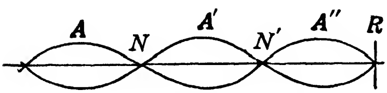

FIG. 82.—Stationary waves.

When a train of incident and reflected waves traverse a card at the same time, a system of *stationary* waves results (Fig. 82). The points of least motion, N and N', are nodes, while the points of maximum transverse motion, A, A', and A'', are antinodes.

The speed of a wave in a cord is

$$v = \sqrt{\frac{F}{m}}$$

where F = stretching force in absolute units (dynes or poundals), and m = mass per unit length in grams or pounds.

Example.—Find the speed of a wave in a flexible cord, the stretching force being 500 g, and the mass per unit length 5 g per cm. *Solution:*

$$v = \sqrt{\frac{980 \times 500}{5}} = 313 \text{ cm/sec.}$$

122. Compressional Waves.—Waves are of two general kinds, transverse and longitudinal or compressional. In those media which possess the property of rigidity (metals for example) the waves may be longitudinal or transverse; in media which do not possess rigidity (as air or water) energy is transmitted only by means of longitudinal waves.

The speed of a wave in an elastic medium is

$$v = \sqrt{\frac{e}{d}},$$

where e is a coefficient of elasticity and d is the density in grams per cm^3. In the case of media possessing rigidity of shape (metals for example) e = coefficient of elasticity (Young's modulus), and for elastic media which do not possess rigidity (fluids) e = coefficient of volume elasticity. Note that e in the equation $v = \sqrt{e/d}$ is expressed in absolute units (dynes per cm. or poundals per sq. in.).

Example 1.—The coefficient of elasticity of copper is about 12×10^{11} dynes per cm^2; and its density 8.8 g per cm^3. Find the velocity of a compressional wave in copper, in centimeters per second and in feet per second. *Solution:* $v = \sqrt{\frac{e}{d}} = \sqrt{\frac{12 \times 10^{11}}{8.8}} = 369{,}300$ cm/sec. $= \frac{369{,}300}{30.48} =$ 12,116 ft./sec.

Example 2.—The coefficient of elasticity of copper is 17.4×10^6 lb. per in.^{2}; its specific gravity is 8.8. Find the velocity of sound in this metal in feet per second. *Solution:* Since in the equation $v = \sqrt{e/d}$, e is expressed in absolute units, and since we wish to find the velocity in feet per second, it will be necessary to reduce 17.4×10^6 lb./sq. in. to absolute units (poundals/ft.^{2}). Now 17.4×10^6 lb./sq. in. $= 17.4 \times 10^6 \times 32 \times 144 = 8064 \times 10^6$ poundals/sq. ft. Also, a specific gravity of $8.8 = 62.5 \times 8.8 = 550$ lb./cu. ft. Then $v = \sqrt{\frac{e}{d}} = \sqrt{\frac{8064 \times 10^6}{550}} = 12{,}100$ ft./sec. Note how closely this value corresponds with that obtained in Example 1.

Example 3.—The coefficient of volume elasticity of water is 0.22×10^{11} dynes per cm^2. Find the velocity of a compressional wave in water, taking the density as 1 g per cm^3. *Solution:* $v = \sqrt{\frac{0.22 \times 10^{11}}{1}} = 148{,}000$ cm/sec.

Problems

449. What must be the value of λ in order that the velocity of a surface water shall be equal to that of an ocean liner, the speed of which is 25 knots per hour? A knot (nautical mile) = 6080 ft.

450. The speed of ripple waves on the surface of mercury, density 13.6 g per cm^3 and surface tension 541 dynes per cm, is 50 cm per sec. Find the wave length.

451. A flexible cord, 20 ft. in length, is stretched with a force of 5 lb. The weight of the cord is 0.5 lb. Find the speed of a wave in this cord.

452. Find the velocity of a compressional wave in an aluminum rod, in centimeter per second.

453. Find the velocity of a compressional wave in brass, in feet per second.

TRANSMISSION OF SOUND

123. Sound Waves.—Physically speaking, sound is that form of vibratory motion which may be perceived by the ear. All sound originates in vibrating bodies. Sound requires for its transmission a medium which is continuous, ponderable (weighable), and elastic. Sound is transmitted from point to point by means of *longitudinal* waves.

The velocity of sound may be computed by means of the equation

$$v = \sqrt{\frac{e}{d}},$$

in which v = velocity of sound; e = coefficient of elasticity of the medium in absolute units, and d = density of the medium. In the case of solids possessing rigidity, such as metals, glass, etc., e = Young's modulus; in liquids e = coefficient of volume elasticity; in gases, as air for example, e = *adiabatic elasticity* = $1.41p$, as explained in Art. 124.

124. Newton's Equation, and Laplace's Correction.—Newton derived the equation

$$v = \sqrt{\frac{p}{d}},$$

in which v = velocity of sound in air; p = pressure of the atmosphere; and d = density of the air. This equation, however, gave results which were only about 80 per cent of the velocity of sound as determined by experiment. Laplace added the correcting factor 1.41 to take account of the coefficient of adiabatic expansion. The equation thus corrected, becomes

$$v = \sqrt{\frac{1.41p}{d}}.$$

125. Correction for Temperature.—The velocity of sound in air at 0°C. is

$$V_o = 1090\ ft./sec. = 332\ m/sec.$$

and for any temperature t,

$$V_t = V_o\sqrt{1 + 0.003{,}665 \times t}.$$

A change in temperature of 1°C. causes a corresponding change in the velocity of sound in air (increase or decrease) = 2 ft./degree = 0.6 m/degree.

Problems

454. Find the velocity of sound in air in feet and meters when the temperature is (*a*) +10°C.; (*b*) −10°C.

455. How far will sound travel in air in ½ min. when the temperature is 68°F.?

456. A given mass of air is contained in a rigid vessel of constant volume. By means of a force pump the mass of air in the tank is doubled. (*a*) How is p within the tank affected? (*b*) How is e affected; (*c*) the density d? (*d*) How is the velocity of sound affected?

457. The air in the tank (problem 456) is heated. How will this affect the velocity of sound, and why?

458. Find the velocity of sound in air when the pressure is 72 cm, and the density is 0.001,28 gram per cm^3.

459. A person approaching a large building at night stamps his foot on the pavement, and 0.8 sec. later hears the echo. How far is the person from the building, assuming the temperature to be 68°F.?

460. Two large buildings are 346.4 ft. apart. The air is still and the temperature is 24°C. A pistol is fired at a distance of 118.8 ft. from one of the buildings and 227.6 ft. from the other. (*a*) How long before the man who fired the pistol will hear an echo? (*b*) When will he hear each of the next three?

461. A cliff 460 ft. distant returns an echo to an observer in 0.8 sec. Find the temperature.

462. The density of air is about 14.4 times that of hydrogen. Find the velocity of sound in hydrogen at 0°C., the pressure in both cases being the same.

463. A given medium has a coefficient of elasticity of 1225×10^6 g per cm^2. The velocity of sound in this medium is 3700 m per sec. Find the density of the medium.

464. If the velocity of sound in given sample of steel, density 7.8, be 5000 m per sec., what is the coefficient of elasticity e in dynes per cm^2? How does this value compare with the value for Young's modulus for steel?

465. Calladon and Sturm found the velocity of sound in water to be 1435 m per sec. From these data find the coefficient of elasticity of water in dynes per square centimeter.

466. On a given day when the temperature is -5°C., the barometer reads 74 cm. The density of the air is 0.001,285. Find the velocity of sound, by two methods, and compare the two results obtained.

467. How does the coefficient of elasticity e, and the density d of water compare with that of air? How do you account for the fact that the velocity of sound in water is about five times as great as in air?

WAVE LENGTH, VELOCITY, AND FREQUENCY

126. Wave Length of Sound.—Sound is transmitted by means of longitudinal (compressional) waves. The relation between wave length, velocity, period, and frequency may be represented by the equation

$$\lambda = vT = \frac{v}{n},$$

in which λ = wave length; v = velocity of sound in air at a given temperature; T = period of vibration; n = frequency. It is important to bear in mind that the frequency is the reciprocal of the period; that is, $n = 1/T$.

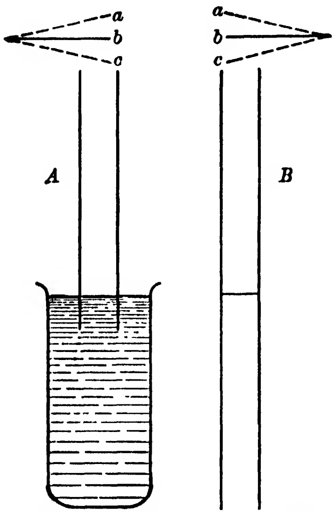

FIG. 83.—Closed and open resonance tubes.

127. Resonance.—If a tuning fork be held above a resonance tube of such a length that the period of the vibrating air column is the same as that of the fork, the two (fork and tube) are said to be in resonance.

A *closed resonance* tube (Fig. 83, *A*) sounding its fundamental, represents one-fourth of the resulting wave length; an *open resonance tube* (Fig. 83, *B*) sounding its fundamental, represents one-half of the resulting wave length. That is,

closed resonance tube = ¼ *wave length,*
open resonance tube = ½ *wave length.*

Example.—A closed resonance tube 10.5 in. in length responds to a given fork, the temperature of the air being 26°C. Find the vibration rate (frequency) of the fork. *Solution:* Wave length $\lambda = 4 \times 10.5$ in. = 3.5 ft.; velocity v at 26°C. = $1090 + 26 \times 2 = 1142$ ft. per sec. Then $\lambda = v/n$, and hence $n = 1142/3.5 = 326$ vibrations/sec.

128. Kundt's Experiment.—Kundt's apparatus (Fig. 84) furnishes a method for measuring the velocity of sound in metals. The metal rod *R*

FIG. 84.—Kundt's apparatus.

is clamped at the middle point. The length of this rod is one-half the wave length λ_m of the sound in the metal. The distance from one dust heap to the next in the tube is likewise one-half the wave length of the sound wave in air; that is, the distance between alternate dust heaps is λ_a. The relation of the velocities of the sound in the two media to the corresponding wave lengths is

$$\frac{V_m}{V_a} = \frac{\lambda_m}{\lambda_a}$$

where V_m = velocity of sound in the metal; V_a = velocity of sound in air; λ_m = the wave length in the metal; λ_a = wave length in air.

Example.—In an experiment with Kundt's apparatus the following data were obtained: Metal rod 1 m in length and clamped in the middle. Density of rod, 10 g per cm³. Average distance between dust piles, 10 cm. Temperature at time of experiment, 22°C. Find (*a*) the velocity of sound in the rod; (*b*) the coefficient of elasticity (Young's modulus) of the metal. *Solution:* (*a*) V_a at 22°C. = 332 + (0.6 × 22) = 345.2 m/sec. = 34,520 cm/sec. Wave length $\lambda_m = 2m$ = 200 cm, and λ_a = 20 cm. Then $V_m/34{,}520 = {}^{200}\!/_{20}$. Hence V_m = 345,200 cm/sec. (*c*) $354{,}200 = \sqrt{e/10}$. Hence $e = 12.5 \times 10^{11}$ dynes/cm².

129. Reenforcement and Interference of Sound.—Two sound waves of the same phase, amplitude, and wave length reenforce each other at every point. Two sound waves of the same amplitude and wave length but of

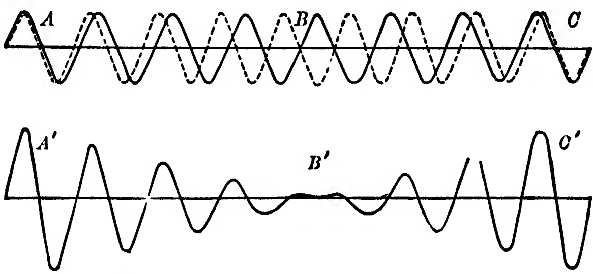

FIG. 85.—Interference of sound waves.

opposite phase may annul each other at every point. Two sound-wave systems, the waves of which are of different lengths, may alternately reenforce each other and interfere with each other, as shown in Fig. 85. The alternate rise and fall in the intensity of sound due to reenforcements and interference give rise to *beats*.

The *number of beats* occurring per second is equal to the *difference in frequency* of the two sounding bodies.

Problems

468. A sounding body makes 100 vibrations per sec. Find the wave length of the disturbance in air at (*a*) + 20°C.; (*b*) −20°C.

469. A tuning fork gives off waves in air 130 cm in length, at 0°C. Find (*a*) the frequency of the fork; (*b*) its period.

470. A string makes 256 complete vibrations per sec. when the velocity of sound is 346 m per sec. Find the wave length.

471. A tuning fork makes 1024 vibrations per sec. and the length of the sound wave given off is 32 cm. Find the velocity of the sound.

472. A cylindrical glass tube is placed vertically in water and its length is adjusted until it responds to a fork making 256 vibrations per sec. when the temperature is 20°C. Find the length of the resonance tube in feet.

473. A closed resonance tube, 11.4 in. in length, responds to a fork making 300 vibrations per sec. Find the temperature.

474. An open resonance tube responds to a fork making 341 vibrations per sec. when the temperature is 15°C. Find (*a*) the wave length of the sound in air; (*b*) the length of the resonance tube.

475. Find the velocity of sound in brass from the following data: A brass rod used to excite the air in a Kundt's tube is 1 m long. The dust heaps which it produces in the tube are 99 mm apart. The temperature is 15°C.

476. The density of the rod (problem 475) is 8.4 g per cm^3. Find the velocity of sound in the rod, Young's modulus for brass being 10×10^{11} dynes per cm^2.

477. The velocity of sound in aluminum is 5100 m per sec. Find the coefficient of elasticity of aluminum, its density being 2.6 g per cm^3.

478. A metal rod of length 125 cm, and density 7 g/cm^3, is clamped at the middle. When struck on the end it gives a note of 1200 vibrations per sec. Find its coefficient of elasticity.

479. Two sounds are produced, making 100 and 120 vibrations per sec., respectively. Both sound wave trains travel with the same velocity. (*a*) When the first sound makes one vibration, the second sound makes how many? (*b*) What fraction of a wave length does the second gain over the first per vibration? (*c*) In what time will the second gain a whole wave length on the first? (*d*) How many times will this occur per second? (*e*) How many beats will occur per second?

INTENSITY, PITCH, AND QUALITY

130. Intensity.—The *intensity* or loudness of sound varies (*a*) with the area of the sounding body; (*b*) the density of the medium; (*c*) directly as the square of the amplitude of vibration; and (*d*) inversely as the square of the distance from the source.

131. Pitch.—*Pitch* is that characteristic of sound which is determined by the number of vibrations per second. Pitch may be measured by means of a siren.

When an observer moves toward or away from a stationary sounding body or when a sounding body moves toward or away from the observer, there occurs an apparent change in the pitch due to the fact that there is a

change (increase or decrease) in the number of waves falling upon the ear per second. This apparent change in pitch is known as the *Doppler effect*. The apparent pitch n' may be expressed in terms of the true frequency, the velocity of sound at the given temperature, and the velocity of the moving body as follows:

a. Sounding Body Stationary.—When the observer is moving toward the sounding body

$$n' = \frac{n(V + v)}{V},$$

and when he is moving away from the sounding body

$$n' = \frac{n(V - v)}{V},$$

where n' = apparent pitch, n = true pitch, V = velocity of sound, and v = velocity of moving body.

b. Observer Stationary.—When the sounding body is moving toward the observer

$$n' = \frac{nV}{V - v},$$

and when the sounding body is moving away

$$n' = \frac{nV}{V + v}.$$

132. Musical Scales.—A diatonic scale consists of a series of eight notes having definite ratios.

Name	DO	RE	MI	FA	SOL	LA	TI	DO
Letter	C	D	E	F	G	A	B	C'
Ratio	1	9/8	5/4	4/3	3/2	5/3	15/8	2
Interval		9/8	10/9	16/15	9/8	10/9	9/8	16/15

FIG. 86.—Diatonic scale.

The *major diatonic scale* in the key of C is derived from the three major triads, having ratios 4:5:6, as follows:

$$\left.\begin{array}{l} C:E:G \\ G:B:D' \\ F:A:C' \end{array}\right\} = 4:5:6$$

Starting with C as a keynote, and assigning to it a value of 256 vibrations per sec., and using the ratios of the major chord, we derive the major scale (Fig. 86) as follows:

	C	D	E	F	G	A	B	C
Key of C.......	256	288	320	341	384	427	480	512

The *minor diatonic scale* is derived from the minor triads, the ratios being 10:12:15

Transposition.—A scale having C as the keynote is sometimes called the natural scale. In order to accommodate different voices and instruments, it is frequently desirable to change the keynote of the scale from C to some other note, as for example, D, F, or G. In order to write the scale in any key we have only to select the vibration number corresponding to that letter, and to multiply this number successively by the fraction $\frac{9}{8}$, $\frac{5}{4}$, $\frac{4}{3}$, etc.

Example.—Suppose that we wish to write the scale in the key of D. We select D from the diatonic scale as our keynote, its vibration number being 288. To get E we multiply 288 by $\frac{9}{8} = 320$; in a similar manner $F = 288 \times \frac{5}{4} = 360$; $G = 288 \times \frac{4}{3} = 438$, and so on.

In physics we assign to middle C 256 vibrations per sec. In music, however, the standard of pitch commonly employed is that which assigns to A 435 vibrations per sec.; this is called the *international standard* of pitch.

133. Tempered Scales.—In order to produce music in different keys on instruments having fixed keyboards, such as the piano and organ, it is necessary to determine upon some arbitrary ratio from note to note. In fixing the ratios from note to note on the piano, musicians have agreed to adopt a system known as that of *equal temperament;* that is, the ratio between all notes is equal. The ratio number selected is $\sqrt[12]{2} = 1.059$.

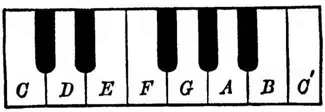

FIG. 87.—Thirteen keys comprising octave on piano.

On a piano there are 13 notes from C to C′, including eight white keys and five black keys. The black keys represent notes called sharps and flats. A sharp is a note having a vibration number higher than that of a given note; a flat is a note having a vibration number lower than that of a given note. Thus the first black key above C (Fig. 87) is the sharp of C and the flat of D. Taking the international pitch of A as 435 vibrations per sec., then C = 258.7; the sharp of C (first black key) = $258.7 \times 1.059 = 274.1$; $D = 274.1 \times 1.059 = 290.3$, and so on.

A tempered scale such as that of the piano is sometimes called a *chromatic scale.*

134. Pitch of Strings.—A string may vibrate as a whole or in parts (segments). The fundamental tone is that given by a string vibrating as a whole; it is the tone of lowest pitch. Overtones are given off by a string vibrating in parts. Harmonics are overtones whose frequencies are exact multiples of the fundamental.

A node in a string is a point of minimum motion.

The relation of the frequency n to the length, stretching force, and density is represented by the equation

$$n = \frac{1}{2L}\sqrt{\frac{F}{m}},$$

in which n = number of vibrations per second; L = length in centimeters; F = force in dynes; and m = mass of the string per unit length.

Example.—A string 1 m in length, and having a mass of 2 g, is stretched by a force of 2 kg. Find the frequency of the string when it is sounding its

fundamental. *Solution:* $L = 100$ cm; $F = 2000 \times 980$ dynes; $m = {}^2/_{100}$ g/cm. Then $n = {}^1/_{200}\sqrt{196{,}000{,}000/2} = 49$ vibrations/sec.

135. Pitch in Pipes.—The length of a *closed* pipe sounding its fundamental is one-fourth the wave length of the sound emitted; the length of an *open* pipe is one-half the wave length.

A node in a pipe is the pointof *minimum motion* and *maximum change of density.*

A closed pipe is capable of producing only those overtones which correspond to odd multiples of the fundamental; that is, 3, 5, 7, etc. An open pipe is capable of producing all the overtones; that is, 2, 3, 4, 5, and so on.

Laws of Pitch.—(*a*) The pitch of a pipe varies inversely as its length. (*b*) The pitch of an open pipe is an octave higher than that of a closed pipe of the same length.

136. Quality of Sound.—Sounds may differ in three respects, namely, in intensity, in pitch, and in quality. Intensity or loudness depends upon the amplitude of vibration; pitch depends upon the frequency (number of vibrations per second); and quality depends upon the form of the wave, that is, upon the number and character of overtones present. In Fig. 88, there are shown two vibrating systems which have the same amplitude of vibration and frequency but different wave forms. The sounds emitted by these have the same loudness and pitch. The quality of the lower system, however, is quite different from that of the upper.

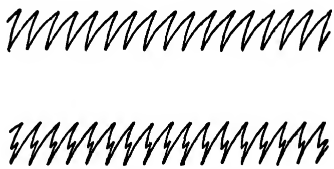

FIG. 88.—Two sound waves of the same pitch and loudness, but of different quality.

ACOUSTICS OF AUDITORIUMS

137. Reflection and Reverberation.—The problem of the acoustical properties of auditoriums is one of great importance. In order that an auditorium may possess satisfactory acoustical properties, it is necessary (*a*) that the sound reaching each individual member of the audience shall be sufficiently loud; (*b*) that the various components of a complex sound shall maintain their proper relative intensities; and (*c*) that the successive sounds in rapidly shifting articulation, as in speaking or in the production of music, shall be clear and distinct. The usual acoustical defects in auditoriums are three, namely, echo, dead points, and reverberation.

The confused sound which comes from the prolongation of a series of sounds resulting from reflections is called a *reverberation.* A reverberation is different from an echo in that in the case of an echo the original sound and the reflected sound (the echo) constitute two distinct impressions; a reverberation, on the other hand, is a prolonged confusion of sounds resulting from repeated reflections. For ordinary room temperature, an echo requires that the reflecting surface (wall or ceiling) shall be at least 55 ft. from the source. Reverberations may arise, and indeed do arise, from reflections from surfaces which are much less than 55 ft. from the source, as is illustrated by the confused sounds resulting from reflections which sometimes occur in auditoriums.

The *period* or *time of reverberation* T is the number of seconds required for the reverberation to die out.

The *coefficient of absorption* of a given substance is 1 divided by the number of square feet of the substance required to produce the same absorbing effect as 1 sq. ft. of open window. The following is a list of the coefficients of absorption of different absorbing materials which are used in the construction and furnishing of auditoriums.

COEFFICIENTS OF ABSORPTION OF SOUND

Absorber	Coefficient
Open window	1 00 unit per square foot
Hair felt, 1 in. thick	0.55 unit per square foot
Stage opening	0 25 unit per square foot
Curtains	0.23 unit per square foot
Carpets	0.20 unit per square foot
Wood	0 061 unit per square foot
Plaster on lath	0 033 unit per square foot
Cork floor tile	0 030 unit per square foot
Brick wall	0.025 unit per square foot
Audience	4 7 units per person
Seats, upholstered	2 0 units per seat
Seats, plain	0.1 unit per seat

The *absorbing power* of a given area is the product of the area in question and its coefficient of absorption. For example, the absorbing power of 10 sq. ft. of hair felt $= a = 10 \times 0.55 = 5.5$ units of absorption.

The problem of controlling the reverberation time is an important one. Sabine worked out the following equation which is now universally employed in preventing and correcting acoustical reverberation defects:

$$T = 0.05\frac{V}{A},$$

where T = reverberation time or period of decay in seconds, V = volume of the auditorium in cubic feet, and A = total absorbing power of the interior of the room, that is, $A = a_1 + a_2 + a_3 + a_4$, etc.

Experience with a number of auditoriums of acceptable acoustic properties makes possible the formulation of the following values of reverberation times in seconds for rooms of different volumes, for a maximum audience in each case:

V Cubic feet	T Seconds
10,000	0.6 to 0.8
25,000	0.8 to 1 1
50,000	0.9 to 1.2
100,000	1.2 to 1.5
200,000	1.4 to 1 7
400,000	1.7 to 2.0
600,000	1.8 to 2 2
800,000	1.9 to 2 3

Example 1.—The volume of a given auditorium is 100,000 cu. ft. The absorbing areas are as follows: floor area, covered with cork tile, 4550 sq. ft.; ceiling, plaster on lath, 4550 sq. ft.; wall area, brick, 2860 sq. ft.; stage opening, 600 sq. ft.; 500 plain wooden seats. Find the toal absorption of this auditorium when empty. *Solution:* Referring to the areas given above and the coefficients of absorption given in the table, we have as approximate values

$$
\begin{aligned}
a_1 &= \text{floor absorption} &&= 4550 \times 0.03 &&= 137 \\
a_2 &= \text{ceiling} &&= 4550 \times 0\ 033 &&= 152 \\
a_3 &= \text{brick wall} &&= 2860 \times 0.025 &&= 72 \\
a_4 &= \text{stage} &&= 600 \times 0.25 &&= 150 \\
a_5 &= \text{seats} &&= 500 \times 0.1 &&= 50 \\
& && \text{Total absorption } A &&= 561
\end{aligned}
$$

Example 2.—Compute the reverberation time T for the auditorium described in Example 1, for a full audience of 500 persons. *Solution:* The total absorption A for the auditorium empty was found to be 561 absorption units. To get the total absorption of the hall when filled with people we shall have to add to the above value the difference between the absorption due to 500 people, which is $500 \times 4.7 = 2350$, and the absorption due to 500 empty seats, as previously computed, giving us a net increased absorption of $2350 - 50 = 2300$ units. The total absorption for a full audience, then, is $A = 561 + 2300 = 2861$. The reverberation time is

$$T = 0.05\frac{V}{A} = \frac{0.05 \times 100{,}000}{2861} = 1.7 \text{ sec.}$$

Problems

480. A siren is set so that its pitch is in unison with a given fork. The number of holes in the siren disc is 36, and it makes 90 revolutions every 10 sec. Find the frequency of the fork.

481. A current of air is blown against the disc of a siren having a row of 30 holes, while the disc is making 3000 r.p.m. (*a*) What is the pitch of the resulting tone? (*b*) If the speed of the siren be doubled how will the pitch be affected?

482. A whistle makes 500 vibrations per sec. when the temperature is 15°C. (*a*) What is the velocity of the sound at this temperature? (*b*) How many sound waves will occur between the whistle and an observer 1120 ft. distant? How many vibrations fall upon his ear (*c*) when he is standing still; (*d*) when moving toward the whistle at the rate of 40 ft. per sec.; (*e*) when moving away from the whistle at the rate of 40 ft. per sec.?

483. The whistle on a train has a frequency of 500 vibrations per sec. The train is moving with a velocity of 30 miles per

hr. The temperature of the air is 20°C. Find the apparent pitch as the train approaches an observer.

484. The vibration frequency of a locomotive whistle is 760 per sec. The velocity of the train is 60 ft. per sec.; the temperature of the air 25°C. What is the vibration frequency of the sound heard by an observer (*a*) on the train; (*b*) on the track ahead of the train; (*c*) on the track behind the train?

485. A locomotive approaches a man standing near the track. The vibration frequency of the bell seems to him to be that of high C (512 vibrations per sec.). After the train has passed him, the pitch seems to be that of A (426.6 vibrations per sec.). The speed of the train is constant and the temperature is 0°C. Find the speed of the train.

486. Middle C is assumed to consist of 256 vibrations per sec. (*a*) Compute the vibration frequency of each of the other three notes of the major chord. (*b*) Compute the vibration frequency of each of the other three notes of the minor chord.

487. Beginning with 288 vibrations per sec., write a scale in the key of D, from D to D' inclusive.

488. If A on the piano has a frequency of 435 vibrations, find the frequency of (*a*) the next black key above A; (*b*) the first black key below.

489. The tones of three forks form a major triad. The middle fork gives a note of 330 vibrations per sec. Find the vibration rate of the two other forks.

490. How will the pitch of a string be affected (*a*) if its length be doubled; (*b*) if its tension be quadrupled; (*c*) if the mass per unit length be increased ninefold?

491. A given string stretched by a force of 1 lb. makes 200 vibrations per sec. (*a*) How will the frequency be affected if the stretching force is increased to 4 lb.? (*b*) How will the pitch be affected?

492. An aluminum wire 1 m in length, and 1 mm in diameter, is stretched by a force of 4 kg. Find the pitch of its fundamental.

493. A string 100 cm long produces middle C as its fundamental tone. A bridge is placed under the string, and its position adjusted until the string produces E of the same octave. (*a*) Where is the bridge placed? (*b*) Where should it be placed to produce the other two notes of the major chord?

494. If the stretching force upon a certain string is 500 g, and it sounds middle C, what will be the force necessary to tune it (*a*) to D of the natural scale; (*b*) to D of the piano scale?

495. A wire 3 ft. long and stretched with a force of 9 lb. makes 300 vibrations per sec. Another wire of the same material and cross-sectional area is 4 ft. long and is stretched with a force of 512 poundals. Find the vibration frequency of the second wire.

496. A wire, 50 cm in length, stretched with a force of 1 kg, has a frequency of 100 as its fundamental tone. What is the weight of the wire in grams?

497. Two wires of the same material and the same size have the same pitch. One is stretched with a force of 4900 g and is 80 cm long; the other is stretched with a force of 8100 g. How long is the second wire?

498. (*a*) What is the relation of the length of a pipe to its pitch? (*b*) What is the relation of the pitch of an open pipe to that of a closed pipe? (*c*) An open pipe of given length is sounding its fundamental. Suppose that a person stops one end by means of a card. How will the pitch be affected?

499. Determine the length of an open organ pipe that is in unison with E above middle C of the piano when the temperature of the air is 25°C. End corrections are not to be considered in this problem.

500. What is the vibration frequency of an open organ pipe 32 ft. long when the temperature is 24°C.? What effect would be produced upon the ear by the waves from a closed pipe of the same length?

501. A long glass tube 5 cm in diameter is so arranged that water may be forced in at the bottom, thus varying the length of the air column above the water. With an E fork (320 vibrations per sec.), strong resonance occurs when the air column is 27.1 cm long. From these data determine the temperature at the time of the experiment.

502. Five square feet of a certain material on the floor of an auditorium has the same absorbing power as 1 sq. ft. of open window. (*a*) What is the coefficient of absorption of this material? (*b*) What is the material?

503. In an auditorium free from echoes and dead points it was found that for full audience the reverberation time was 1.5 sec. The volume of the room was 90,000 cu. ft. How many

square feet of hair felt would have to be placed on the walls in order to make the acoustic properties satisfactory ($T = 1.2$) for a full audience?

504. The total absorption, including that of a full audience of 500 persons, of an auditorium of volume V is 3000 units. The period of reverberation is 1.5 sec. What is the value of T when this auditorium is empty, the seats being upholstered?

CHAPTER VIII

MAGNETISM AND ELECTROSTATICS

MAGNETISM

138. Magnetic Poles.—For convenience we shall speak of the north-seeking pole of a magnet as the N-pole, or N; the south-seeking pole, as the S-pole, or S.

Like poles repel; unlike poles attract.

A magnetic *pole* of *unit strength* is one which at a distance of 1 cm in a vacuum repels an equal and similar pole with a force of 1 dyne.

139. Coulomb's Law.—The magnitude and sense of the force of attraction or repulsion between two poles is given by Coulomb's law, which may be expressed in equational form as,

$$F = \pm \frac{mm'}{\mu d^2},$$

in which F = force in dynes; m and m' = the respective pole strengths; d = distance in centimeters between the poles m and m'; and μ = permeability of the medium. The sign + indicates repulsion between the poles m and m'; the sign $-$, attraction.

140. Permeability.—The *permeability* of a medium is that property which modifies the action of magnetic poles placed in the medium.

Permeability (μ) varies greatly with different media, and for a given medium, such as iron, it varies also with intensity of the magnetizing field. For a vacuum $\mu = 1$; for air at 20°C. under a pressure of 1 atmosphere, μ = 1.000,005. In the case of iron the permeability may be as high as 2000 and over.

141. Intensity of Magnetic Field.—The intensity of a magnetic field at a point may be measured by the force which it exerts on a unit pole at that point. In other words, magnetic field intensity H is force per unit pole; that is $H = F$ per unit pole $= +\frac{(m \times 1)}{\mu d^2}$, or the field intensity at a point p with reference to the pole m is

$$H = \pm \frac{m}{\mu d^2},$$

where d is the distance of the unit pole from the pole m or m'.

The *unit of field intensity* is the gauss. A *gauss* is a field intensity of 1 dyne per unit pole.

Magnetic field intensity is a vector quantity, having the magnitude, direction, and sense of the force acting on unit positive pole; the signs + and $-$, therefore, have directional significance.

Example.—Two magnetic poles, $m = +200$ units and $m' = -200$ units, are 16 cm apart. Find the magnitude, direction, and sense of the intensity

of the field at a point p, 10 cm from each (Fig. 89). *Solution:* In order to find the intensity of the field H, consider that a positive unit pole ($p = +1$) is placed at the point p 10 cm from m and m', respectively. Cos $\theta = 0.8$. The magnetic intensity of p with respect to m is $H = +\frac{200}{10^2} = +2$ dynes per unit pole, represented in direction and sense by the line pa. The + sign indicates repulsion between m and p. Likewise the intensity of field with

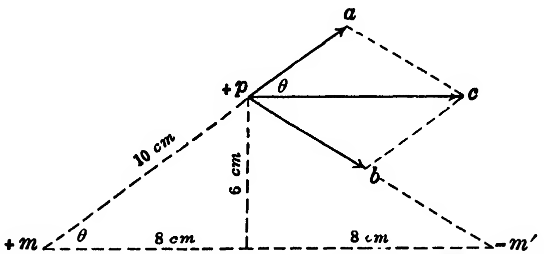

FIG. 89.—Magnitude, direction, and sense of magnetic field.

respect to m' is $H = -\frac{200}{10^2} = -2$ dynes per unit pole, represented by the line pb. The − sign in this case indicates attraction. The resultant of the two vector quantities pa and pb is $pc = 2 \times 2 \cos \theta = 2 \times 2 \times 0.8 =$ 3.2 dynes per unit pole, the direction and sense being represented by the line pc.

142. Magnetic Moment.—A magnet makes an angle α with the direction of a magnetic field (Fig. 90). The torque $\mathfrak{T}$ in dyne centimeters acting upon the magnet is

$$\mathfrak{T} = Hml \sin \alpha = MH \sin \alpha.$$

The factor M is called the *magnetic moment;* it is the product of the strength of one of the poles into their distance apart; that is, $M = ml$.

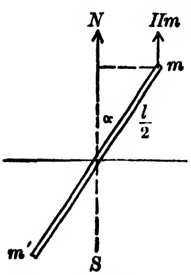

FIG. 90.—Magnetic moment.

143. Magnetic Induction and Magnetic Flux.—The magnetic field intensity H at a given point may be likened to a stress in an elastic body; the *magnetic induction* B may be likened to the corresponding strain. The relation of the magnetic induction B to the field intensity H, and to the permeability μ of the medium is expressed by the equation

$$\textit{magnetic induction} = B = \mu H.$$

It is sometimes desirable to represent magnetic induction graphically by lines, called *lines of induction.* In the case of a magnet, the lines of induction (sometimes called lines of force) are closed curves coming out of the N-pole and entering again at the S-pole. In this sense, induction B may be defined as the number of lines of magnetic induction per unit area.

Magnetic flux ϕ is the total number of lines passing through a given area A; that is,

$$\text{magnetic flux} = \phi = BA.$$

Engineers generally speak of H as the *field*, and of B as the *flux density*.

144. Terrestrial Magnetism.—The three important factors to be considered with reference to the earth's magnetism, for a given time and place, are (*a*) the magnetic declination, (*b*) the magnetic dip, (*c*) the intensity of the field. These three elements of terrestrial magnetism are of such great commercial and scientific importance that our government, through the Coast and Geodetic Survey, and other authorized branches of service, maintains permanent stations for the collection and tabulation of data relating thereto.

Magnetic declination at a given place is the angle which the magnetic needle makes with the geographical meridian at that place. At Ann Arbor, Mich., for example, the angle of declination is at present about 2° W. This means that the N-pole of the magnetic needle points west of true north by 2°. True north from a given point (in the northern hemisphere) is the direction represented by a meridian connecting the point and the north geographic pole.

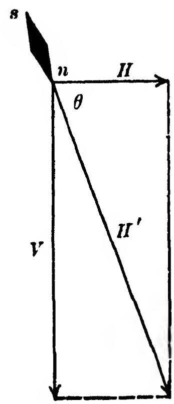

FIG. 91 Magnetic dip.

Magnetic dip is the angle which a dipping needle, when placed in the magnetic meridian, makes with a horizontal line. In Fig. 91 the angle of dip is designated by θ. At the magnetic equator $\theta = 0$; at the magnetic pole, $\theta = 90°$.

If for any given place we let the total intensity of the field, *measured in the direction of the field*, be H', then we may resolve this intensity into a horizontal component H, and a vertical component V, such that

$$H' = \sqrt{H^2 + V^2},$$

and the angle of dip θ (Fig. 91) may be determined by means of the equation

$$\tan\theta = \frac{V}{H}.$$

The *horizontal component* of the earth's field H may be determined by means of a small bar magnet suspended at its midpoint so as to vibrate freely in the given magnetic field. The equation is

$$T = 2\pi\sqrt{\frac{I}{MH}},$$

where T = period of vibration of the magnet; I = moment of inertia of magnet; M = magnetic moment; H = horizontal component of the field.

145. Summary of Magnetic Units and Symbols.—1. The unit of magnetic pole is that pole which at a distance of 1 cm in a vacuum will repel an equal and like pole with a force of 1 dyne.

2. Magnetic intensity of field $= H = \pm\frac{m}{\mu d^2}$.

3. Magnetic moment $= M = ml$.

4. Magnetic induction $= B = \mu H =$ number of lines of induction per cm².
5. Magnetic flux $= \phi = BA =$ total number of lines of induction passing through a given area A.

Problems

NOTE.—In the solution of the following problems we shall assume, unless stated to the contrary, that the permeability of air is equal to unity, that pole strengths are expressed in c.g.s. units, and that Coulomb's law holds.

505. Explain the meaning of each term in the following equations. (*a*) $F = \pm\frac{mm'}{\mu d^2}$; (*b*) $H = \pm\frac{m}{\mu d^2}$; (*c*) $B = \mu H$; (*d*) $\phi = BA$; (*e*) $\tan\theta = V/H$.

506. How far from an N-pole of 10 units must an S-pole of 20 units be placed so that the attraction shall be 2 dynes?

507. A magnetic pole of 40 units acts with a force of 32 dynes upon another pole 5 cm away. Find the strength of the other pole.

508. Two magnetic poles of strengths +18 and −24 units, respectively, attract each other with a force of 3 dynes when placed in air. Find the distance between them.

509. Find the magnitude, direction, and sense of the intensity of the magnetic field H midway between the two poles of problem 508.

510. A magnetic needle of length 20 cm and magnetic moment M 200 lies in a magnetic field of intensity 2 gausses in such a position that the torque acting upon it is 100 dyne centimeters. Find (*a*) the angle (Fig. 90) that the magnet makes with the lines of the field, and (*b*) the strength of the pole m.

511. The intensity of a magnetic field, the cross-sectional area of which is a rectangular figure 5 by 6 cm, is 2 gausses. The permeability of the medium is 5. Find (*a*) the induction B; (*b*) the flux ϕ.

512. Consider a right-angled triangle of base AB 8 cm, and altitude AC 6 cm. Assume that a magnetic pole of +10 units is placed at A; a pole of −40 units at B; and one of +30 units at C. Find the force (*a*) of attraction between A and B; (*b*) of repulsion between A and C; (*c*) the attraction between B and C.

513. (*a*) Make a drawing to show the magnitude, direction, and sense of the resultant force at A, due to the action of the three poles of problem 512. (*b*) Find the numerical value of the resultant force.

514. Make a drawing to show the magnitude, direction, and sense of the field H midway between A and B (problem 512) due to the three poles.

515. Find the magnitude of the field at A (problem 512), assuming that the $+10$ unit pole at A has been eliminated.

516. The poles of a given magnet are 16 cm apart, and lie at the extremities of the base of a right-angled triangle, the altitude of which is 12 cm. The pole strength m is 40 units. Find the magnitude of H at a point forming the third vertex of the triangle, and show by means of a sketch its direction and sense.

517. A given magnet of pole strength m lies on the diameter of a circle, the poles being on the circumference. Illustrate by diagram the magnitude, direction, and sense of H at a point on the circumference 30° from the x-axis.

518. What force must be applied to a magnet whose magnetic moment is 500 to hold it in an east and west position if the distance between the poles is 25 cm and H is 0.18 gauss?

519. At a certain place the angle of dip θ is 70°, and the horizontal component of the earth's field H is 0.19 gauss. Find the vertical component V.

520. At Ann Arbor, Mich., the vertical component of the earth's field V is 0.54 gauss, and the angle of dip 71° 30′. Find the magnitude of the earth's magnetic force H' at this place.

521. A magnet of length 12 cm, pole strength 20, is suspended by a fine wire so as to rest in a horizontal plane and in the magnetic meridian. The horizontal component of the magnetic field is 0.18 gauss. When the upper end of the wire is twisted through 90°, the magnet is deflected 30° from the meridian. Find the torque tending to restore the magnet to the meridian.

522. Two places A and B, are 10 miles apart, as measured on a magnetic meridian. Between these places there is a straight line AB. The angle of declination at A is 2°. C is on the same parallel of latitude as B and is directly north of A, as measured on a geographical meridian. Find the distance from B to C, measured in feet.

523. According to the 1910 report of the U. S. Great Lakes' Survey the horizontal component of the earth's field at the lower end of Lake Michigan was 0.1871 gauss, and the angle of dip 72° 28′. Find the vertical component of the earth's field from these data.

524. It was found that a magnet suspended horizontally at Bristol, England, made 110 complete vibrations in 5 min., and that the same magnet at St. Helena made 112 vibrations in 4 min. Find the ratio of the values of H at the two places.

525. A magnet suspended to vibrate in a horizontal plane is caused to oscillate in two places. At the first it makes 100 vibrations in 5 min. The horizontal component H of the earth's field at the first place is 0.18 gauss; at the second place it is 0.16 gauss. How many vibrations will the magnet make in 5 min. at the second place?

ELECTROSTATICS

146. Positive and Negative Charges.—When a glass rod is rubbed with silk the glass is said to be positively charged, the silk having an equal negative charge. When a vulcanite rod is rubbed with flannel the vulcanite is negatively charged, the flannel having an equal positive charge. A positive charge of electricity then is of the nature of that developed on glass when rubbed with silk; negative electricity is of the nature of that developed on vulcanite when rubbed with flannel. Whenever a body is charged with electricity of a given sign (+ or −) an equal charge of the opposite sign is always developed on some other body.

A positive charge is represented by $+Q$ and a negative charge by $-Q$. Surface density σ is the charge per unit area; that is, $\sigma = Q/A$.

147. The Electron Theory.—The electron theory maintains that each atom of which matter is composed is made up of two distinct parts, (a) a positively charged nucleus called the *proton*, and (b) one or more electrons rotating around the nucleus. An *electron* is the elementary unit of negative electricity. All electrons carry *negative* charges of the same amount. The nucleus of the atom always carries a positive charge. In each atom there are, under ordinary conditions, just a sufficient number of electrons to neutralize the positive charge on the nucleus. The hydrogen atom, for example, carries one elementary positive charge in the nucleus, and around this nucleus there rotates one electron. The helium atom has two positive charges in the nucleus and two electrons.

Electrons are quite easily dissociated from the influence of the positive nucleus of the atom. When a glass rod is rubbed with silk, the friction of the silk against the rod pulls electrons from the glass. The silk has thus gained electrons and is therefore negatively electrified; the glass rod, having lost electrons, is positively electrified.

An excess of electrons on a body means a negative charge; a deficiency of electrons, a positive charge.

148. Coulomb's Law of Attraction and Repulsion.—Electric charges of like sign repel; charges of unlike sign attract. The force of attraction or repulsion, as given by Coulomb's law, is

$$F = \pm \frac{QQ'}{kd^2},$$

in which F = force in dynes; Q and Q' = the two charges; d = distance in centimeters between the points at which the charges may be considered to be concentrated; and k = the dielectric constant of the medium. The signs + and −, as in the corresponding case in magnetism, are directional in nature, the positive sign indicating repulsion and the negative sign attraction.

For a vacuum the dielectric constant k is taken as unity. For air k is very nearly unity.

The dielectric constant of a vacuum = 1; of air = 1.0006; paraffin = 2; mica = 6. The dielectric constant of air is usually taken as 1.

For dielectric constants, see Table XXVI, Appendix.

149. Electrostatic Unit Quantity.—*Unit quantity* is that charge which at a distance of 1 cm in a vacuum will repel a similar and like charge with a force of 1 dyne.

150. Intensity of Electric Field.—The intensity E of an electrostatic field at a given point is measured by the force which is exerted on a unit positive charge at that point. As in magnetism ($H = F/m$), so also in electrostatics, $E = F/Q$. From Coulomb's law we may then write

$$E = \pm \frac{Q}{kd^2}.$$

The intensity of an electrostatic field E at any point is a *vector quantity*, having the same direction and sense as the force acting upon a unit positive charge at the given point.

Example.—Two charges Q and Q' are at a distance of 30 cm apart in air ($k = 1$). The charge Q is +100 units; Q' is −800 units. Find the magnitude, direction, and sense of the electrostatic field at a point p between the two charges, and 10 cm from Q. *Solution:* Consider a unit positive charge to be placed at p. Then the intensity of the field at p with respect to Q is $E = +\frac{100}{10^2} = +1$, the + sign indicating repulsion. Also, the intensity of the field at p with respect to Q' is $E' = -\frac{800}{20^2} = -2$, the − sign indicating attraction. The sum of these two vectors = 3 dynes, having the direction and sense p to Q'.

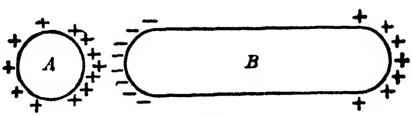

Fig. 92.

151. Electrostatic Induction.—Charges may be represented by the signs + and − (Fig. 92) or by directional lines representing electrostatic lines of force (Fig. 93).

When a charged body, A of Fig. 92, or A' of Fig. 93, is brought near another body, B or B', a charge of the opposite sign is induced in the near side of the second body and a charge of the same sign on the far side.

As in magnetic induction, electrostatic induction corresponds to a strain in an elastic medium, while intensity of field corresponds to a stress. In

magnetism the relation of strain to stress is expressed by the equation $B = \mu H$; in electrostatics we write $D = kE$, where D is the electrostatic induction; E the electrostatic field intensity; and k the dielectric constant.

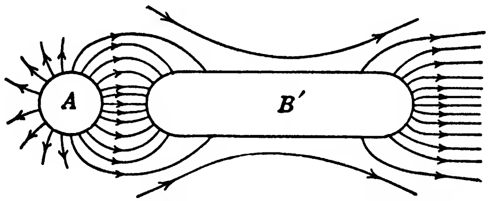

FIG. 93.—Electrostatic charges by induction.

If we agree to represent unit induction by one line per square centimeter, then the total number of lines of induction passing through a sphere of unit radius, described about a point charge of Q units as a center, is $4\pi Q$.

152. Electrostatic Potential.—Consider two points p and p' in an electrostatic field due to a charge Q. The difference of potential V between the two points is numerically equal to the work done in moving a unit positive charge from p' to p. Since potential is measured in terms of work, we have

$$V = Fd = \pm Q \times 1 \times \frac{d}{kd^2} = \pm\frac{Q}{kd},$$

in which the sign of Q (+ or −) determines whether the potential V is positive or negative. Since potential is not a vector quantity, the + or − signs indicate only positive or negative work; that is, the sign tells us whether work was done *on* or *by* the unit + charge. The symbol d is the distance through which the unit + charge is moved.

Since a charge Q uniformly distributed upon a sphere acts as if it were concentrated at the center, it follows that d, in the equation above, is the distance from p to the center of the sphere upon which Q is uniformly distributed.

Example 1.—The charge Q (Fig. 94) is + 120 units. Find the work in ergs required to move a unit positive charge from p' to p. The distance from

FIG. 94.

p' to $p = d' = 5$ cm, and $d = 15$ cm. *Solution:* The potential at $p = V = Q/d = {}^{120}\!/_{15} = 8$, and the potential at $p' = V' = {}^{120}\!/_{20} = 6$. The difference of potential between p and $p' = V - V' = 2$ ergs per unit quantity.

Example 2.—To find the potential at a point due to a number of concentrated charges, Q, Q', etc. In this case we get the sum of the potentials at p with reference to Q, Q', etc., *taking each with its proper sign* as illustrated by the following example. Two concentrated charges Q and Q' are 30 cm apart. Find the potential at a point p, 10 cm from Q, and 20 cm from Q', when $Q = +100$ and $Q' = -800$. *Solution:* Let V be the potential at p,

due to the charge Q. $V = +\frac{Q}{d} = +\frac{100}{10} = +10$. This means that 10 units (ergs) of work would be expended upon a unit charge in moving it from p to Q. Let V' be the potential at p due to Q'. Then $V' = -\frac{Q'}{d} = -{}^{80}\!/_{20} = -40$. This means that 40 units of negative work would be expended on a unit positive charge in moving from p to Q'. The total potential then $= V + V' = (+10) + (-40) = -30$.

153. Capacitance.—The relation of a given charge Q to the capacitance and potential of the conductor is

$$C = \frac{Q}{V}$$

where C = capacitance of the conductor, and V = the potential to which it is raised by the charge Q. The *capacitance of a conductor* is numerically equal to the quantity required to raise it to unit potential.

154. Capacitance of Conductors.—The capacitance of a conductor depends upon its shape, its contiguity to other conductors, and the dielectric constant k of the medium concerned. The capacitance of the following conductors is in each case expressed in c.g.s. units.

Capacitance of a sphere $= C = kr$, where k = dielectric constant, and r = radius of sphere.

Concentric spherical condensers $= C = krr'/(r - r')$, where r and r' are the radii of the two spheres.

Plate condenser $= C = kA/4\pi t$, where A = total area in square centimeters of the dielectric between the conductors; and t = thickness of the dielectric in centimeters.

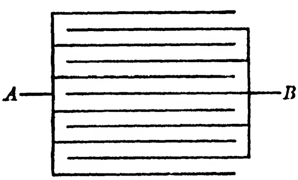

FIG. 95.

Cable $= C = kl/2\log_e\,(r'/r)$, where l = length of cable in centimeters; k = dielectric constant of the sheath; r' = radius of the sheath or envelope; and r = radius of the core.

Single wire $= C = l/2\log_e\,(l/r)$, where l = length in centimeters; r = radius of wire in centimeters.

Aerial twin wires $= C = kl/4\log_e\,(d/r)$, where l = length in centimeters; d = distance in centimeters between the wires; r = radius of each wire.

Example.—A plate condenser consists of 11 sheets of tinfoil, as shown in Fig. 95, the area of each sheet being 400 cm². The conductors (sheets of tinfoil) are separated by glass of thickness 5 mm, having a dielectric constant 8. Find the capacitance of this condenser in c.g.s. units. *Solution:* The 11 sheets of tinfoil enclose 10 thicknesses of the dielectric, the area of each being 400 cm². From the equation $C = kA/4\pi t$ we may write $C = (8 \times 10 \times 400)/(4 \times \pi \times 0.5) = 16{,}000/\pi$ c.g.s. units.

155. Capacitance of Condensers in Series and in Parallel. *Series.*—The capacitance of condensers connected in series (Fig. 96) as expressed in reciprocal quantities, is

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C''} + \cdots$$

Parallel.—The capacitance of condensers in parallel (Fig. 97) is

$$C = C' + C'' + \ldots$$

Example.—Given three condensers, each having a capacitance of 6 units, to find the capacitance of the system when the three are connected (*a*) in series; (*b*) in parallel. *Solution:* (*a*) $1/C = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$; then $C = 2$. (*b*) $C = 6 + 6 + 6 = 18$.

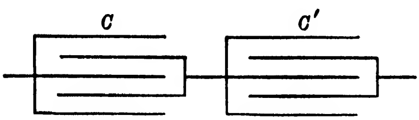

FIG. 96.—Condensers in series.

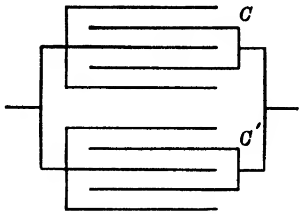

FIG. 97.—Condensers in parallel.

156. Energy Expended in Charging Condensers.—It may be shown that the energy W required to charge a condenser with a quantity Q to a potential V is

$$W = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where W is expressed in ergs when Q, C, and V are expressed in c.g.s. units.

157. Loss of Energy Due to Dividing a Charge.—Let us take as a special case for illustration two spheres A and B, having equal radii r. Sphere A is charged with a quantity Q to a potential V. It is then brought into contact with B by means of a thin wire. A spark appears when the contact is made between the spheres, thus indicating a loss of energy in the form of heat. The energy of the charge on A before contact with B is $W = QV/2 = Q^2/2C = Q^2/2r$. Since the two spheres have equal radii their capacities are equal. Therefore, after contact, Q will be equally distributed over the two spheres, and the energy of each will be $W = (Q/2)^2/2r$. The energy due to the charge on the two spheres is now $W = (Q/2)^2/2r + (Q/2)^2/2r = Q^2/4r$. The loss of energy is therefore

$$\frac{Q^2}{2r} - \frac{Q^2}{4r} = \frac{Q^2}{4r}.$$

158. Summary of Electrostatic Units and Symbols.—1. Unit quantity is that charge which at a distance of 1 cm in a vacuum will repel an equal and like charge with a force of 1 dyne.

2. Surface density $= \sigma = Q/A$.

3. The force of attraction or repulsion $= F = \pm\frac{QQ'}{kd^2}$ dynes.

4. Intensity of electrostatic field $= E = \pm\frac{Q}{kd^2}$ dynes per unit pole.

5. Electrostatic induction $= D = kE$.

6. Electrostatic potential $= V = Q/kd$ ergs per unit pole.

7. Capacitance $= C = Q/V$.

Problems

NOTE.—In the solution of these problems we shall consider that the dielectric constant of air = 1.

526. Explain the meaning of each term in the following equations:

$$\sigma = \frac{Q}{A};\ F \pm \frac{QQ'}{kd^2};\ E = \pm\frac{Q}{kd^2};\ D = kE;\ V = \pm\frac{Q}{kd};\ C = \frac{Q}{V}.$$

527. Two point charges, Q and Q', are 10 cm apart. Find the force acting between them (*a*) when $Q = +100$ and $Q' = +100$; (*b*) when $Q = -100$ and $Q' = -100$; (*c*) when $Q = +100$ and $Q' = -100$.

528. Find the intensity E of the field at a point half way between Q and Q', under the conditions given in problem 527.

529. Find the potential at the midpoint between Q and Q', conditions as in problem 527.

530. A sphere of radius 10 cm and free from all disturbing conditions is charged with 1000 c.g.s. units. Find (*a*) the density of the charge on the sphere; (*b*) its capacitance.

531. Find (*a*) the potential of the sphere (problem 530); (*b*) the energy of the charge.

532. Find the potential of the sphere (problem 530) at a point 10 cm from its surface.

533. Suppose that the sphere of problem 530 is made to divide its charge with a similar uncharged sphere. Find the energy due to the charge on both spheres. How do you explain the apparent loss of energy?

534. Sphere A has a charge of $+20$ and an equal sphere B a charge of -10. The two spheres, which have radii of 10 cm, are brought into contact for a moment and are then separated to a distance of 20 cm between their surfaces. Find the magnitude and direction of the force acting between them.

535. Two equal spheres charged one with $+20$ and the other with -15 units of electricity are placed at a certain distance apart. They are then brought into contact and afterward placed in their original positions. Find the ratio of the forces acting between them before and after contact.

536. Consider a straight line on which there are three points A, B, and C, such that the distance from A to B is 70 cm and from B to C 30 cm. A charge of $+50$ units is placed at A, and -20 units at B. Find the magnitude, direction, and sense of the field intensity E at a point (*a*) midway between A and B; (*b*) at C.

537. Find the potential V at the two points mentioned in problem 536.

538. Given three condensers, each having a capacitance of 300 electrostatic c.g.s. units. Find the capacitance of the system when the condensers are connected (*a*) in series; (*b*) in parallel.

539. Find the charge on each system (problem 538) when the charging potential is 300 units. What is the charge on each condenser for the conditions given in (*a*) and (*b*)?

540. Find the energy of the charge on each system under the conditions of problem 538.

541. Find the capacitance of a condenser made of two concentric spheres having radii of 10 and 8 cm, respectively, when the space between the spheres is filled (*a*) with air; (*b*) with oil having a dielectric constant 2.

542. A condenser consists of two parallel circular plates. The radius of each plate is 10 cm and their distance apart 1 mm. Find the capacitance of the condenser when (*a*) the dielectric is air; (*b*) the dielectric is sulphur.

543. Two small spheres are charged with +400 and +100 units of electricity, respectively, and placed 100 cm apart. (*a*) Find the neutral point in the field, that is, the place where the intensity of electric field is zero. (*b*) If the 100 units were negative, where would the neutral point be?

544. Two metal spheres, one having a diameter six times as great as the other, are connected by a long thin wire and electrified. Compare their electric potentials, capacitances, charges, surface densities, and energies.

CHAPTER IX

ELECTRICITY (*Continued*)

CURRENT, QUANTITY, RESISTANCE, AND ELECTROMOTIVE FORCE

159. Electric Currents.—Static electricity is a charge at rest upon a body. Current electricity, on the other hand, is electricity in motion. In other words, an *electric current* consists of a stream of electrons flowing from a point of given potential to a point of lower potential.

In Fig. 98 there is shown a section of a simple *voltaic* or *galvanic* cell. The electrodes, Cu and Zn, are immersed in an electrolyte consisting of a dilute solution of acid in water. The copper (+ electrode) is positively charged and the zinc (− electrode) is negatively charged. When the electrodes are connected by means of a conductor, a current flows around the circuit.

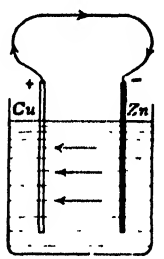

Fig. 98 Simple voltaic cell.

Electric currents are of two kinds, namely, (*a*) direct current (d.c.) and (*b*) alternating currents (a.c.). The problems of this chapter and the one that follows deal mainly with direct currents.

160. Direction of the Current.—It has been the established custom, ever since electrical currents have been known, to think of the current as flowing in the external circuit from the positive electrode to the negative electrode, as from the copper to the zinc. In the light of our present knowledge of electrons, however, this practice turns out to be embarrassing. The zinc electrode, in the case given, is negatively electrified and therefore must contain an excess of electrons, while the copper electrode, which is positively electrified, has a deficiency of electrons. Now the electric current, according to present notions, is a stream of electrons, flowing from the region with an excess of electrons (zinc) to the region with a deficiency of electrons (copper). The electric current flows through the metal conductor, but exactly in the reverse direction to that which we are accustomed to give.

Meanwhile, there has been written an immense number of books on all aspects of electric currents, using the customary way of describing the direction of currents. In all the practical books on electricity, including electricians' manuals, handbooks on radio, etc., the currents are described as passing through the conductor from the positive electrode to the negative one. Since any attempt to change our way of describing the current now would simply produce hopeless confusion, we shall continue to follow the established custom and will speak of currents as passing in the external circuit from the positive to the negative electrode. The student should

bear in mind, however, that the stream of electrons actually flows in a direction exactly opposite to that of the current as described.

161. Current Strength.—The electrical units of current strength, quantity, resistance, and electromotive force may be defined in three ways, as (*a*) c.g.s. units, (*b*) practical units, and (*c*) international units. The magnetic effect of a current furnishes a means of defining current strength. If a conductor carrying a current be bent into a loop or coil of radius r, the loop will have the property of a magnet, one side or face of the coil being an N-pole and the other side an S-pole. In the case of a circular current of this sort, where we consider the proportionality factor between H and I to be unity, the intensity of the magnetic field H at the center of the coil is

$$H = \frac{2\pi NI}{r} = \frac{2\pi NI'}{10r},$$

where H = intensity of field in gausses; N = number of turns of wire in the coil; I = current in c.g.s. units; I' = current in practical units (amperes); and r = radius of coil in centimeters.

The equation $H = 2\pi NI/r = 2\pi NI'/10r$ gives us a means of defining I and I' in terms of H and r.

The *electromagnetic c.g.s. unit* of current is that current which, flowing through an arc of unit length, in a circle of unit radius, will produce a unit magnetic field at the center of the circle.

The *practical unit* of current strength is the *ampere*. An ampere = 10^{-1} c.g.s. units.

A very close approximation to the ampere, as defined above, may be made experimentally by means of the silver coulometer. This unit is known as the *international ampere*, which is defined as that unvarying current which will deposit silver from a standard solution of silver nitrate at the rate of 0.001,118,00 g per sec.

The ampere (10^{-1} c.g.s. units) and the international ampere are so nearly identical that no discrimination is ordinarily made between these units.

162. Quantity.—Electrostatically, quantity may be defined in terms of the equation $Q = CV$. In current electricity, quantity is the amount of electricity conveyed by unit current in unit time.

The *electromagnetic c.g.s. unit* of quantity is the quantity transferred by 1 c.g.s. unit of current in 1 sec.; that is,

$$Q = It,$$

where I and t represent c.g.s. unit values.

The *practical unit* of quantity is the *coulomb*, which is the quantity of electricity transferred by 1 amp. in 1 sec. One coulomb = 10^{-1} c.g.s. unit of quantity.

163. Resistance.—Electrical resistance is that property of a conductor by virtue of which the energy of a current is transformed into heat. Resistance R is defined in terms of the equation $W = RI^2t$, in which W is the heat energy in ergs generated by I c.g.s. unit of current in t sec., and R is the resistance in c.g.s units.

The *c.g.s. unit* of resistance is that resistance by which heat energy equal to 1 erg is produced per second by unit current.

The *practical unit* of resistance is the *ohm*. One ohm $= 10^9$ c.g.s. units of resistance.

The *international ohm* is a near approximation to the ohm (10^9 c.g.s. units) as determined by the following specifications: "The international ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 g in mass, of constant cross-sectional area, and of length 106.300 cm."

In the practical application of these units no distinction is ordinarily made between the ohm (10^9 c.g.s. units) and the international ohm.

164. Laws of Resistance.—The resistance of a conductor is a function of its length, cross-sectional area, kind of material, and temperature.

Metric Units.—For a given temperature, the resistance R in ohms is

$$R = \frac{kl}{a},$$

where l = length in centimeters; a = cross-sectional area in square centimeters; and k is the resistivity (specific resistance) in ohm-centimeters. *Resistivity* may be defined as the resistance in ohms of a conductor having a cross-sectional area of 1 cm², and a length of 1 cm.

For resistivity values, see Table XXVII, Appendix.

English Units.—For a given temperature, the resistance R in ohms is

$$R = \frac{kl}{d^2},$$

where l = length in feet; d = diameter in mils; and k = resistance in ohms per mil-foot. A "mil" is a thousandth of an inch; a "circular mil" (d^2) is a mil squared; a "mil-foot" represents a conductor 1 ft. in length and $^1\!/_{1000}$ in. in diameter.

For values of k in ohms per mil-foot, see Table, XXVII, Appendix.

Example 1.—Find the resistance of 50 m of aluminum wire, the radius of which is 0.5 mm. *Solution:* Referring to Table XXVII, Appendix, we find that k for aluminum $= 2.6 \times 10^{-6} = 0.000{,}002{,}6$ ohm-cm. The area $a = \pi r^2 = \pi(^5\!/_{100})^2 = 25\pi/10{,}000$. Then $R = 0.000{,}002{,}6 \times 50 \times 100 \times 10{,}000/25\pi = 5.2/\pi$ ohms.

Example 2.—Find the resistance of 1 mile of copper wire, the diameter of which is 0.03 in. *Solution:* From Table XXVII, Appendix, we find that k (ohms per mil-foot) for copper is 9.5. The length of $l = 5280$ ft. Diameter $d = 0.03 = {^{30}\!/_{1000}} = 30$ mils; hence $d^2 = 900$. Then $R = kl/d^2 = 9.5 \times 5280/900 = 55.7$ ohms.

For wire gauge values, see Table XXVIII, Appendix.

165. Change of Resistance with Change of Temperature.—In general the resistance of a metallic conductor *increases* with increase of temperature, in accordance with the equation $R_t = R_o(1 + \alpha t)$, where α is the temperature coefficient of resistance. For pure metals α is nearly 0.004 per degree C. The temperature coefficient (α) for alloys is very much lower than that of pure metals.

The resistance of carbon and electrolytes *decreases* with increase of temperature.

166. Resistances in Series and in Parallel.—Resistance of conductors in series and in parallel may be expressed in equational form as,

$$\textit{Series, } R = R' + R'' + \cdots$$

$$\textit{Parallel, } \frac{1}{R} = \frac{1}{R'} + \frac{1}{R''} + \cdots$$

167. Conductance and Conductivity.—*Conductance* is the reciprocal of resistance; *conductivity* (specific conductance) is the reciprocal of resistivity. The terms conductance and conductivity are, in general, applied to electrolytes.

Example.—The resistance of a cylindrical column of $CuSO_4$ solution, of cross-section 2 cm^2, and length 6 cm, is 60 ohms. (*a*) What is the conductance? (*b*) the conductivity? *Solution:* (*a*) Conductance $= 1/R = 1/60$ reciprocal ohms. (*b*) $R = kl/a$, hence $k = 60 \times 2/6 = 20$ ohm-cm. Then conductivity $= 1/20 = 0.05$ reciprocal ohm.

168. Electromotive Force.—The electromotive force (e.m.f.) of an electric generator (battery or dynamo) is its capacity for generating electric pressure or potential. *Electromotive force* is that which produces a difference of potential in a circuit, and therefore it is the cause of the flow of an electric current. Electromotive force is analogous to water pressure. It should be noted, however, that it is not pressure; neither is it a force; e.m.f. is measured in terms of work, ergs.

The *c.g.s. unit* of e.m.f. is the difference of potential $(V - V')$ produced at the terminals of unit resistance when traversed by unit current.

The *practical unit* of e.m.f. is the *volt*, which is equivalent to 10^8 c.g.s units.

The *international volt* is the electric pressure which, when steadily applied to a conductor whose resistance is an international ohm, will produce a current of an international ampere.

The *Weston standard cell*, when set up according to standard specifications, has an e.m.f. of

$$E_t = 1.0183 - 0.000,04(t - 20) \text{ volts,}$$

where t is the temperature in degrees C.

Example.—What is the e.m.f. of a Weston standard cell at 30°C.? *Solution:* $E_{30} = 1.0183 - 0.000,04 \times 10 = 1.0183 - 0.0004 = 1.0179$ volts.

169. The E.M.F. of Some Commonly Used Cells.—It is frequently convenient to know the e.m.f. of some of the cell types that are in common use today. It must be noted that the following e.m.fs. are only approximate working values.

Name of Cell	E.m.f., Volts
Daniell cell	1.1
Leclanché cell	1.5
Dry cell	1.4
Lead storage cell	2.1
Edison storage cell	1.2

170. Thermoelectromotive Force.—*Thermoelectromotive force* (t.e.m.f.) is the e.m.f. due to a difference of temperature at the juncture of two substances. For example, if a copper-iron juncture be heated so that the

difference in temperature between A and B (Fig. 99) is 200°C. there will be developed in the system an e.m.f. of about 0.0027 volt, directed at the hot juncture from the copper to the iron.

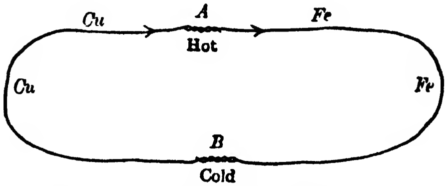

FIG. 99.—Thermoelectromotive force.

Thermoelectric power is the variation of thermoelectromotive force per degree change in temperature. Thermoelectric power is usually given in microvolts. A *microvolt* is one millionth of a volt.

For thermoelectric power, see Table XXIX, Appendix.

Example.—Reference to Table XXIX, Appendix, reveals the fact that the thermoelectric power of iron is +17.5 microvolts per degree C., and that of German silver, −12 microvolts. Find the thermoelectromotive force of 20 German silver-iron couples in series, the difference of temperature between junctures being 80°C. *Solution:* Since the value for iron (+17.5) is on the positive side of the zero point in the series, and the value for German silver (−12) is on the negative side, the total difference between the two values is 29.5 microvolts = 0.000,029,5 volt per degree. The total thermoelectromotive force then is E = 0.000,029,5 × 80 × 20 = 0.0472 volt.

Problems

545. Given a circular coil of five turns, of mean radius 2 cm, carrying a current of 10 c.g.s. units. Find (*a*) the intensity of the field at the center of the coil due to the current; (*b*) the force in dynes with which this field will repel a 5-unit positive magnetic pole.

546. Assume that a current of 10 amp. flows through the coil (problem 545). Find (*a*) the current strength in c.g.s. units; and (*b*) the intensity of the field at the middle of the coil due to the current; (*c*) the force in dynes with which a 10-unit magnetic pole will be urged along the axis.

547. What must be the radius of a circular loop of wire such that the field due to a current of 20 amp. will repel a +5-unit pole at the center of the coil with a force of 20 dynes?

548. A circular coil of wire CC' (Fig. 100) lies with its vertical face in the plane of the earth's field. Assume that the horizontal component of the earth's field is 0.18 gauss; that the needle,

of pole strength m and length l, is short as compared with diameter of coil; number of turns in the coil around the needle, 10; radius of the coil, 10 cm; the angle α, 30°. Find the current in amperes flowing through the coil.

549. A uniform current deposits from a standard silver nitrate solution 5.031 g of silver in half an hour. Find the current in (*a*) amperes; (*b*) c.g.s. units.

550. What quantity of electricity is required to deposit 0.6708 g of silver from a standard $AgNO_3$ solution? Give your results in (*a*) practical units; (*b*) c.g.s. units.

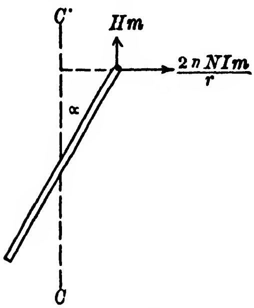

FIG. 100.—Magnetic forces at center of a coil.

551. Find the resistance of 20 m of iron wire, diameter 0.4 mm, at 18°C.

552. Find the resistance of 10 miles of iron wire having a diameter of 150 mils.

553. A copper wire is 10 ft. in length; its diameter is 0.025 in. What is (*a*) its diameter in mils? (*b*) its resistance in ohms?

554. Find the resistance of 6 m of pure platinum wire, B. & S. gauge No. 40, at (*a*) 32°F.; (*b*) 32°C. (See Table XXVIII, Appendix.)

555. (*a*) Find the resistance of 100.6 ft. of No. 30 copper wire at 0°C. (*b*) What will be the resistance of this wire if both its length and diameter be doubled?

556. The resistance of No. 12 copper wire is 1.6 ohms per 1000 ft. Find the resistance of No. 00 copper trolley wire.

557. The resistance of a cube of a given sample of copper 1 cm on each edge, density 8.9 g per cm³, is 1.6 microhms. Find the resistances of 1.78 kg of this sample of copper when drawn into wire, B. & S. gauge, No. 20.

558. A wire is stretched uniformly until its length is doubled. Compare its resistance before and after stretching.

559. Three conductors have resistances of 2, 4, and 6 ohms, respectively. Find the resistance of the system when connected (*a*) in series; (*b*) in parallel.

560. Enumerate all the resistances that can be obtained from three coils of resistances 2, 4, and 6 ohms, respectively, by the various ways in which they may be connected, all three coils being always in use.

561. The two circular, parallel platinum electrodes of a conductivity cell are 0.5 cm apart. The cross-sectional area of each electrode is 1 cm². With this cell it was found that at 20°C. the resistance of a tenth normal ($n/10$) solution of KCl between the electrodes was 42.92 ohms. Find (a) the conductance of the $n/10$ KCl solution; (b) the conductivity.

562. Find the area of the electrodes of a conductivity cell in order that the resistance of an $n/10$ KCl solution shall be 64.38 ohms, when the electrodes are set 3 cm apart, temperature and other conditions to be the same as in problem 561.

563. One juncture of an iron-constantan couple consisting of a single element is placed in ice; the other juncture in an oil bath, the temperature of which is 55°C. Find the thermoelectromotive force generated.

564. It is desired to install a copper-constantan thermopile in a chimney in such a way that one set of junctures is within the chimney and the other set outside. The average difference of temperature maintained between the two faces of the system is 95°C. How many elements will be required to furnish an e.m.f. as great as that of a dry cell (e.m.f. = 1.34)?

OHM'S LAW AND ITS APPLICATIONS

171. Ohm's Law.—One of the most important generalizations in electricity is that known as Ohm's law, which holds for constant currents, and which may be stated in equational form as

$$I = \frac{E}{R}, \text{ or } E = IR.$$

This law implies that when E and R are once fixed, the current strength

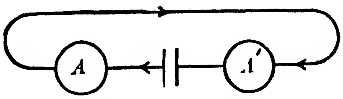

FIG. 101.—Current of same strength throughout the circuit.

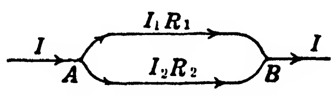

FIG. 102.—Currents in divided circuits.

is the same at every point throughout the circuit, the conductors of which are in series. Thus for a given current containing a fixed E and R (Fig. 101) the current strength, as registered by ammeters A and A', will be the same at both A and A'. If, on the other hand, the circuit contains resistances in parallel (Fig. 102) the current $I = I_1 + I_2$.

172. Potential Difference and E.M.F.—There frequently exists in the minds of students a good deal of confusion as to the distinction between *potential difference* and *electromotive force*. Potential difference (p.d. or IR drop) and e.m.f. are both measured in terms of volts. In order to get at the

meaning of these terms (p.d. and e.m.f.), let us consider a circuit (Fig. 103) which includes an electric cell with its internal circuit and the external conductor $ABCD$. The circuit is closed and a current I flows around the circuit. Let the cell resistance $r = 1$ ohm, the resistance $AB = 2$ ohms, $BC = 5$ ohms, $CD = 2$ ohms. The total resistance of the circuit $= 1 + 2 + 5 + 2 = 10$ ohms. Let the current $I = 0.5$ amp.

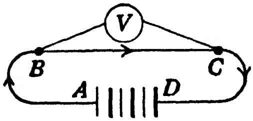

Fig. 103.

If a voltmeter V be connected to the points B and C it will measure the fall of potential (p.d.) in volts between these points; that is, p.d. $= E = IR = 0.5 \times 5 = 2.5$ volts. In a like manner we may determine the p.d. between AB or CD or any other two points we choose in the conductor AD.

Potential difference is the IR drop between any two points in the circuit; electromotive force, on the other hand, is equal to the IR drop around the *entire circuit;* that is, e.m.f. $= 0.5 \times (1 + 2 + 5 + 2) = 0.5 \times 10 = 5$ volts.

173. Fall of Potential over Conductors in Parallel.—A problem of special importance to the student is that of the fall of potential over conductors in parallel, especially as it furnishes a method of determining the current strength in the various branches of the conductors in a parallel system. The important point to note in this connection is this: *The fall of potential over each branch of a parallel system from A to B* (Fig. 102) *is the same as the fall of potential over the entire system from A to B.* If R be the resistance of the parallel circuit of which R_1 is the resistance of one branch, and R_2 the resistance of another branch, then $E = IR = I_1R_1 = I_2R_2$.

Example.—Two conductors of 4 and 12 ohms, respectively, are connected in parallel to the points A and B. A current of 4 amp. flows from A to B, a part passing through each conductor. Find (a) the resistance from A to B; (b) the fall of potential from A to B; (c) the current through each conductor. *Solution:* (a) $1/R = 1/4 + 1/12 = 1/3$. Hence $R = 3$ ohms. (b) Fall of potential from A to $B = E = IR = 4 \times 3 = 12$ volts. (c) $I_1 = 12/4 = 3$ amp.; $I_2 = 12/12 = 1$ amp. *Verification:* $I = I_1 + I_2 = 3 + 1 = 4$ amp.

174. Cells in Series and in Parallel.—In connection with this subject, it is important to bear in mind that the e.m.f. of a cell depends only upon (a) the nature of the electrolyte, (b) the kind of electrodes used, and (c) the temperature. The e.m.f. is entirely independent of the size of the plates. It is also important to note, on the other hand, that the internal resistance of a battery is *inversely* proportional to the size of the plates.

Cells in Series.—The current I for n cells of equal e.m.f. in series, where R is the external resistance and r is the internal resistance of one cell, is

$$I = \frac{nE}{(R + nr)}.$$

Cells in Parallel.—The current I for n cells in parallel, where E is the e.m.f. of a single cell, is

$$I = \frac{E}{\left(R + \frac{r}{n}\right)}.$$

Multiple Series.—Cells may be connected in multiple series, as shown in Fig. 104, in which n is the number of cells in each series and m is the number of series. For multiple series the current is

$$I = \frac{nE}{\left(R + \frac{nr}{m}\right)},$$

in which R = external resistance; and nr/m = internal resistance of the system. It is evident that I will be a maximum when the denominator of the fraction is a minimum. It may be shown that $R + \frac{nr}{m}$ is a minimum

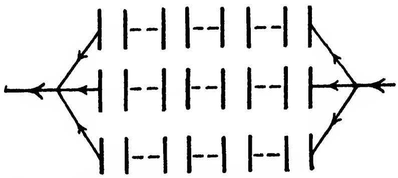

FIG. 104.—Cells in multiple series; $n = 4$, $m = 3$.

when $R = nr/m$. It should be noted, however, that the conditions for maximum current in multiple series, as given above, do not hold if there is a counter e.m.f. in the circuit; nor does it hold for a grouping of cells for quickest action when there is an e m f. of self-induction in the circuit.

175. Kirchhoff's Laws.—Kirchhoff's laws, which follow as a deduction from Ohm's law, may be stated thus:

I. *In any network in a closed circuit containing a source of e m.f., the sum of the currents flowing away from a given point is equal to the sum of the currents flowing to the point.* Suppose, for example, that we select the point A (Fig. 105). Then $I_1 = I_2 + I_3$, that is, $I_1 - I_2 - I_3 = 0$. Also for point B, $I_2 = I_4 + I_5$, or $I_2 - I_4 - I_5 = 0$; and for point C, $I_6 - I_3 - I_4 = 0$. This may be expressed in general terms as

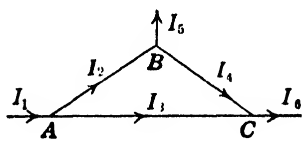

FIG. 105.

$$\Sigma I = 0,$$

which means that there can be no accumulation of electricity at any point in a closed circuit.

II. *In any closed network, the algebraic sum of the IR drops around any given circuit is equal to the algebraic sum of the electromotive forces; that is,* $I_1R_1 + I_2R_2 + \cdots = E_1 + E_2 + \cdots$, or

$$\Sigma IR = \Sigma E,$$

in which the IR drops over the various lines of the circuits are taken with their proper signs, being positive (+) when taken *with* the current and negative (−) when taken *against* the current. For example, the potential difference between A and C (Fig. 105) is $+I_3R_3$ when taken with the cur-

rent from A to C and $-I_3R_3$ where taken against the current from C to A. In other words, ΣIR and ΣE always represent algebraic sums.

In order to make clear the application of Kirchhoff's laws, let us consider the following cases:

Example 1.—Two cells having equal e.m.f. and resistances are connected in parallel at the points A and C (Fig. 106*a*). Let $E_1 = E_2 = 1.5$ volts and $R_1 = R_2 = 2$ ohms. Find the current through an external resistance $D = R_3$ of 4 ohms. *Solution:* (*a*) Since $E_1 = E_2 = e$, and $R_1 = R_2 = r$, we may apply the special equation for parallel circuits given in Art. 174; that is, $I = E/\left(R + \frac{r}{n}\right) = 1.5/\left(4 + \frac{2}{2}\right) = 0.3$ amp. (*b*) Let us now solve the problem by using Kirchhoff's laws. We have four possible circuits, namely, $ABCD$, $AB'CD$, $ABCB'$, and $AB'CB$, from which we may obtain two independent and two dependent equations. Taking C as a point of reference, we have two currents flowing to the point and one away from it, and hence, by Kirchhoff's first law, $I_1 + I_2 = I_3$; that is, $I_1 + I_2 - I_3 = 0$, which is an inde-

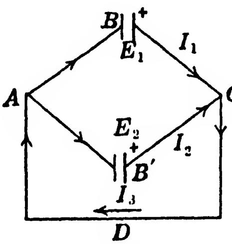

Fig. 106*a*.

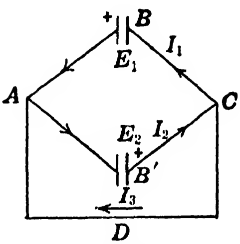

Fig. 106*b*.

pendent equation. Now, since we have three unknowns (I_1, I_2, I_3), we must write three independent equations involving these factors. These are

$$I_1 + I_2 - I_3 = 0. \tag{1}$$
$$2I_1 + 4I_3 = 1.5. \tag{2}$$
$$2I_2 + 4I_3 = 1.5. \tag{3}$$

Multiplying Eq. (1) by -2 gives us

$$-2I_1 - 2I_2 + 2I_3 = 0, \tag{4}$$

which with Eq. (2) gives

$$-2I_2 + 6I_3 = 1.5. \tag{5}$$

Eliminating I_2 from Eqs. (3) and (5), we have

$$10I_3 = 3,$$

from which $I_3 = \frac{3}{10} = 0.3$ amp., a value which corresponds to that obtained by the special method employed in Solution *a*.

Under the conditions given in Example 1, namely, that $E_1 = E_2$ and $R_1 = R_2$, it is evident that the special solution given in *a* is the simpler of the two. If, however, the value of E_1 is different from that of E_2, or if R_1 is different from R_2, or if one of the cells is reversed, the solution by Kirchhoff's laws becomes necessary, as illustrated by the following examples:

Example 2.—Suppose that in a set-up similar to that shown in Fig. 106*a* we have given $E_1 = 1.5$, $R_1 = 2$, and $E_2 = 2$, $R_2 = 3$, the resistance of the external circuit being, as before, $R_3 = 4$, to find the direction and magnitude of I_3. *Solution:* Since the currents from the + electrodes of the two cells flow toward C, $I_1 + I_2 = I_3$, the direction of the current I_3 in the external circuit is from C to D to A. The magnitude of I_3 may be calculated from the following three independent equations:

$$I_1 + I_2 - I_3 = 0. \tag{1}$$
$$2I_1 + 4I_3 = 1.5. \tag{2}$$
$$3I_2 + 4I_3 = 2. \tag{3}$$

Eliminating I_1 from Eqs. (1) and (2) gives

$$-2I_2 + 6I_3 = 1\ 5. \tag{4}$$

Eliminating I_2 from Eqs. (3) and (4), we have

$$26I_3 = 8.5,$$

from which $I_3 = 8.5/26 = 0.327$ amp.

Example 3.—Let cell E_1 now be reversed, as shown in Fig. 106*b*, the other factors remaining the same as given in Example 2. Find the direction and magnitude of the current I_3 in the external circuit. *Solution:* The current I_1 flows away from C and I_2 flows toward C. We cannot, however, in this case determine by inspection whether the current I_3 flows away from or toward C because we do not know the respective magnitudes of I_1 and I_2. It is necessary then to make an assumption. In order therefore to serve as a basis for our computation we shall *assume* that I_3 flows *away* from C, as shown by the arrow above D (Fig. 106*b*). This assumption will not affect the numerical value of I_3 thus obtained; it will only determine its sign. If for I_3 we obtain + value, our assumption as to direction is correct; if, on the other hand, I_3 comes out with a − value, our assumption is incorrect, and consequently the true direction is the reverse of that assumed. According to our assumption that I_3 flows away from C, $I_2 = I_1 + I_3$, or $-I_1 + I_2 - I_3 = 0$. Also, in the path $CBAD$, we move from C to B to A in the direction of the current, and from A to D back to C again *against* the current, it follows that I_1R_1 is positive ($+1.5I_3$) and I_3R_3 is negative ($-4I_3$). In a like manner for the path $CDAB'$, in which we move *with* the current in both lines, we have $+3I_2$ and $+4I_3$. Our three independent equations are therefore

$$-I_1 + I_2 - I_3 = 0. \tag{1}$$
$$2I_1 - 4I_3 = 1.5. \tag{2}$$
$$3I_2 + 4I_3 = 2. \tag{3}$$

Eliminating I_1 from Eqs. (1) and (2) gives us

$$2I_2 - 6I_3 = 1.5. \tag{4}$$

Eliminating I_2 from Eqs. (3) and (4),

$$26I_3 = 0.5,$$

from which $I_3 = -0.5/26 = -0.019$ amp. The negative sign tells us that the current I_3 flows in the opposite direction to that assumed. The direction of I_r is therefore from A to D to C, and its magnitude is 0.019 amp.

Problems

565. Consider a straight horizontal line *ABCD*. Three resistances $R_1 = 4$ ohms, $R_2 = 6$ ohms, $R_3 = 12$ ohms, are connected in *series* between *B* and *C*. An ammeter between *A* and *B* reads 2 amp. (*a*) What is the total resistance between *B* and *C*? (*b*) What is the fall of potential (*IR* drop) from *B* to *C*? (*c*) What is the *IR* drop over each conductor? (*d*) What will be the reading of the ammeter if it is connected in the line between *C* and *D*?

566. Assume now that the three resistances of problem 565 are connected in *parallel* between *B* and *C*. The current through the ammeter, which is between *A* and *B*, is 4 amp. Make a drawing to illustrate the set-up. (*a*) What is the total resistance from *B* to *C*? (*b*) What is the *IR* drop from *B* to *C*? (*c*) What is the *IR* drop over each conductor? (*d*) What is the current through each conductor?

567. Given three conductors *A*, *B*, and *C*, having resistances of 2, 3, and 6 ohms, respectively, and a battery, having an e.m.f. of 7 volts and an internal resistance of 3 ohms. Find (*a*) the current flowing through the battery when *A*, *B*, and *C* are connected in series across the terminals; (*b*) the *IR* drop over each conductor.

568. The conductors *A*, *B*, and *C* (problem 567) are in parallel. Find (*a*) the current through the battery; (*b*) the *IR* drop over each conductor.

569. Three resistances, $R_1 = 2$ ohms, $R_2 = 5$ ohms, $R_3 = 8$ ohms, are connected in parallel between the points *A* and *B*. The total current in the three parallel lines is $I = I_1 + I_2 + I_3 = 5$ amp. Find (*a*) the resistance from *A* to *B*; (*b*) the *IR* drop from *A* to *B*; (*c*) the fall of potential over each of the three lines; (*d*) the current through each line.

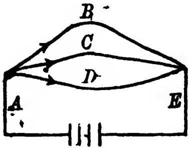

FIG. 107.

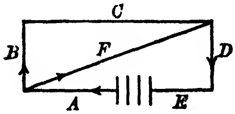

FIG. 108.

570. Consider Fig. 107. The resistance of *A* is 2 ohms; *B*, 3 ohms; *C*, 4 ohms; *D*, 6 ohms; *E*, 2 ohms. The e.m.f. of the

battery is 6 volts, resistance is $\frac{2}{3}$ ohm. Find (a) the fall of potential over A; (b) the fall of potential over B, C, D; (c) the Ir drop over the battery.

571. The resistances of the various conductors of Fig. 108 are as follows: A and E are 2 ohms apiece; B and D are 3 ohms apiece; C is 5 ohms; and F is 8 ohms. The e.m.f. of the battery is 12 volts, internal resistance 1 ohm. Find the fall of potential over C.

572. Three wires, A, B, C, each having a resistance of 100 ohms, are connected in series across the terminals of a generator G (Fig. 109) of constant e.m.f. of 580 volts, and negligible internal resistance. A voltmeter of resistance 900 ohms is connected across the terminals of B. Find the reading of the voltmeter, the resistance of a and b being negligible.

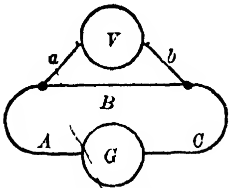

FIG. 109.

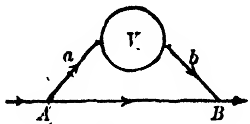

FIG. 110

573. Consider Fig. 110. The resistance from A to B is 10 ohms. The resistance of the voltmeter is 100 ohms. The resistance of the connecting wires a and b is negligible. A current of 5 amp. flows through AB. Find (a) the fall of potential from A to B; (b) the current through the voltmeter V; (c) the fall of potential over the voltmeter.

574. Suppose that the voltmeter (Fig. 110) has a resistance of 94 ohms, the connecting wires, a and b, have a resistance of 2 ohms each, and the resistance of AB is 2 ohms. The voltmeter registers 18.8 volts. Find (a) the current through the voltmeter; (b) the current in a and b; (c) the fall of potential from A to B; (d) the current over AB.

575. A storage battery of constant e.m.f., consisting of five cells, has a total internal resistance of 0.5 ohm. Connected to its terminals is a wire of 10 ohms resistance. The current flowing through the wire is 1 amp. Find (a) the terminal potential (fall of potential over external circuit); (b) internal drop of potential; (c) e.m.f. (fall of potential around entire circuit).

576. Consider conditions of problem 575. A second wire of 10 ohms resistance is put in from terminal to terminal of the battery parallel with the first wire. (*a*) What is the resistance of the two wires in parallel? (*b*) What current flows through the battery? (*c*) What is the drop of potential over the external circuit?

577. A battery, e.m.f. 20 volts, internal resistance 0.2 ohm, has connected across its terminals a wire of 100 ohms resistance. A voltmeter of resistance 490 ohms is connected to this wire at two points A and B, the resistance between A and B being 10 ohms. Find the reading of the voltmeter.

578. If the voltmeter of problem 577 be removed, (*a*) what will be the current in the wire; (*b*) the fall of potential over AB?

579. Five dry cells, each having an e.m.f. of 1.5 volts, and an internal resistance of 0.1, 0.2, 0.3, 0.4, 24 ohms, respectively, are connected in series. (*a*) What current will the five cells furnish through an external resistance of 5 ohms? (*b*) What current will be furnished through the same resistance if the last cell be cut out?

580. A storage cell, e.m.f. = 2.1, r = 0.1 and a Daniell cell, e.m.f. = 1.1, r = 2.9, are set in opposition. Find the current furnished through a resistance of 7 ohms.

581. A battery, e.m.f. 20 volts on open circuit, internal resistance 5 ohms, is delivering a current of 2 amp. Find (*a*) the external resistance; (*b*) the terminal potential.

582. Calculate the resistance of a galvanometer shunt in terms of the resistance of the galvanometer if it is desired to have one-fiftieth of the current go through the galvanometer.

583. A galvanometer having a resistance of 5000 ohms is shunted with 100 ohms. A certain deflection of the galvanometer is obtained with a battery of constant e.m.f. when the resistance of the rest of the circuit is 2000 ohms. What add'' tional resistance must be inserted to produce the same deflecti when the shunt is removed?

584. Consider Fig. 111. Assume that the bridge is in balan that is, the fall of potential over CD is zero. (*a*) How d the fall of potential over AC compare with that over AD? How does the current over AC compare with that over AD? How does the IR drop over CB compare with that over B, 3 (*d*) How does the current over CB compare with that ove of the

585. The potential difference between the terminals of a cell is 1.5 volts when the cell is furnishing no current. When its terminals are connected by a wire of 2 ohms resistance, the terminal potential falls to 1.2 volts. Find its internal resistance.

586. Given four cells, the e.m.f. of which is 1.5 each and the internal resistance 2 ohms each. Find the current furnished through a resistance of 100 ohms when the cells are connected in (*a*) series; (*b*) parallel. (*c*) Solve for conditions (*a*) and (*b*) when the external resistance is 1 ohm.

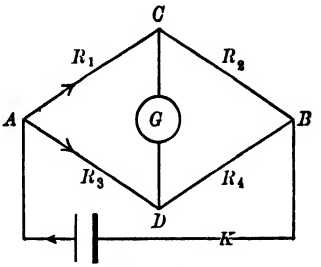

FIG. 111.

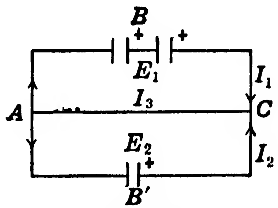

FIG. 112.

587. How should four cells be connected to produce a maximum current through a 2-ohm coil, if each cell has an e.m.f. of 1.6 volts and an internal resistance of 2 ohms? What is the maximum current?

588. A battery of three dry cells is connected up as shown in Fig. 112. In line ABC, $E_1 = 1.2 + 1.3 = 2.5$ volts, $R_1 = 6$ ohms; in line $AB'C$, $E_2 = 1.5$ volts, $R_2 = 4$ ohms; line AC, $R_3 = 2$ ohms. By means of Kirchhoff's laws find the magnitude and direction of the current I_3.

589. Find the magnitude and direction of the current in AC when cell E_2 is reversed.

CHAPTER X

ELECTRICITY (*Continued*)

ELECTROLYSIS

176. Dissociation.—Acids, bases, and salts when placed in water undergo, to a certain degree, dissociation into positive and negative ions. The following reactions illustrate a few typical cases of dissociation in water.

HCl in water $= \overset{+}{H} + \overset{-}{Cl}$	Na_2SO_4 in water $= \overset{+}{Na} + \overset{+}{Na} + \overset{--}{SO_4}$
KCl in water $= \overset{+}{K} + \overset{-}{Cl}$	$CuSO_4$ in water $= \overset{++}{Cu} + \overset{--}{SO_4}$
HNO_3 in water $= \overset{+}{H} + \overset{-}{NO_3}$	$ZnCl_2$ in water $= \overset{++}{Zn} + \overset{-}{Cl} + \overset{-}{Cl}$
$AgNO_3$ in water $= \overset{+}{Ag} + \overset{-}{NO_3}$	$NaOH$ in water $= \overset{+}{Na} + \overset{-}{OH}$
H_2SO_4 in water $= \overset{+}{H} + \overset{+}{H} + \overset{--}{SO_4}$	KOH in water $= \overset{+}{K} + \overset{-}{OH}$

177. Electrolytic Reactions.—When a current is passed through an electrolytic cell (Fig. 113) the ions in the electrolyte are deposited upon the electrodes, the positive ions (cathions) appearing on the cathode C and the negative ions (anions) on the anode A. The positive ions always go *with* the current while the negative ions go *against* the current. The amount (mass) of a given ion which is deposited depends upon the strength of the current and the time that it flows.

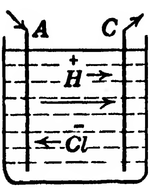

FIG. 113.—Electrolytic cell.

178. Chemical Equivalents.—The *chemical equivalent* of a substance is its atomic weight divided by its valence. The valence of the radicals given in the preceding article is in each case indicated by the number of charges (positive or negative) which the radical carries. For example, the valence of H and of Ag is 1; the valence of Cu and of Zn is 2, and so on.

The following table gives the symbol, atomic weight, valence, and chemical equivalent of a number of elements. For the sake of convenience in computation, values are given to the first decimal place only. See also atomic weights, Table XXX, Appendix.

A *faraday* is the quantity of electricity ($Q = It$) required to liberate a chemical equivalent (gram ion) of a given substance. A faraday = 96,500 coulombs. This means that the passage of 96,500 coulombs through an electrolytic cell will liberate 1 g of H, 8 g of O, 35.5 g of Cl, 107.9 g of Ag, and so on.

Symbol	Atomic wt.	Valence	Chemical eqv.
H.	1 0	1	1 0
O.	16.0	2	8 0
Cl.	35.5	1	35 5
K.	39 1	1	39.1
Na.	23.0	1	23 0
Ag.	107.9	1	107 9
Cu.	63 6	2	31 8
Ni.	58.7	2	29.4
Zn.	65.4	2	32 7

179. Electrochemical Equivalent.—The *electrochemical equivalent* of a substance is the mass in grams deposited by a current of 1 amp. flowing for 1 sec. By international agreement the electrochemical equivalent of silver is 0.001,118 g per coulomb.

180. Faraday's Laws of Electrolysis.—The laws governing the deposition of ions by electrolysis, first announced by Faraday, may be stated as follows:

I. *The mass of a substance liberated from an electrolyte at either the anode or the cathode is directly proportional to the strength of the current and the time which it flows.* This is equivalent to saying that the mass liberated is proportional to the *quantity* of electricity that passes through the cell.

II. *If the same quantity of electricity be passed through a number of electrolytic cells, each containing a different electrolyte, the masses liberated at the electrodes are proportional to their chemical equivalents.*

Law I may be expressed in equational form as

$$M = ZIt,$$

where M is the mass in grams liberated, Z is its electrochemical equivalent of the element, I the current in amperes, and t the time in seconds.

Example.—A current of 0.5 amp. flows through a cell containing a solution of $AgNO_3$ for 1 hr. How much silver is liberated at the cathode? *Solution:* $M = 0.001{,}118 \times 0.5 \times 60 \times 60 = 2.01$ g.

Law II gives us a means of computing the mass of an element deposited in terms of chemical equivalents. Let C and C' represent the respective chemical equivalents of two given substances. Then

$$\frac{M}{M'} = \frac{C}{C'}.$$

Example.—A given quantity of electricity passes through two electrolytic cells, one containing $AgNO_3$ and the other containing $CuSO_4$. Five grams of silver are deposited in the one cell. How many grams of copper (Cu) will be deposited in the other cell? *Solution:* $5/M' = 107.9/31.8$. Hence $M' = 1.47$ g of copper.

If as a basis for computation we start with the electrochemical equivalent of silver as 0.001,118 and its chemical equivalent as 107.9, it follows from law II that the electrochemical equivalent for any element may be computed

in terms of its chemical equivalent. If we let Z and C be the electrochemical equivalent and the chemical equivalent, respectively, of a given element, then

$$\frac{0.001,118}{Z} = \frac{107.9}{C}.$$

Example.—The electrochemical equivalent of Ag is 0.001,118 and the chemical equivalent is 107.9. Find the electrochemical equivalent of H, its chemical equivalent being 1. *Solution:* $0.001,118/Z = 107.9/1$. Hence the electrochemical equivalent of $H = Z = 0.000,010,36$ g per coulomb.

Problems

590. (*a*) What is the distinction between chemical equivalent and electrochemical equivalent? (*b*) What is the distinction between a farad and a faraday?

591. The passage of one faraday (96,500 coulombs) of electricity through an electrolyte will liberate how many grams of hydrogen; oxygen; silver?

592. From the data given in Arts. 178 and 179, compute the electrochemical equivalents of copper, nickel, zinc.

593. A Daniell cell furnishes a current of 0.1 amp. for 1 hr. (*a*)How much zinc goes into solution from the negative electrode? (*c*) How much copper is deposited on the positive electrode?

594. (*a*) What time will be required to deposit 1 g of silver electrolytically with a current of 2 amp.? (*b*) How much copper will be deposited by the same current in the same time?

595. What time will be required to decompose electrolytically 90 g of water by a current of 2 amp.? How many grams of oxygen will be liberated?

596. In a certain test it was found that a current of 4.5 amp. would decompose 18 g of water in 12 hr. From these data compute the electrochemical equivalent of oxygen.

597. A current passes through two electrolytic cells in series; one contains a solution of silver nitrate, the second a solution of copper sulphate. It is found that 2.7 g of silver are deposited. Calculate the mass of copper deposited.

598. How much will a metal plate be increased in weight if it be nickel-plated by a current of 0.5 amp. running 5 hr., taking the valence of nickel as 2?

599. A deposit of 6.445 g of Cu is made by a current flowing for 1 hr. through a copper coulometer. What quantity (coulombs) of electricity passed through the cell?

600. An ammeter is calibrated by means of a silver coulometer, the ammeter and coulometer being in series. A constant current is passed through both instruments for 1 hr. The reading of the ammeter during this time is 0.76 amp.; the amount of silver deposited in the platinum bowl of the coulometer is 3.0186 g. Find (*a*) the error in the ammeter reading; (*b*) the percentage of error.

ELECTRIC HEAT AND POWER

181. Electric Energy Expended as Heat.—The energy expended by a current in the form of heat is proportional to the square of the current and the time which it flows; that is,

$$\textit{energy} = W = I^2Rt = EIt.$$

When I, R, and t are given in c.g.s. units, W is expressed in *ergs*. When, however, I is the current in amperes, R is ohms, and t is seconds, the energy is expressed in *joules*.

Since 1 cal. = 4.186×10^7 ergs = 4.186 joules, 1 joule = 1/4.186 = 0.24 cal., and consequently the heat developed is

$$H = \textit{amperes}^2 \times \textit{ohms} \times \textit{seconds} \times 0.24 = \textit{calories}.$$

$$B.t.u. = \frac{\textit{calories}}{252}.$$

182. Electric Power.—As formulated in the preceding article, *electrical energy* = $W = I^2Rt = EIt$. Power is the expenditure of energy per unit of time; that is

$$\textit{power} = \frac{W}{t} = I^2R = EI.$$

When I, R, and E are given in c.g.s. units, power is expressed in *ergs per second*. When, on the other hand, I is expressed in amperes, R in ohms, and E in volts, then power is expressed in *joules per second*. Since the expenditure of energy at the rate of a joule a second is a watt,

$$\textit{amperes}^2 \times \textit{ohms} = \textit{volts} \times \textit{amperes} = \textit{watts}.$$

Since 1 hp. is equivalent to 746 watts, we may therefore write

$$\textit{horsepower} = \frac{\textit{watts}}{746}$$

183. Watt-hours. Kilowatt-hours.—The total amount of energy expended by a current may be measured (*a*) in terms of the amount of metal deposited in an electrolytic cell; (*b*) in terms of the quantity of heat generated; and (*c*) in terms of power multiplied by the time. The last-named method is the one most usually employed in measuring the total energy expended by a current. The units are the *watt-hour*, or more commonly the *kilowatt-hour*. A kilowatt-hour is 1000 watt-hours. We may therefore write

$$\textit{watt-hours} = \textit{watts} \times \textit{time in hours},$$

and

$$\textit{kilowatt-hours} = \frac{\textit{watt-hours}}{1000}$$

Example.—A current of 0.5 amp. flows through an incandescent lamp for 5 hr. Find (*a*) the power expended in watts; (*b*) the energy expended in watt-hours; (*c*) in kilowatt-hours; (*d*) in ergs; (*e*) joules. *Solution:* (*a*) $EI = 110 \times 0.5 = 55$ watts; (*b*) $55 \times 5 = 275$ watt-hr.; (*c*) $275/1000 = 0.275$ kw-hr. (*d*) $55 \times 10^7 \times 5 \times 60 \times 60 = 99 \times 10^{11}$ ergs; (*e*) $(99 \times 10^{11})/10^7 = 99 \times 10^4$ joules.

Problems

601. A current of 5 c.g.s. units flows through a conductor for 2 min. The energy expended in heat is 24 joules. Find (*a*) the energy expended in ergs; (*b*) the resistance of the conductor in c.g.s. units; (*c*) in ohms.

602. Suppose that we have a battery of five storage cells. A wire of 10 ohms resistance is connected to the terminal of the battery. A current of 0.8 amp. flows through the circuit. Find the heat developed in the wire in 10 min. in (*a*) calories; (*b*) joules; (*c*) ergs.

603. A car heater, supplied with a pressure of 550 volts, uses a current of 5 amp. Find (*a*) the resistance of the heater; (*b*) the calories developed per hour.

604. How much energy, in joules, is used by a 55-watt lamp burning 10 min.?

605. A bank of incandescent lamps takes 10 amp. at 110 volts, 80 per cent of the energy received being given off as heat. Find the heat given off in 1 hr. in (*a*) calories; (*b*) joules.

606. Two wires, *A* and *B*, of the same material and length are connected in series. The diameter of *A* is to that of *B* as 2:1. The resistance of *A* is 2 ohms. The applied e.m.f. is 10 volts. Find the heat in calories generated in *B* during 10 sec.

607. An electrolytic cell for the purification of copper has a resistance of 0.01 ohm. How much heat will be developed in this cell per minute at a time when 10 g of copper are being deposited per minute?

608. Find the horsepower required to operate 150 incandescent lamps, each taking 0.45 amp. at 110 volts.

609. Find the current used by an 8-hp. 500-volt motor if its efficiency is 90 per cent.

610. Find the current used if 11 kw. are to be transmitted (*a*) at 110 volts; (*b*) at 2200 volts.

611. For the conditions given in problem 610, compare the heat losses in the lines in a given time, assuming that the resistances of the two lines are the same.

612. A storage battery has an e.m.f. of 60 volts and an internal resistance of 0.8 ohm. When it is being charged a current of 20 amp. is used. (*a*) What is the applied e.m.f., and (*b*) what is the rate (power) at which energy is stored in the battery?

613. Assuming an efficiency of 50 per cent, what current would be used by a 220-volt electric hoist when raising 2200 lb., at the rate of 30 ft. per min.?

ELECTROMAGNETICS

184. Magnetic Effect of a Current.—When an electric current flows through a conductor there is set up in the medium around the conductor magnetic lines of force. The direction and sense of these lines may be determined by the right-hand rule, which is stated as follows: *Grasp the conductor with the right hand, the thumb pointing in the direction of the current,*

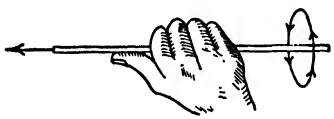

FIG. 114.—Thumb indicates direction of current; fingers, direction of magnetic field.

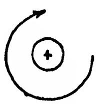

FIG. 115. "In" current.

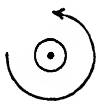

FIG. 116. "Out" current.

and the fingers will indicate the sense of the lines of magnetic induction (Fig. 114). If, on the other hand, we consider that we are looking at the end of the conductor, the right-hand rule again gives us the sense of the magnetic field. In Fig. 115 there is represented a conductor, viewed "end-on," in which the curent is represented as flowing into the paper; the lines of force are clockwise in sense. In Fig. 116 the current is represented as coming out; the field is counterclockwise in sense. It should be noted that a cross (Fig. 115) representing the feathered end of an arrow, indicates an *in* current; a dot (Fig. 116) representing the point of the arrow, indicates an *out* current.

185. Deflection of Magnetic Needle.—A magnetic needle in a magnetic field always tends to set itself in such a position that the lines of force of the field enter the S-pole and come out of the N-pole. Suppose that a wire carrying a current is placed in a north-south position directly above a magnetic needle. The needle is now acted upon by two fields, the earth's field in a north-south direction, and the field due to the conductor, at right angles to the earth's field. The position of the needle is such that the respective torques due to the two fields are equal to each other.

Problems

614. Draw a line to represent a magnetic needle, the N-pole being to the right. Draw a second line AB, parallel to the first, representing a conductor carrying a current flowing from left to right, that is, from A to B. Make drawings now to indicate the direction of the deflection of the N-pole of the needle when (*a*) the conductor AB is above the needle; (*b*) below the needle; (*c*) beside the needle, and between it and the observer.

615. A wire carrying a current lies in a north-south direction. Find the direction and sense of the current for the following deflections of the N-pole of the needle: (*a*) needle above the wire, N-pole to the east; (*b*) needle on left side of the wire, N-pole down.

616. A circular coil of wire carrying a current lies in a north-south vertical plane. The current flows in a clockwise sense where observed from the west. On which side of the coil (east or west) must an N-pole be placed in order that repulsion shall occur?

617. A circular conductor lies in the plane of the paper. Determine by the right-hand rule the polarity of the face of the coil toward the observer when the current is (*a*) in a counter-clockwise sense; (*b*) clockwise sense.

186. Magnetic Field in a Solenoid.—The magnetic intensity in the interior of a solenoid whose length is great in comparison with its cross-section, or of a solenoid bent into the form of a closed ring, may be shown to be

$$H = 4\pi nI = \frac{4\pi nI'}{10},$$

where H = intensity of the field in gausses; I = current strength in c.g.s. units; I' = current strength in amperes; n = number of turns per centimeter length of the solenoid.

187. Magnetic Induction.—Magnetic induction B is equal to the permeability μ multiplied by the intensity of the magnetizing field H; that is, $B = \mu H$, from which we may write $\mu = B/H$.

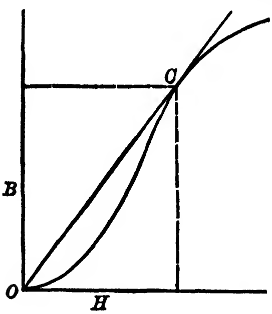

FIG. 117.—Magnetization curve, showing point of maximum permeability.

A study of the magnetization curve of a given sample of iron (Fig. 117) reveals the fact the μ is not a constant but varies from point to point, being greatest at the point of tangency of the line OC with the curve.

188. Magnetic Flux.—Magnetic induction B may be considered as the number of lines of magnetic induction passing through a substance per unit

area. Magnetic flux ϕ through a given area A is the total number of lines passing through the area; that is, $\phi = BA$.

The c.g.s. unit of flux is the *maxwell*.

Example.—A helix 1 m long containing 2000 turns, and having a cross-sectional area of 10 cm^2, carries a current of 10 amp. The length of the helix is sufficiently great so that the end effects may be ignored with reference to the field near its middle point. The value of μ is 5. Find (*a*) the intensity of the field H near the middle of the helix; (*b*) the induction B; (*c*) the flux ϕ. *Solution:* (*a*) The number of turns per centimeter length $n = {}^{2000}\!/\!_{100} = 20$. $H = 4\pi \times 20 \times {}^{10}\!/\!_{10} = 80\pi$ gausses. (*b*) $B = 5 \times 80\pi = 400\pi$. (*c*) $\phi = 400\pi \times 10 = 4000\pi$ lines of induction.

189. Magnetomotive Force.—If a magnetic pole m be carried once around the magnetic circuit within a closed solenoid against the magnetic field, W units of work will be done. *Magnetomotive force* (m.m.f. = Ω) is numerically equal to the work done in carrying a unit pole once around the magnetic circuit; that is, m.m.f. = W/m. If we let l equal the length of the magnetic circuit, n the number of turns of wire per unit length, and N the total number of turns ($n = N/l$), then since $W = Fl$, and $F = Hm$, we may write

$$\textit{magnetomotive force} = \Omega = \frac{W}{m} = 4\pi NI = \frac{4\pi NI'}{10},$$

where I = current in c.g.s. units, and I' = current in amperes.

The c.g.s. unit of m.m.f. is the *gilbert*, which is equal to 1 erg per unit pole. The practical unit of magnetomotive force is the *ampere-turn* NI', where N is the number of turns in the coil, and I' is the current in amperes.

190. Magnetic Reluctance.—In the case of a magnetic circuit we have an equation which is very similar to Ohm's law ($I = E/R$) for electrical circuits,

$$\phi = \frac{\Omega}{\mathcal{R}}$$

in which ϕ = magnetic flux ($\phi = BA$); Ω = magnetomotive force; and $\mathcal{R}$ = *magnetic reluctance* = $l/\mu A$, where μ is the permeability of the medium, l the length of the magnetic circuit, and A its cross-sectional area. Then magnetic reluctance is

$$\mathcal{R} = \frac{\Omega}{\phi} = \frac{l}{\mu A}$$

The c.g.s. unit of reluctance is the *oersted*.

Example.—An insulated wire is wrapped in the form of a spiral around a circular iron ring, the mean length of which is 60 cm and the cross-sectional area is 4 cm^2. The number of turns of wire per centimeter is 3. A current of 5 amp. flows through the wire of the solenoid. The permeability μ of the iron for the given conditions is 400. Find (*a*) the intensity of the magnetic field H; (*b*) the magnetic induction B; (*c*) the flux ϕ; (*d*) the magnetic reluctance $\mathcal{R}$; (*e*) the magnetomotive force by two methods. *Solution:* (*a*) $H = 4\pi \times 3 \times {}^{5}\!/\!_{10} = 6\pi$ gausses. (*b*) $400 \times 6\pi = 2400\pi$, lines of induction/cm^2. (*c*) $\phi = 2400\pi \times 4 = 9600\pi$ lines. (*d*) $\mathcal{R} = 60/(400 \times 4) = 0.0375$. (*e*) *M.m.f.* $= 4\pi NI = \phi\mathcal{R} = 360\pi$ ergs/unit pole.

191. Magnetization Curves.—In Fig. 118 there are shown the magnetization curves for three different samples of iron.

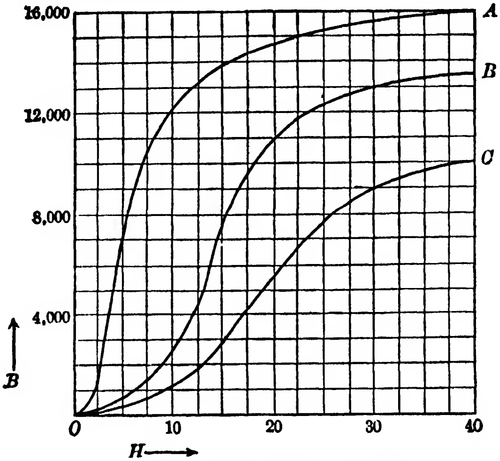

FIG. 118.—Magnetization curves.

192. Magnetic Hysteresis.—*Hysteresis* is a lagging of the induction behind the magnetizing field. A hysteresis curve (Fig. 119) is a loop mapped out during a complete cycle represented by the closed curve $pab'p'a'bp$. Starting at o the field is increased from o to $+H$, that is, from zero to 40 gausses. The magnetization is represented by the broken line op. The

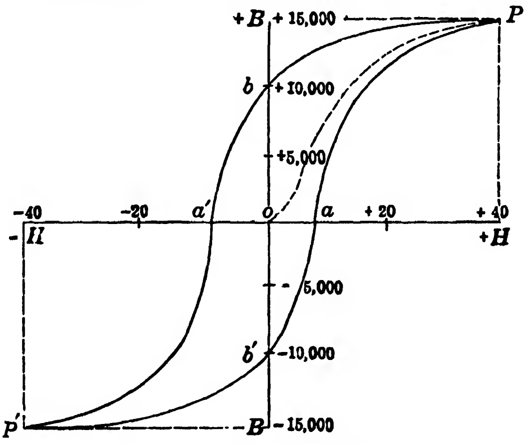

FIG. 119.—Hysteresis curve.

field intensity is now reduced from $+H$ to zero, reversed and carried to $-H$, then back to zero, then reversed again and taken to $+H$, thus completing the cycle represented by the closed curve pp'. When the field is at a the induction is zero; when $H = o$ the induction is represented graphically by the lines $ob' = ob$, equal in this case to 10,000 lines per cm².

The *remanence* is the induction B which remains in the iron when the field H is reduced to zero, represented by the line ob. The *coercive force* is the negative field intensity required to reduce the induction to zero, represented by the line oa'.

Problems

618. Explain each term in the following equations: $H = 4\pi nI$; $B = \mu H$; $\phi = BA$; $m.m.f. = 4\pi NI$; $\mathcal{R} = l/\mu A$; $\phi = \Omega/\mathcal{R}$.

619. A solenoid of cross-sectional area 5 cm², having 1200 turns of wire, and bent in the form of a ring of mean radius 10 cm, carries a current of 5 amp. Consider the permeability of the air within the coil to be unity. Find (*a*) the intensity of the field H within the coil; (*b*) the induction B; (*c*) the flux ϕ.

620. Suppose that the solenoid of problem 619 is filled with an iron core of permeability 500, for the given conditions. Find (*a*) H; (*b*) B; (*c*) ϕ; (*d*) $\mathcal{R}$; (*e*) Ω.

621. A current of 5 amp. flows through a solenoid consisting of 500 turns of wire, and bent in the form of a ring having a mean radius of r cm. Find the radius r such that H shall equal 50 gausses.

622. A solenoid of $25/\pi$ turns per cm and a c.s.a. of 5 cm², bent in the form of a ring having a mean radius of 8 cm, carries a current of 5 amp. Find the m.m.f. (*a*) in gilberts; (*b*) in ampere turns.

623. A solenoid, the axis of which is a straight line 1 m in length, contains 2000 turns of wire. The c.s.a. is 40 cm². The length of this solenoid is sufficiently great so that end effects may be ignored. A current of 5 amp. flows through the wire. Assuming that the permeability is equal to unity, find the magnetic flux in maxwells.

624. Since B/H is the tangent of an angle (Fig. 117), show that μ will be a maximum for that point on the magnetization curve when a straight line drawn from O touches the curve tangentially on its upper side.

625. From Fig. 118 find the permeability μ for (*a*) curve A, $H = 40$; (*b*) curve B, $H = 20$.

626. Compute the values of μ for curve C (Fig. 118) when (*a*) $H = 30$; (*b*) $H = 40$.

627. Consider the hysteresis curve (Fig. 119). (*a*) What is the remanence at the point b? (*b*) What is the approximate value of the coercive force at a'?

193. Electromagnetic Induction.—If a magnet be thrust into a coil of wire (Fig. 120), an induced e.m.f. is set up in a given sense; if the magnet be withdrawn an e.m.f. is set up in an opposite sense. In general, whenever a conductor cuts across the lines of force of a magnetic field, an induced e.m.f. is set up in the conductor. It is important to know the relation which exists between the motion of the conductor or field and the direction and sense of the induced e.m.f. There are a number of rules which may be employed to advantage to determine the direction and sense of the induced e.m.f., two of which are:

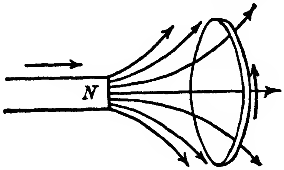

FIG. 120.—Induced e.m.f. in coil.

Rule 1.—For the case of a circular conductor, the following rule is serviceable: Consider that the observer is looking in the direction and sense of the field, that is, from left to right in the case shown in Fig. 120. An *increase* of the number of lines threading through the coil gives an *indirect* (counterclockwise) e.m.f., a *decrease* in the number of lines gives a *direct* (clockwise) e.m.f.

Rule 2.—The right-hand rule, which was employed to determine the direction and sense of the magnetic field about a conductor, may be used also to determine the sense of the induced e.m.f.

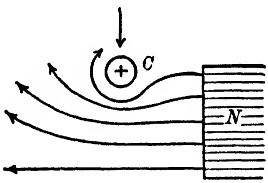

FIG. 121.—"In" e.m.f.

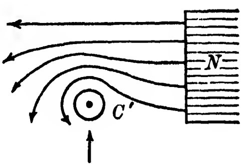

FIG. 122.—"Out" e.m.f.

In Fig. 121 we have an end view of a metal rod falling vertically through a magnetic field. Each line on being cut by the conductor C may be conceived of as tending to wrap itself around the rod. *Grasp the conductor with the right hand, the fingers being in the direction and sense of the lines of force of the field, and the thumb will extend in the direction and sense of the induced e.m.f.* According to this rule the e.m.f. in the conductor C (Fig. 121) is directed *into* the paper; the e.m.f. in C' (Fig. 122) is directed *out.*

Problems

628. By means of the right-hand rule determine the direction and sense of the induced e.m.f. when the conducting rod R (Fig. 123) moves in the direction (*a*) from R to A; (*b*) from R to B; (*c*) R to C; (*d*) R to D; (*e*) R to E; (*f*) R to F.

629. Find the direction and sense of the induced e.m.f. when the conductor (Fig. 124) moves in the circular path (*a*) from

A to B; (b) C to D. (c) At what two points does the sense of the induced e.m.f. change?

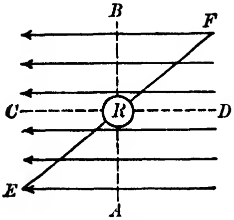

FIG. 123.

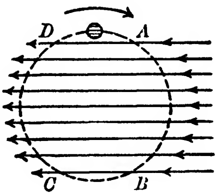

FIG. 124.

194. Magnitude of the Induced E.M.F.—Quantitative experiments have shown that the relation of the induced electromotive force E to the number of turns N of the conductors and the time rate of change of the flux is

$$E = -\frac{N(\phi - \phi')}{t} \text{ c.g.s. units} = -\frac{N(\phi - \phi')}{(t \times 10^8)} \text{ volts,}$$

where N = the total number of turns in the coil, the minus sign indicating that the induced e.m.f. is opposed to the action producing it; $\phi - \phi'$ = the *change* of flux in the time t seconds. Special note should be made of the fact (a) that E represents *average* values of the e.m.f. during the time t, and (b) that $(\phi - \phi')/t$ may represent either a time rate of change of flux through a given coil, or a time rate of cutting of lines of force in the magnetic field.

Example 1.—A given coil of wire consisting of five turns, encloses 1000 lines of induction. The flux changes from 1000 to 200 in 0.1 sec. Find (a) the time rate of change of flux; (b) the average E in c.g.s. units; (c) in volts. *Solution:* (a) $(\phi - \phi')/t = (1000 - 200)/0.1 = 8000$ lines per sec. (b) $E = 5 \times 8000 = 40{,}000$ c.g.s. units. (c) $E = 40{,}000/10^8 = 0.0004$ volt.

Example 2.—A single straight conductor 1 m in length is moved across a magnetic field at right angles to the direction of the lines of force with a uniform speed of 1 m in 2 sec. The induction B of the field is 800. Find the average E induced in the conductor in (a) c.g.s units; (b) in volts. *Solution:* When the conductor moves across the field through a distance of 1 m = 100 cm, the area enclosed = $A = 100 \times 100 = 10{,}000$ cm². Flux $\phi = BA = 800 \times 10{,}000 = 8{,}000{,}000$ lines. The time involved is $t = 2$ sec. Then (a) $E = 8{,}000{,}000/2 = 4{,}000{,}000$ c.g.s. units; (b) $E = 4{,}000{,}000/10^8 = 0.04$ volt.

195. Self-induction.—When a current is changing (increasing or decreasing) there is set up in the circuit at every instant a counter e.m.f. of self-induction, which tends to oppose the change. This opposition to change in an electromagnetic system is formulated by Lenz's law, which may be stated as follows: *When a change occurs in an electromagnetic system, that thing happens which tends to oppose the change.* Thus, if an N-pole is thrust into a closed circuit, (Fig. 125) the induced current will be of such a nature as to produce in the coil an n-pole, which will tend to oppose the thrusting in of the N-pole of the magnet; in a like manner when the N-pole is with-

drawn from the coil, the induced current will produce an s-pole, thus again opposing the change.

The e.m.f. of self-induction may be expressed by the equation

$$E = -\frac{N(\phi - \phi')}{t} \text{ c.g.s. units} = -\frac{N(\phi - \phi')}{(t \times 10^8)} \text{ volts,}$$

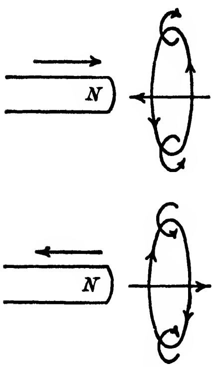

FIG. 125.

where E = average e.m.f. of self-induction in the time t; N = total number of turns of wire in the coil; and $\phi - \phi'$ = change of flux in t sec. The *minus* sign indicates that the induced e.m.f. is opposed to the action producing it. In the equations which follow this sign will be omitted.

196. Coefficients of Induction. *Self-induction.*—The flux ϕ in a given coil multiplied by the number of turns N of the coil is called the *coil flux, $N\phi$*. Now the coil flux is proportional to the current I flowing in the coil, therefore by introducing the proportionality factor L we may write $N\phi = LI$, where L is the *coefficient of self-induction*, or the *inductance*, as it is sometimes called.

Starting with the equation $N\phi = LI$, we may define L in terms of two equations as follows: First, by differentiating with respect to t, we have

$$\frac{Nd\phi}{dt} = \frac{Ldi}{dt}.$$

But

$$\frac{Nd\phi}{dt} = \frac{N(\phi - \phi')}{t} = E,$$

from which we have

$$L = \frac{E}{(di/dt)}.$$

We may therefore define L as the ratio of the induced e.m.f. to the time rate of change of current.

Second, let $N = nl$, where n is the number of turns per unit length of the coil and l is its length, and $\phi = BA = \mu HA = 4\pi\mu nAI$. Then by substituting these values of N and ϕ in the equation $N\phi = LI$, we have

$$nl \times 4\pi\mu nAI = LI,$$

from which

$$L = 4\pi\mu n^2lA \text{ c.g.s units} = \frac{4\pi\mu n^2lA}{10^9} \text{ henrys.}$$

Mutual Induction.—The *mutual induction* of two coils is the ratio of the e.m.f. induced in one of the coils to the time rate of change of current in the other. The equation is

$$M = \frac{E}{(di/dt)}$$

in which E = induced e.m.f. in one coil; di/dt = time rate of change of the current in the other coil; and M = coefficient of mutual induction.

The value of M may be calculated in terms of the constants of the coils, provided all the lines of force due to the current in one helix pass through all the coils of the other helix. This condition may be secured in a practical way by winding the helices on an iron ring, such as is shown in the Faraday transformer ring of Fig. 153, page 189. The coefficient of mutual induction may then be expressed in terms of the equation

$$M = 4\pi\mu n_1 n_2 lA \text{ c.g.s. units} = \frac{4\pi\mu n_1 n_2 lA}{10^9} \text{ henrys,}$$

where μ = permeability of the medium enclosed by the coils; n_1 = number of turns per cm of one helix and n_2 the number of turns per cm of the other; l = mean length of the ring = $2\pi r$, where r = radius of the ring; and A = cross-sectional area of the core.

197. Units of Inductance.—Self-induction L and mutual induction M are physical quantities of the same nature, and hence the same unit may be used for both.

The c.g.s. *unit of inductance* is an inductance such that a change of one c.g.s unit of current per second will give rise to one c.g.s unit of e.m.f.

The *practical unit* of inductance is the *henry*, which is equal to 10^9 c.g.s. units of inductance. The henry is an inductance such that a current changing at the rate of 1 amp. per sec. will give rise to an induced e.m.f. of 1 volt.

Example 1.—In a coil the inductance of which is 5×10^6 c.g.s. units, a current of 12 amp. drops to 2 amp. in $1/10$ sec. Find the induced e.m.f. developed in volts. *Solution:* $L = \frac{5 \times 10^6}{10^9} = \frac{1}{200}$ henry. $E = \frac{L(I - I')}{t}$ $= (1/200)(12 - 2)/(1/10) = 0.5$ volt.

Example 2.—A helical coil of wire of cross-sectional area 4 cm² bent into the form of a ring of mean radius $60/\pi$ cm, consists of 3 turns per centimeter. Find the inductance of this coil (a) in c.g.s units and (b) in henrys, when it is filled with a substance having permeability of 200. *Solution:* $L = 4\pi\mu n^2 lA = 4\pi \times 200 \times 9 \times 120 \times 4 = 3{,}456{,}000\pi$ c.g.s. units = 0.003456π henry.

198. Energy Stored in the Field.—If we assume that in a circuit of self-inductance L the current rises from zero to I in t sec., then the average current during the time is $I/2$, and the quantity of electricity flowing through the circuit is $Q = It/2$. Also, since the time rate of change of the current (di/dt) is uniform, the induced e.m.f. is constant during the time t, and $E = LI/t$. Now the energy stored in the system during the time t (that is, while the current is rising from zero to I) is represented by the quantity $EQ = (LI/t) \times (It/2) = LI^2/2$, and hence we may write

$$EQ = W = \frac{LI^2}{2},$$

where W = energy stored in the circuit of inductance L, due to the rise of the current from zero to I. W is expressed in *ergs* when L and I are in c.g.s. units, and in *joules* when L is in henrys and I is in amperes.

199. Relation of Practical to Electrostatic and Electromagnetic Units.—The relation of the practical units of current, quantity, resistance, potential,

capacitance, and inductance to the corresponding electrostatic units (e.s.u.) and electromagnetic units (e.m.u.) is given in the following table.

PRACTICAL, ELECTROSTATIC, AND ELECTROMAGNETIC UNITS

Practical units	Electrostatic c.g.s. units	Electromagnetic c.g.s. units
1 ampere	10^{-1}	3×10^{9}
1 coulomb	10^{-1}	3×10^{9}
1 ohm	10^{9}	$1/(9 \times 10^{11})$
1 volt	10^{8}	$1/300$
1 farad	10^{-9}	9×10^{11}
1 microfarad	10^{-15}	9×10^{5}
1 henry	10^{9}	$1/(9 \times 10^{11})$

Problems

630. Explain each term in the following equations:

$$E = -\frac{N(\phi - \phi')}{t} = -\frac{N(\phi - \phi')}{(t \times 10^{8})} = -\frac{Nd\phi}{dt} = -\frac{Ldi}{dt}; \quad L = \frac{E}{(di/dt)} = 4\pi\mu n^2 lA = \frac{4\pi\mu n^2 lA}{10^9}; \quad E = \frac{Mdi}{dt}.$$

631. State Lenz's law, and make drawings to illustrate application of the principle.

632. A conductor cuts across a uniform magnetic field of area 1 m², having an induction (B) of 500 lines, in 0.1 sec. Find the average induced e.m.f. in (*a*) c.g.s. units; (*b*) volts.

633. A helical coil a meter long and 50 cm² in cross-sectional area has 10,000 turns. Compute the inductance of this coil, assuming that $\mu = 1$.

634. If the current of the coil (problem 633) increases from 0 to 5 amp. in 0.1 sec., what will be the average induced current?

635. (*a*) If 110 volts be applied to the coil of problem 633, at what rate will the current increase at the instant the circuit is closed? (*b*) Why will the rate of increase be less immediately afterward?

636. An iron ring whose magnetization curve is B (Fig. 118) has a mean radius of 8 cm and a sectional area of 10 cm² and is wound with 600 turns of wire. Find the current in the wire which will produce a flux of 130,000 lines of induction in the iron.

637. (*a*) Find the coefficient of self-induction of the coil (problem 636) when the flux is 130,000. (*b*) Find the counter

e.m.f. due to self-induction at the instant the flux reaches a value of 130,000, assuming that the current is changing at the rate of 25 amp. per sec.

638. Find the energy in joules stored in the magnetic field of the ring under the conditions stated in problem 637.

639. The mean radius of a Faraday transformer ring is 10 cm; cross-sectional area 10 cm^2; permeability under given conditions 2000; number of turns in the primary per unit length 2; total number turns in the secondary 50. Find the mutual inductance M in (*a*) c.g.s. units; (*b*) henrys.

CHAPTER XI

ELECTRICITY (*Continued*)

ALTERNATING CURRENT PHENOMENA

200. Comparison of Direct and Alternating Currents with Respect to Use.—The subject of alternating currents is not only one of the most interesting phases of electricity but it is in certain respects the most important, because of the almost universal use today of this type of current in the affairs of everyday life.

Direct currents have to be used for electrolytic processes such as silver plating, nickel plating, electrotyping, and copper refining. They are used for charging storage batteries, and for certain manufacturing operations where speed control of the motor is an important factor. Also, on most street car systems, but not all, d.c. power is used.

Aside from the uses mentioned above and a very few others, alternating currents are coming into almost universal use, and for the following reasons:

1. Since on a.c. generators slip rings are used, no commutators being necessary as in the case of d.c. generators, sparking at the brushes is reduced to a minimum. This makes it possible to produce pressures of higher voltage with an a.c. generator than can be safely done with a d.c. generator. Direct current generators may be built with capacities as high as 5000 kw. at pressures as high as 500 volts. Alternating current generators, on the other hand, may be built with capacities of 50,000 kw. or more, at pressures as high as 15,000 volts. The advantages to be derived from large capacity in a generator unit are that the larger the unit the more efficient it is, and the lower is its cost per kilowatt of capacity.

2. The second advantage in the use of alternating currents is that by means of a transformer these currents may be "stepped up" for high-tension long-distance transmission purposes, and then "stepped down" to such voltages as the customer may desire. Transmission voltage may be as high as 100,000 to 200,000 volts. If pressures higher than these are attained, the current is likely to jump from the wires to the poles or towers or other near-by objects. The usual step-down voltage employed for house lighting is 110 volts.

3. The third advantage is that, by the use of alternating currents of high voltage, power may be transmitted more economically than by direct currents. The economy in transmission comes from the fact, first, that because of the extremely high voltages employed, smaller wires may be used, and second, there is less heat loss. The higher the voltage the lower may be the current for a given power unit, and the lower the current the less is the heat loss. Indeed, the main advantage in the use of alternating currents is economy of transmission.

201. The A.C. Generator.—A simple a.c. generator is shown in Fig. 126 and a diagrammatic section of the field elements in Fig. 127. In the generator shown in Fig. 126, six armature wires cut the field twice during each revolution. In Fig. 127 only one wire is considered.

Whenever lines of magnetic force are cut by a conductor, or whenever the number of lines of force threading through a conductor in the form of a coil

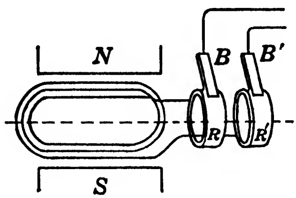

Fig. 126.—Diagram of a simple a. c. generator.

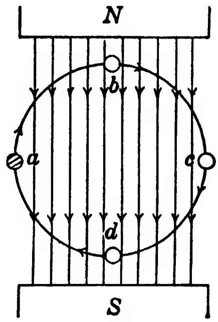

Fig. 127.

is varied, an e.m.f. is always generated in the conductor. It is immaterial whether the conductor moves and cuts the lines or whether the lines move so as to be cut by the conductor. In d.c. generators, it is always the conductors which move. In other words, d.c. generators have rotating armatures. In a.c. generators, especially of the larger capacities, it is usually the

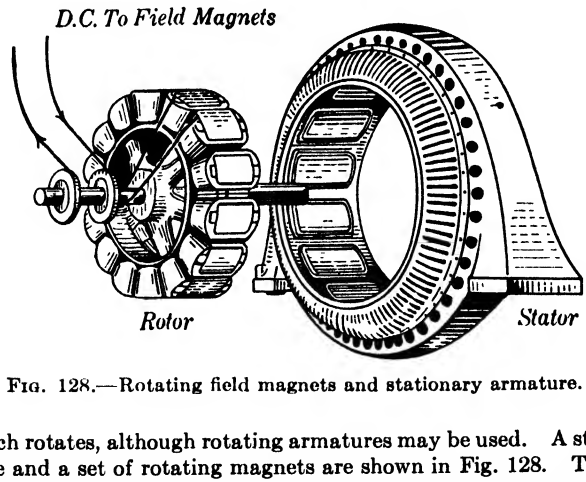

Fig. 128.—Rotating field magnets and stationary armature.

field which rotates, although rotating armatures may be used. A stationary armature and a set of rotating magnets are shown in Fig. 128. The wheel with its magnets is called the *rotor;* the stationary armature, the *stator.* The adjacent poles in a rotating field have opposite polarities; that is, if one is an N-pole, the next is an S-pole, and so on around the entire series. The magnets of the rotating field are excited by a direct current from a d.c.

generator. The direct current is conducted into the field through metal rings which are attached to the shaft of the rotor.

The advantage of the rotating field lies in the fact that the armature is stationary and the high-voltage current can be taken directly from the armature and delivered to the external circuit without the aid of any sliding contacts. Also, the insulation necessary for high voltage is easier to construct and maintain.

202. Voltage and Current Phases.—When an armature coil (Fig. 126) passes through the field generated by successive N- and S-poles, as from *a* to *b* to *c* to *d* and back to *a* again (Fig. 127) there results an alternating e.m.f., and, if the circuit be closed, an alternating current (a.c.). The rise, fall, and reversal of an alternating e.m.f. or current may be pictured graphically by a curve (Fig. 129). That portion of the curve from *a* to *a′* represents one *cycle*, and from *a′* to *a″* another *cycle*. The number of cycles per second is the *frequency, n*. A 60-cycle system is one in which the e.m.f. rises and falls 60 times a second; that is, $n = 60$.

The term "phase," as employed in electricity, is used to denote the condition of change at any point in a cycle. In engineering practice, phase

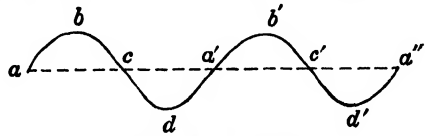

FIG. 129.—Single-phase curve.

is measured in electrical degrees. An *electrical degree* is $1/360$ of a cycle. For example, at the point *a* (Fig. 129) where the alternating e.m.f. or current is just beginning its positive impulse, the phase is zero; at *b* where the e.m.f. or current has reached a maximum positive value, the phase is 90°; at *c* it is 180°; at *d* 270°; and at *a′*, where the cycle is completed, it is 360°. Here the cycle begins to repeat its various phases.

The above statements of phase relations refer in each case to *electrical degrees*. The student should not confuse electrical degrees with mechanical degrees. An electrical degree is $1/360$ of a *cycle;* a mechanical degree is $1/360$ of a *circle*. Since one cycle is completed for every passage of an armature coil through a magnetic field generated by consecutive N- and S-poles, it follows that for one revolution of the armature through 360 mechanical degrees the number of cycles developed depends upon the number of coils in the armature and the number of pairs of poles in the field.

Two alternating e.m.f. or currents may be in phase or out of phase. Two e.m.f. or currents are in phase when any point on one curve has the same number of electrical degrees as the corresponding point on the other curve.

203. Kinds of A.C. Generators with Respect to Phase.—Alternating current generators may be one, two, three phase, or more, depending on the number and manner of connection of the coils of the armatures and the number of poles in the field. A *single-phase* generator may be represented by a single coil rotating in a magnetic field (Fig. 126). The resulting e.m.f. or current curve consists of a single series of cycles (Fig. 129).

In the *two-phase* generator the armature consists of two coils which are set at right angles to each other. If each coil is connected to a separate pair of slip rings, the generator will deliver two separate and distinct currents. These currents differ in phase by 90° (Fig. 130).

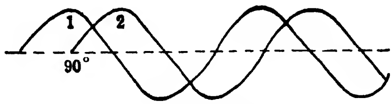

FIG. 130.—Two-phase curves.

The armature of a simple *three-phase* generator has three coils symmetrically placed 120° apart. If the three coils are connected respectively to three different sets of rings, a generator of this type will deliver three separate currents. This, however, would require six line wires, which for long-distance transmission would involve an unnecessary expense because of the large amount of wire that would have to be used. In order to avoid the use of six wires in three-phase systems, armature coils have been devised so that only three wires are necessary. During each series of cycles one wire serves successively as a return circuit for each of the other two. Three-phase generators give three current curves differing in phase from each other

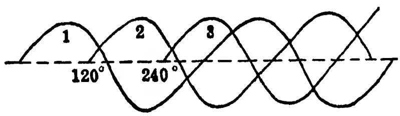

FIG. 131.—Three-phase curves.

by 120° (Fig. 131). In commercial practice three-phase generators are today almost exclusively used. For lighting purposes a 60-cycle current is used. If a lower frequency is employed, a flickering light is produced which is hard on the eyes. For power purposes, however, such as the operation of electric motors, 25-cycle currents give the most satisfactory results.

Problems

640. Consider Fig. 127 in which a single conductor *a* is shown in its various positions (*a*, *b*, *c*, *d*) during one complete revolution in the field *NS*. The corresponding e.m.f. or current curve is given in Fig. 129. At what two positions during a complete revolution have the e.m.f. and current (*a*) minimum values; (*b*) maximum values? (*c*) Give reasons for your answers to *a* and *b*.

641. During the first half revolution in a clockwise sense (from *a* to *b* to *c*), Fig. 127, what are the direction and sense of the induced e.m.f. and current, in or out? (*b*) During the second half turn?

642. At what two points does the induced e.m.f. (*a*) have zero values? (*b*) Change from positive to negative values?

643. Assume that the field flux between N and S (Fig. 127) is 20,000 lines. (*a*) How many times are the lines cut by the conductor a during one revolution? (*b*) This is equivalent to a total cutting of how many lines?

644. The six conductors shown in Fig. 126 make 600 revolutions per minute (r.p.m.) in a field of 20,000 lines of force. The magnitude of the induced e.m.f. developed is proportional to the field strength, the speed of rotation of the armature, and the number of conductors involved. (*a*) What is the period T (time of one revolution)? (*b*) How will the induced e.m.f. in this case compare with that developed by the single conductor of problem 643, the period in each case being the same?

645. (*a*) Explain the meaning of the terms "rotor" and "stator." (*b*) What are some of the advantages of using rotating magnetic fields in a.c. generators?

646. (*a*) By means of a curve explain the meaning of *cycle.* (*b*) Distinguish between electrical and mechanical degrees.

647. (*a*). Explain the meaning of *phase.* (*b*) Draw curves to represent two alternating currents differing in phase by 90°.

VALUES OF E AND I

204. Methods of Representing Alternating E.M.F. and Currents.—A harmonic alternating e.m.f. or current is one which obeys the laws of simple

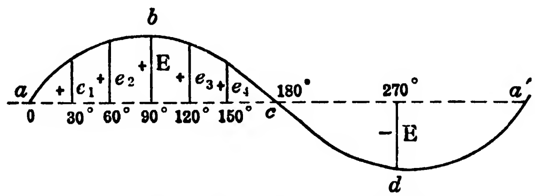

FIG. 132.—Alternating e.m.f. curve.

harmonic motion. An harmonic e.m.f. and current may be represented by means of (*a*) a sine curve, (*b*) a clock diagram, or (*c*) an algebraic equation.

(*a*) In Fig. 132 we have represented an harmonic alternating e.m.f. The letters e_1, e_2, e_3, etc., represent instantaneous values; +E and −E represent maximum values. A similar curve may be used to represent an alternating current. The portion of the curve shown in Fig. 132 represents a cycle, in which e_1 is the instantaneous value at the 30° phase, e_2 the instantaneous value at the 60° phase, E is the instantaneous (maximum) value at the 90° phase, and so on to the end of the cycle.

(*b*) The cycle shown above may also be represented by means of the rotation of a line (Fig. 133) which is numerically equal to **E**, and which rotates in a counterclockwise sense. This is known as the clock diagram method. When the phase is zero, **E** lies on the horizontal axis, as shown in Fig. 133, *A*; *B* illustrates the 30° phase; *C*, the 120° phase.

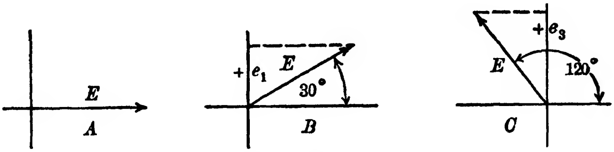

FIG. 133.—Use of clock diagram to illustrate an alternating *E* or *I*.

(*c*) The third method of representing an harmonic electromotive force and current is by means of the equations

$$e = \mathbf{E} \sin \omega t,$$
$$i = \mathbf{I} \sin \omega t,$$

where e and i = instantaneous values; **E** and **I** = maximum values; ω = angular velocity (radians per second) $= 2\pi/T$; and ωt = phase angle.

205. Average Values of E and I.—Consider the values of e of the sine curve for a half cycle, as shown in Fig. 132. The quantities e_1, e_2, etc., represent numerical values of the sines of a series of angles, varying from 0° to 90° to 0°.

An average value for e (or i) may be found approximately by getting the mean of a series of these values throughout the half cycle. The true average for the half cycle, as determined by methods of the calculus, is

$$AE = 0.636\mathbf{E},$$
$$AI = 0.636\mathbf{I},$$

where AE and AI = average values; and **E** and **I** = maximum values

206. Effective Values of E and I.—When an alternating current or e.m.f. passes through each half cycle, it changes from zero to a maximum value and to zero again, when reversal takes place. In each half cycle there is a *maximum value*, and the average of all the changing values is the *average value*. Now it so happens that a.c. ammeters and voltmeters are calibrated to give neither of these values, but record what are called *effective values*. What is meant by an *effective current*, for example, may be made clear by a consideration of the ampere.

An alternating current really has no unit of its own, so we measure it in terms of the d.c. unit, the ampere. To have the a.c. ampere exactly equal the d.c. ampere, it must produce the same effect as the d.c. ampere. Now an ampere is defined as that steady current which will deposit 0.001,118 g of silver from a standard solution in 1 sec. But an alternating current is not a steady current and neither will it deposit any silver from a solution, since whatever it deposits during one half cycle, it takes off the next half. Accordingly, in order to compare the alternating with the direct current, we must use some other property which both kinds of current possess. The most natural property is the *heating effect* of each. Therefore an alternating cur-

rent is said to be equivalent to a direct current when it produces the same heating effect under exactly similar conditions. This value is called the *effective value* of an alternating current.

The effective value of an alternating e.m.f. or current, designated by the letters E and I, represents the square root of the mean square of all the values taken during a given half cycle. It may be shown that

$$E = 0.707\mathbf{E},$$
$$I = 0.707\mathbf{I},$$

where E and I = effective values; and $\mathbf{E}$ and $\mathbf{I}$ = maximum values.

Average e.m.f. $AE = 0.636\mathbf{E}$ and average current $AI = 0.636\mathbf{I}$, and since $\mathbf{E} = E/0.707$ and $\mathbf{I} = I/0.707$ it follows that

$$E = 1\ 112 \times AE$$
$$I = 1.112 \times AI.$$

Example.—In a given half cycle representing an alternating e.m.f. curve the following ten values for e were found: 20, 60, 80, 90, 95, 90, 80, 60, 20, 0, where 95 represents the maximum $\mathbf{E}$. (*a*) Using these data, find by the square root of the mean square method the approximate effective e.m.f. (*b*) Using the equation, find the true effective e.m.f. *Solution:* (a) Approximate e.m.f. = $(20^2 + 60^2 + 80^2 + 90^2 + 95^2 + 90^2 + 80^2 + 60^2 + 20^2 + 0)/10 = 4602.5$. The square root of this mean square = $\sqrt{4602.5} = 67.8$. (b) By use of the equation we have $E = 0.707\ \mathbf{E} = 0.707 \times 95 = 67.2$ = true effective e.m.f.

207. Summary.—The student should note carefully the following symbols which are employed to represent harmonic e.m.f. and currents:

e and i = instantaneous values of e.m.f. and current,
$\mathbf{E}$ and $\mathbf{I}$ = maximum values,
E and I = effective (square root of mean square) values.

Problems

648. Illustrate and explain three ways of representing an harmonic e or i, as follows: (*a*) sine curve; (*b*) clock diagram; (*c*) equation.

649. Define and illustrate the following: instantaneous, maximum, average, and effective values of e.m.f. and current.

650. (*a*) Find the instantaneous value of an harmonic e.m.f. at 30°, the maximum value of which is 110 volts. (*b*) Find the maximum value of an harmonic e.m.f., the instantaneous at 45° is 60 volts. (*c*) Find the maximum value of an a.c. the instantaneous value of which at the 60° phase is 8 amp.

651. (*a*) Find the effective value of an a.c. whose maximum value is 70 amp. (*b*) An a.c. at 30° has an instantaneous value of 4 amp. To what d.c. is this equivalent?

652. An a.c. flowing through a conductor of resistance 50 ohms for 10 min. develops 28,800 cal. of heat. Find (*a*) the effective value of the current; (*b*) the maximum value.

653. The maximum value of an a.c. for a half cycle is 10 amp. (*a*) What is the square root of the mean square value? (*b*) What reading will an a.c. ammeter in this line show?

654. An a.c. voltmeter is connected to the points *AB* of a straight wire through which there flows an a.c. The resistance between the points *AB* is 40 ohms. The maximum value of the current per half cycle through *AB* is 5 amp. What is the reading of the voltmeter?

INDUCTANCE AND CAPACITANCE IN A.C. CIRCUITS

208. Non-inductive Circuits.—When an alternating e.m.f. is impressed on a non-inductive circuit, that is, one that contains resistance R only (no inductance L) there results an alternating-current which is in phase with the impressed e.m.f. (Fig. 134).

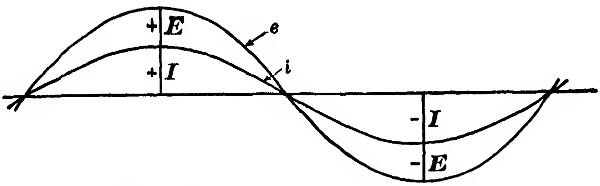

FIG. 134.—Current and e.m.f. in phase.

In this case Ohm's law applies; that is,

$$E = RI,$$

where E and I are effective values (volts and amperes) and R is expressed in ohms.

In a circuit containing inductance or capacitance or both, Ohm's law does not hold. For example, when the circuit contains both resistance R and inductance L, the governing equation for instantaneous values is

$$e = Ri + L\frac{di}{dt}.$$

209. Effects of Inductance.—When a coil of wire is placed in an a.c. circuit, two very important effects may result. (*a*) First, the a.c. current is choked down due to the back e.m.f. of self-inductance in the coil. (*b*) Second, the inductance throws the e.m.f. and current out of phase, the current lagging behind the e.m.f. Inductance tends to cause the current to *lag behind* the e.m.f.; that is to say, the current in an inductive circuit comes to its maximum value at a point which is at a greater number of electrical degrees on the axis than that of the position of maximum e.m.f. Lag is

measured in electrical degrees. In Fig. 135 the current is pictured as lagging behind (to the right of) the e.m.f. by 30°.

The angle of lag is of great significance in the computation of power in an a.c. circuit.

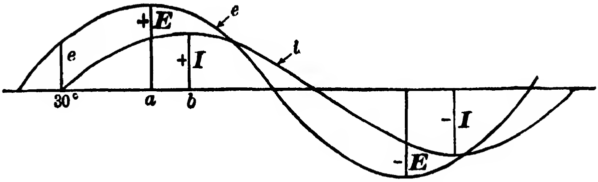

FIG. 135.—Current lagging behind the e.m.f. by 30°.

210. Angle of Lag.—When an harmonic e.m.f. is impressed on a system containing resistance R and inductance L in series, there results an alternating current which lags behind the impressed e.m.f. by an angle ϕ, called the *angle of lag*. In Fig. 135 the angle of lag $= ab = 30°$.

Starting with the equation $i = \mathrm{I} \sin \omega t$, we get by differentiation an expression for the induced e.m.f., $e = -\frac{Ldi}{dt} = -L\omega\mathrm{I} \cos \omega t$. Thus it appears that i is a sine function of the phase angle ωt and the induced e is a cosine function of the same angle. This means that i and e differ in phase by 90 deg. And further, it may be shown from the solution of the differential

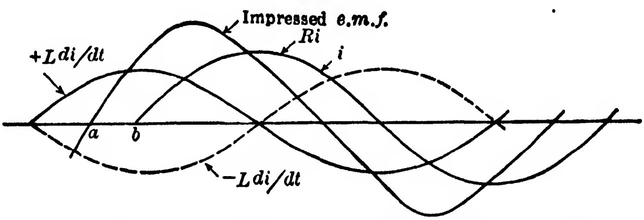

FIG. 136.—Electromotive forces due to R and L in series.

equation $e = Ri + \frac{Ldi}{dt}$ that $-\frac{Ldi}{dt}$ lags behind the current by a quarter of a period (90°), as shown in Fig. 136. The generator then must furnish an Ri e.m.f. in phase with the current, and it must also furnish an e.m.f. having a value of $+\frac{Ldi}{dt}$ to overcome the $-\frac{Ldi}{dt}$ of self-induction. The impressed e.m.f. is the sum of these two e.m.f.'s. The current thus *lags behind* the impressed e.m.f. by an angle represented by ab (Fig. 136).

Now for maximum values of the current and the induced e.m.f. we have $R\mathrm{I}$ and $+L\omega\mathrm{I}$, where $R\mathrm{I}$ is the component of the impressed e.m.f. in phase with the current, and $+L\omega\mathrm{I}$ is the component required to overcome the $-L\omega\mathrm{I}$ of self-induction. These two e.m.f.'s. are at right angles to each other, and therefore may be represented as in the diagram of Fig. 137. The maximum impressed e.m.f. is then $\mathrm{E} = \mathrm{I}\sqrt{R^2 + L^2\omega^2}$.

The angle of lag ϕ may then be found from the equation:

$$\tan \phi = \frac{L\omega}{R}.$$

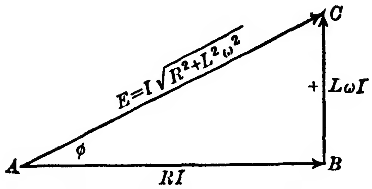

FIG. 137.—E.m.f. diagram.

211. Angle of Lead.—When an harmonic e.m.f. is impressed on a system containing resistance R and capacitance C in series, there results an alternating current which *leads* the impressed e.m.f. by an angle ϕ' which is called the angle of lead.

Starting with the equations $e = q/C$, and $i = I \sin \omega t = dq/dt$, and integrating $dq = I \sin \omega t dt$, we have $q = -\frac{I \cos \omega t}{\omega}$. Now the $e = \frac{q}{C}$ of the condenser is in phase with the charge q, and since i and q are sine and cosine functions of ωt, respectively, it follows that i and e differ in phase by 90°. It may be shown further that $e = +\frac{q}{C}$, the instantaneous e.m.f. of the condenser, leads the current by a quarter of a period (Fig. 138).

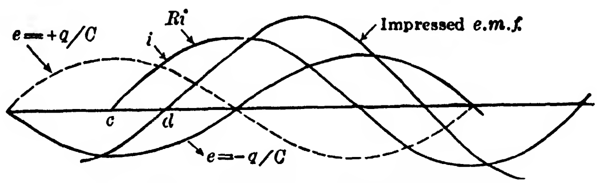

FIG. 138.—E.m.f. due to R and C in series.

Again, as in the case in which we considered the factor of inductance, the generator must supply two e.m.f's., an Ri e.m.fs. in phase with the current, and an $e = -\frac{q}{C}$ to overcome the $+\frac{q}{C}$ of the condenser. The impressed e.m.f., then, is the sum of the Ri and $-\frac{q}{C}$ values. It thus appears that the current *leads* the impressed e.m.f. by an angle represented by cd (Fig. 138).

From the equation $q = -\frac{I \cos \omega t}{\omega}$, we may write for maximum values, $Q = \frac{\mathbf{I}}{\omega}$. And since $Q = CE$, we have $\mathbf{E} = -\frac{\mathbf{I}}{C\omega}$, where $\mathbf{E}$ is the maximum e.m.f. furnished by the generator to overcome the $+\frac{\mathbf{I}}{C\omega}$ of the condenser. The generator then must furnish two e.m.fs., $R\mathbf{I}$ and $-\frac{\mathbf{I}}{C\omega}$, which are in quadrature, and which may be represented as in Fig. 139. In this case $\mathbf{E} = \mathbf{I}\sqrt{R^2 + (1/C\omega)^2}$.

The angle of lead ϕ' may be found from the equation

$$\tan \phi' = \frac{1}{C\omega R}.$$

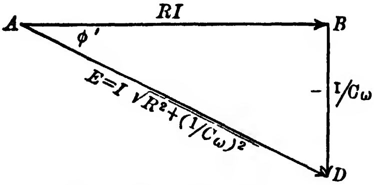

FIG. 139.—E.m.f. diagram.

212. Summary.—In Arts. 210 and 211 we have dealt only with maximum values of e.m.f. and current, that is with **E** and **I**. Since, however, the effective values are equal to a constant times the corresponding maximum values, we may write our equations as follows:

Given R and L in series in an a.c. system, we have

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + L^2\omega^2},$$

or

$$E = I\sqrt{R^2 + L^2\omega^2};$$

and given R and C in series in an a.c. system, we have

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2},$$

or

$$E = I\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}.$$

When the system contains all three factors, R, L, and C, then the equations become

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2},$$

or

$$E = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2},$$

in which **E** and **I** = *maximum* values, and E and I = *effective* values.

When **I** or I, R, L, C are given in c.g.s. units, **E** and E = c.g.s. units; when **I** or I is expressed in amperes, R in ohms, L in henrys, C in farads, then **E** and E = volts. In the above equations R is the total resistance, inductive and non-inductive.

213. Reactance and Impedance.—Consider the equation

$$E = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}.$$

The term $L\omega$ is called the *inductive reactance*, and $1/C\omega$ is called the *capacity reactance*. The expression $\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$ or $\sqrt{R^2 + L^2\omega^2}$ or

$\sqrt{R^2 + (1/C\omega)^2}$ is the *impedance*. Reactance and impedance are measured in ohms.

Example 1.—A 60-cycle a.c. circuit contains in series a resistance of 21 ohms, an inductance of 0.5 henry, and a capacitance of 40 mf. Find (*a*) the inductive reactance; (*b*) the capacity reactance; (*c*) the impedance. *Solution:* (*a*) Inductive reactance $= L\omega = 2\pi nL = 6.28 \times 60 \times 0.5 = 188.4$ ohms. (*b*) Capacity reactance $= 1/C\omega = 1/(2\pi nC) = 1/(6.28 \times 60 \times 0.000{,}040) = 66.3$ ohms. (*c*) Impedance $= \sqrt{R^2 + \left(L\omega - \dfrac{1}{C\omega}\right)^2} = \sqrt{441 + 14{,}884} = 123.8$ ohms.

Example 2.—What effective E will be required to maintain an effective current I of 5 amp. in a circuit having a resistance of 5 ohms, and containing a helical coil 1250 turns, length 25 cm, mean cross-sectional area $10/\pi$ cm², when μ is considered as equal to 20, and the frequency n of the alternating system is 60 cycles per sec. *Solution:* $L = 4\pi\mu n^2 Al/10^9 = (4\pi \times 20 \times 2500 \times 10 \times 25)/(\pi \times 10^9) = 0.05$ henry. The angular velocity $\omega = 2\pi n = 120\pi$. Then $E = 5\sqrt{25 + 355.32} = 97.5$ volts.

Example 3.—A 60-cycle alternating e.m.f. of 110 volts (effective) is applied to a system having a resistance of 7 ohms, and a condenser of capacitance C. The current I is 5 amp. Find (*a*) the reactance resistance, and (*b*) the value of C in microfarads. *Solution:* (*a*) $E = 110 = 5\sqrt{7^2 + \dfrac{1}{\omega^2 C^2}}$. The reactive resistance $1/\omega C = 20.85$ ohms. (*b*) $\omega = 2\pi n = 120\pi$. Then $1/120\pi C = 20.85$. Hence $C = 0.000{,}121$ farads $= 121$ mf.

214. Power in an A.C. System.—In a d.c. system the power in watts is equal to the product of volts times amperes; that is, $P = EI$. In an a.c.

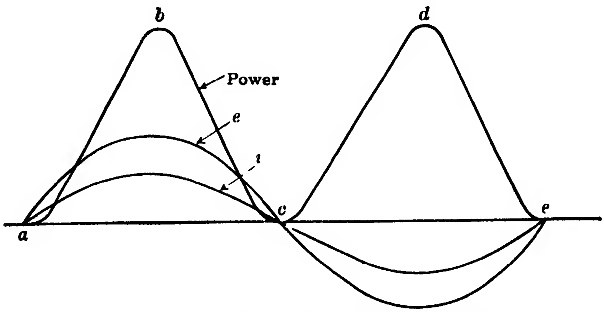

FIG. 140.—Power curves, e and i in phase.

system, however, this is not necessarily the case, because of the fact that the current I may be in phase or out of phase with E. In the case of the alternating current the power expended at any instant is $p = ei$, in which e is the instantaneous e.m.f. in volts, and i the instantaneous current in amperes. Since e and i may be in phase or out of phase, it follows that a consideration of power values involves two cases, namely, (*a*) when e and i are in phase (Fig. 140), and (*b*) when e and i are out of phase (Fig. 141). In Fig. 140, we have represented a power curve for e and i in phase. The ordinates of

the power are obtained for any given point by multiplying the ordinates of e and i for that point. The curve *abcde* is all on the positive side of the time axis.

In Fig. 141, we have a power curve for an e and i out of phase; that is, i lags behind e by an angle of 30°. In this case some of the ordinates ($e \times i$) are positive and some are negative. This gives part of the curve on the positive side of the axis and part on the negative side. The positive part of the power curve represents power delivered to the circuit; the negative part of the curve represents power given back from the circuit.

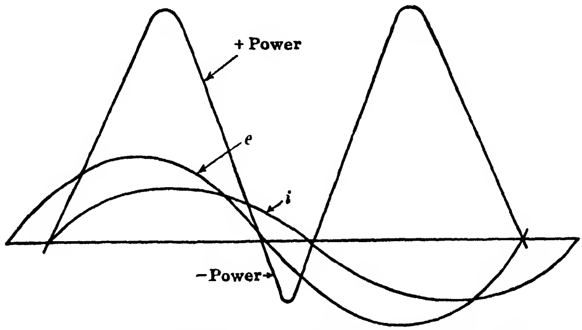

FIG. 141.—Power curve, e and i differing in phase.

The power delivered by an a.c. system is

$$P = EI \cos \phi$$

when P = power in watts; E = effective volts; I = effective current; ϕ = phase difference between e and i (angle of lag or lead). The quantity $\cos \phi$ is called the *power factor*.

Example.—An alternating e.m.f. of 110 volts is applied to an inductive system in which $R = 60$ ohms, and $L = 0.1$ henry. The frequency is 60 cycles per sec. Find the power expended by the current. *Solution:* $I = E/\sqrt{R^2 + L^2\omega^2} = 110/\sqrt{3600 + 0.01 \times 14{,}400\pi^2}$; hence $I = 1.56$ amp. To find the angle of lag ϕ, we write $\tan \phi = L\omega/R = 0.6283$, whence $\phi = 32.15°$. $\cos \phi = 0.8467$. Power $= 110 \times 1.56 \times 0.8467 = 145.3$ watts.

Problems

655. Make drawings to illustrate the fundamental relations between an alternating harmonic e and i (*a*) in phase; (*b*) i lagging behind e; (*c*) i leading e.

656. Make drawing to illustrate (*a*) i lagging behind e by 30°; (*b*) lagging behind e by 60°; (*c*) leading e by 90°.

657. Drawing to illustrate (*a*) i leading e by 30°; (*b*) leading e by 60°; (*c*) lagging behind e by 90°.

658. Drawing to illustrate power curve when e and i are (*a*) in phase; (*b*) out of phase.

659. Having given the maximum RI and the induced e.m.f., explain how to find the impressed e.m.f. by the vector method. Write the equation and explain each term.

660. Explain how to find the angle of lag due to (*a*) induction in the circuit. Explain $\tan \phi = L\omega/R$.

Given a coil of 1000 turns, length 20π cm, cross-sectional area 20 cm^2, resistance 5 ohms, permeability unity. Upon this coil there is impressed an alternating e.m.f. (effective) of 110 volts, having a frequency of 60 cycles.

661. Find the inductance L in (*a*) c.g.s. units; (*b*) henrys.

662. Find the angular velocity ω, in radians per second.

663. Find (*a*) the inductive reactance; (*b*) the angle of lag ϕ in degrees.

664. Find the effective value of the current.

665. Find the maximum value of (*a*) the current; (*b*) the e.m.f.

666. Find the instantaneous value of the current in its 30° phase; (*b*) 120° phase.

667. When the current is passing through its zero phase, what is the instantaneous value of the impressed e.m.f.?

668. What capacitance would have to be put in series with the inductance in order to annul the effect of the latter?

669. Find the power expended upon this coil, in watts.

Given a coil of 3000 turns, 30π cm in length, $50/\pi$ cm^2 in cross-sectional area. The resistance of the coil is 6 ohms. The permeability of the medium within the coil is 10. The coil is connected to a 60-cycle 110-volt circuit.

670. Find the inductance of the coil in practical units.

671. Find the phase difference (angle of lag) between e and i.

672. Find the effective value of the current.

673. Find the power expended upon the coil.

Given a 60-cycle 110-volt circuit in which there are connected in series a resistance of 20 ohms and a condenser of 120 mf. capacity.

674. Find (*a*) the capacity reactance of the system; (*b*) the impedance; (*c*) the current flowing in the system.

675. Find the power conveyed by the current.

676. A non-inductive resistance of 20 ohms, a 120-mf. condenser, and a 6-ohm coil, having an inductance of 0.006 henry, are connected in series with a 60-cycle 110-volt circuit. Find the current in the system.

677. (*a*) Find the phase difference between the current and the e.m.f., problem 676. (*b*) Does the angle represent a lag or a lead?

CHAPTER XII

ELECTRICITY (*Continued*)

ELECTRIC GENERATORS

215. Dynamos.—A dynamo is a machine for transforming mechanical energy into electric energy, or for transforming electrical energy. When a dynamo is used to transform mechanical energy into electrical energy it is called a *generator;* when it is used to transform electrical energy into mechanical energy it is called a *motor.* Dynamos are of two general classes, namely, direct current (d.c.) and alternating current (a.c.) dynamos. When a dynamo is used as a generator, the e.m.f. and current *in the armature* is

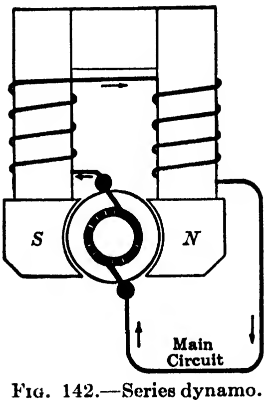

FIG. 142.—Series dynamo.

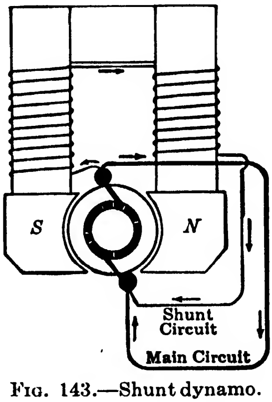

FIG. 143.—Shunt dynamo.

always alternating in character, whether the machine is of the a.c. or the d.c. type. The essential difference between d.c. and a.c. generators lies in the fact that in the former (d.c. generators) the alternating current of the armature is changed into a direct current in the external circuit (line) by means of a commutator, while in the latter (a.c. generators) the alternating armature current is conveyed directly to the line as an alternating current.

With respect to field winding, d.c. generators are of three types, (*a*) series wound (Fig 142), (*b*) shunt wound (Fig 143), and (*c*) multiple wound.

In the d.c. problems which follow in this text we shall consider only those which have to do with simple series-wound and shunt-wound machines.

216. The Ideal Simple Dynamo.—There is shown in Fig. 144 the essential elements of an ideal simple dynamo, having two poles, NS, and an armature, *ac*, consisting of a single coil.

Let us consider that this dynamo is operating as a generator, and that the armature is rotating in a clockwise sense (Fig. 145). The vertical compo-

nent of the velocity of a given conductor is $v \sin \alpha$. In this case we may write

$$\textit{instantaneous e.m.f.} = NBlv \sin \alpha,$$

where N = numbers of conductors (2 in this case) cutting the field; B = magnetic induction (number of lines per cm²); l = length in centimeters of the conductors cutting across the field; v = linear velocity in centimeters per second of the conductor; and α = angle that the face of the armature coil makes with a line at right angles to the field.

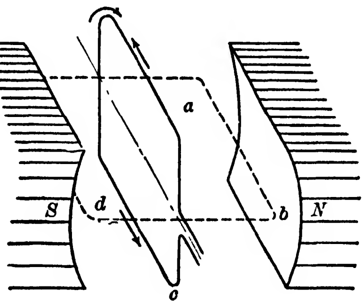

FIG. 144.—Ideal simple dynamo.

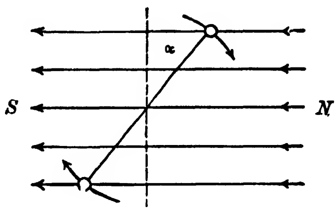

FIG. 145.—Rotation of armature in magnetic field.

If we consider the average e.m.f. developed during any number of revolutions of the armature, we have

$$E = \frac{N\phi}{t} \text{ c.g.s units} = \frac{N\phi}{(t \times 10^8)} \text{ volts},$$

in which N = number of conductors in the armature, or coil; ϕ/t = total number of lines of induction cut per second, or the change of flux per second. It is important to note that for each revolution of the armature, each conductor cuts all the lines of the field *twice*.

Example.—Consider an armature coil consisting of a single loop moving in a circular path through a uniform field. The induction of the field is 50 lines per cm². The length of one conductor (conductor *a*, Fig. 144, for example) is 30 cm. The radius of the circular path (Fig. 145) is 10 cm, and therefore the cross-sectional area of the armature is $20 \times 30 = 600$ cm². The coil makes 3600 r.p.m. Find (*a*) the instantaneous value of the e.m.f. when the coil has advanced 30° from the vertical position as shown in Fig. 145; (*b*) the average induced e.m.f. in volts. *Solution:* (*a*) There are 2 conductors, hence $N = 2$. Also, 3600 r.p.m. = $^{3600}\!/_{60}$ = 60 r.p.s., and consequently $v = 2\pi r \times 60 = 2\pi \times 10 \times 60 = 1200\pi$ cm/sec. Then $e = NBlv \sin \alpha = 2 \times 50 \times 30 \times 1200\pi \times 0.5 = 1{,}800{,}000\pi$ c.g.s. units = $18 \times 10^5 \times \frac{\pi}{10^8} = 0.018\pi$ volt. (*b*) $N = 2$, and $\phi = BA = 50 \times 20 \times 30 = 30{,}000$. The time of one revolution $t = {}^{60}\!/_{3600} = {}^{1}\!/_{60}$ sec. Then since each conductor cuts the field *twice*, $E = N\phi/(t \times 10^8) = 2 \times 30{,}000 \times 2 \times 60/10^8 = 0.072$ volt.

217. The Fundamental Equation of the Generator.—For purposes of illustrating and explaining the fundamental equation of the generator, we shall consider a two-pole machine of the ring armature type, Fig. 146. The armature rotates in a clockwise sense. By means of the right-hand rule we may show that the e.m.f. at a is directed toward the brush A; and in b, the

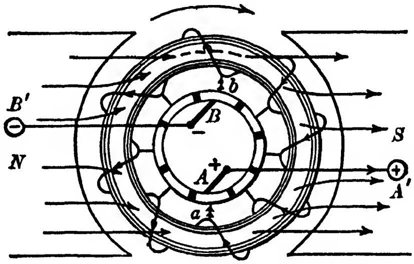

FIG. 146.—Two-pole, two-path dynamo.

e.m.f. is directed away from brush B. Then A' is the positive terminal of the machine and B' is the negative terminal.

It will be noted that in a dynamo of the type shown in Fig. 146 there are *two* parallel paths through the armature from terminal to terminal (A' to B'). In a four-pole dynamo of the corresponding type (Fig. 147) there are *four* parallel paths from terminal to terminal.

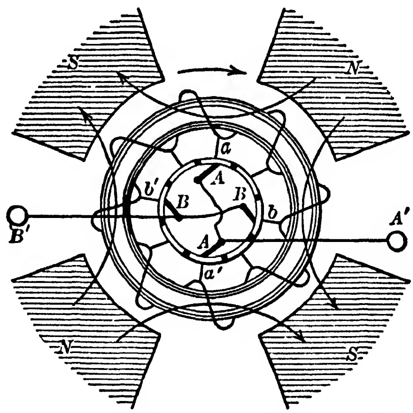

FIG. 147.—Four-pole, four-path dynamo.

The equation which expresses the relation between the e.m.f. of the dynamo on one hand, and the flux, the number of poles, the number of paths in parallel between the terminals, and the speed of the armature on the other is called the fundamental equation of the generator, and it is written

$$E = \frac{pN\phi n}{p'} \text{ c.g.s units} = \frac{pN\phi n}{(p' \times 10^8)} \text{ volts,}$$

where E = the e.m.f. developed in the armature; p = number of poles; N = the number of conductors; ϕ = flux per pole; n = number of revolutions per second; p' = number of electrical paths in parallel between the terminals.

Example.—In a given four-pole dynamo, of the type shown in Fig. 147, the flux per pole is 250,000 lines; the number of conductors on the outside of the armature is 200; the speed is 1200 r.p.m. Find the e.m.f. *Solution:* The frequency $n = {}^{1200}\!/_{60} = 20$ r.p.s., and $p' = 4$. Then

$$E = \frac{pN\phi n}{(p' \times 10^8)} = 4 \times 200 \times 250{,}000 \times \frac{20}{4 \times 10^8} = 10 \text{ volts.}$$

218. Generator Efficiency.—The efficiency of a dynamo (generator or motor) may be expressed as the ratio of the power output to the power input; that is

$$\textit{efficiency} = \frac{\textit{output}}{\textit{input}}$$

in which *input* = *output* + *power losses.* The power losses are in general of two kinds, namely, (*a*) copper losses and (*b*) stray power losses. *Copper losses* = $I_a^2R_a$ heat losses in the armature + $I_f^2R_f$ heat losses in the field. *Stray power losses* = S = hysteresis and eddy-current losses + friction losses.

Generator efficiency may be expressed (*a*) in terms of commercial (true) efficiency, or (*b*) in terms of electrical efficiency.

Commercial efficiency = output/(output + copper losses + stray power losses) = $EI/(EI + I_a^2R + I_f^2R_f + S)$.

Electrical efficiency = output/(output + copper losses) = $EI/(EI + I_a^2R + I_f^2R)$.

Example.—The line output of a shunt-wound generator is 50 amp. at 110 volts. The stray power loss S = 400 watts. The armature resistance R_a = 0.2 ohm, and the field resistance R_f = 55 ohms. Find (*a*) the current in the field; (*b*) the current in the armature; (*c*) the commercial efficiency; (*d*) the electrical efficiency. *Solution:* (*a*) The field current $I_f = E/R_f = {}^{110}\!/_{55} = 2$ amp. (*b*) Since in a shunt-wound dynamo the field and the line are in parallel, the armature current I_a = line current + field current = 50 + 2 = 52 amp. (*c*) Commercial efficiency =

$$\frac{110 + 50}{(110 + 50 + 52^2 \times 0.2 + 2^2 \times 55 + 400)} = 82.5 \text{ per cent.}$$

(*d*) Electrical efficiency $= \dfrac{110 \times 50}{(110 \times 50 + 52^2 \times 0.2 + 2^2 \times 55)} = 87.8$ per cent.

Problems

678. A rectangular loop of five turns of wire, 20 by 20 cm, lies in a vertical position with its face at right angles to the direction of a magnetic field, the induction of which is 2000 lines per cm². The armature (rectangular loop) rotates about a median axis, lying in its face, 360 times per min. Find the instan-

taneous value of the e.m.f. (*a*) when the loop is 30° from the vertical position; (*b*) 60°; (*c*) 90°.

679. Find the average e.m.f. developed per revolution (loop of problem 678) (*a*) in c.g.s. units; (*b*) volts.

680. The loop of problem 678 rotates from a vertical to a horizontal position in 0.1 sec. Find the average current in the coil, assuming that the resistance is 4 ohms.

681. Heat is measured in terms of *effective* amperes. What heat will be developed in the coil of problem 680 in 1 min.?

682. A coil of 100 turns of wire is wound on a square frame, the mean area enclosed being 900 cm². The coil is revolved in a horizontal field of uniform intensity of 80 gausses in air. It makes 600 r.p.m. Find (*a*) the instantaneous e.m.f. when the coil is horizontal; (*b*) the average e.m.f. per revolution.

683. If a circular coil of wire were made to spin about a vertical diameter first at the magnetic equator, second at the magnetic pole of the earth, explain the e.m.f. that would be developed in each case.

684. A two-pole generator has pole faces of area 240 cm². There are 200 conductors on the armature in the form of two parallel circuits. The speed is 1200 r.p.m. The e.m.f. developed is 96 volts. Find the induction in the air gap.

685. Consider a generator of the type shown in Fig. 147, and assume that the armature rotates in clockwise sense. Determine the positive terminal (A' or B') of the dynamo.

686. A four-pole dynamo having a four-path armature, has a flux in the air gap at each pole of 2,250,000 lines of induction. There are 672 conductors on the armature. The speed is 900 r.p.m. Calculate the average e.m.f. of the machine.

687. A four-pole generator has an armature of the Gramme ring type. There are four brushes and four parallel paths for the current in the armature. The total flux from each pole is 2,500,000 lines of induction. There are 600 turns of wire on the ring. At what speed is it running, if it develops 225 volts?

688. Given a shunt-wound generator which delivers a line current of 50 amp. at 110 volts. The field resistance is 22 ohms; the armature resistance is 0.15 ohm. The stray power loss is 500 watts. Find (*a*) the commercial efficiency; (*b*) the electrical efficiency.

689. Consider that the generators, Figs. 142 and 143, run at constant speed. If the resistance in the main circuit (line) of

each be reduced what will be the effect on the field and the voltage in each case?

ELECTRIC MOTORS

219. Kinds of Motors.—An electric motor is a machine for transforming the energy of an electric current into the energy of mechanical motion. With respect to its essential elements, a motor does not differ in principle from that of a generator, except that in one case (generator) the armature is rotated by mechanical means, thus generating a current; while in the other case (motor) a current is forced through the armature, thus causing it to rotate, due to a distortion of the magnetic field. Any ordinary d.c. generator, if connected up with a source of e.m.f., will operate as a motor; and, on the other hand, any d.c. motor may act as a generator.

With respect to the kind of current they are designed to take, motors may be classified as d.c. or a.c. motors.

220. D.C. Motors.—When a conductor carrying a current is placed in a magnetic field, the field is distorted (Figs. 148, 149). This distortion of the

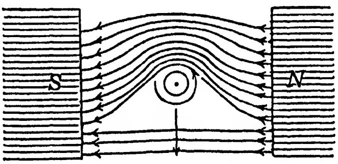

FIG. 148.—Current out; motion down.

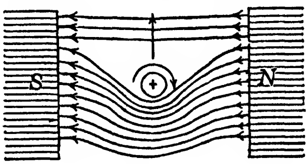

FIG. 149.—Current in; motion up.

field exerts upon the conductor a *side-push*, which is technically known as the *current-magnetic-field-force*.

Suppose for example, that there is an *out* current in the conductor, giving rise to a distorted field (Fig. 148). The crowded lines above the conductor tend to push it down. When the current is reversed, that is, when there is an *in* current (Fig. 149) the lines are crowded below the conductor and consequently the side-push is upward. Thus it appears that the conductor is always pushed away from the side where the field is strongest.

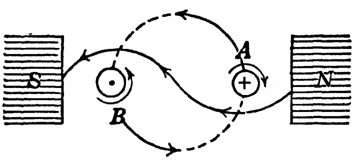

FIG. 150.—Diagram of an electric motor.

In Fig. 150 there is shown a diagrammatic section of an electric motor, in which the field is distorted downward at the right and upward at the left. Now the tendency of the distorted lines of the field is to straighten, thus causing the armature to rotate in a counterclockwise sense.

The direction of rotation of the armature of a d.c. motor may be reversed (*a*) by reversing the current in the armature or (*b*) by reversing the polarity of the field. If both the armature current and the polarity of the field magnets be reversed simultaneously, no change in the direction of rotation will occur.

221. Counter or Back E.M.F. in a Motor.—When the ar[illegible] from the motor rotates in a magnetic field, it cuts lines of force and therefore [illegible] act as a generator. This gives rise to a counter or back e.m.f. in the arm[illegible]ture, which opposes the applied e.m.f. at the brushes. The faster a motor runs the greater is the back e.m.f., and consequently the less armature current it takes. In general we let E = applied e.m.f. and E' = counter e.m.f.

Series Motor.—In the case of a series motor, the line current I = armature current I_a = field current I_f. Then

$$E = E' + IR_a + IR_f$$

Shunt Motor.—In a shunt motor the armature and field currents are not the same. The field current $I_f = E/R_f$, while the armature current depends upon the back e.m.f. developed and the armature resistance. In this case we have

$$E = E' + I_aR_a = I_fR_f.$$

Counter e.m.f. may be expressed in terms of flux and speed as follows

$$E' = \frac{pN\phi n}{p' \times 10^8} \text{ volts,}$$

where E' = back e.m.f.; p = number of poles and p' = number of parallel armature paths; N = number of armature conductors; ϕ = magnetic flux = total number of lines of force passing through armature; and n = number of armature r.p.s.

222. Motor Efficiency.—The efficiency of a motor, like that of any other machine for transforming energy into useful work, is output/input. But in this case *output = input − power losses.* Motor efficiency may be expressed as (*a*) commercial (true) efficiency, or as (*b*) armature efficiency or the efficiency of conversion, as it is sometimes called.

Commercial efficiency = (input − copper losses − stray power losses)/input = $(EI - I_a^2R_a - I_f^2R_f - S)/EI$, where E = applied e.m.f.; I = line current; I_a = armature current; R_a = armature resistance; I_f = field current; R_f = field coil resistance; and S = stray power losses.

Armature efficiency = $E'I_a/EI$, where E' = back e.m.f.; I_a = armature current; and EI = applied power.

Example.—A shunt motor running at full-load speed takes 62 amp. at 110 volts. The stray power loss is 400 watts. The armature resistance is 0.02 ohm, and the field resistance is 55 ohms. Find (*a*) the commercial efficiency; (*b*) the efficiency of conversion. *Solution:* The field current I_f = $^{110}/_{55}$ = 2 amp.; the armature current therefore = 62 − 2 = 60 amp. (*a*) $C.E. = (EI - I_a^2R_a - I_f^2R_f - S)/EI = (110 \times 62 - 60^2 \times 0.02 - 2^2 \times 55 - 400)/(110 \times 62)$ = 89.8 per cent. (*b*) The counter e.m.f. $E' = E - I_aR_a = 110 - 60 \times 0.02 = 108.8$ volts. Hence efficiency of conversion = armature efficiency $= \dfrac{E'I_a}{EI} = \dfrac{108.8 \times 60}{110 \times 62}$ = 95.7 per cent.

223. Work Done by a Motor.—The work done by a motor is a function of the counter e.m.f. developed in the armature, the current flowing through the armature, and the time; that is

$$W = E'I_at,$$

= back e.m.f.; I_a = armature current; and t = seconds. When
I_a are in c.g.s. units, W = ergs; when E' is in volts and I_a is in
peres, W = joules. Joules/1.355 = foot-pounds.

224. Torque.—The torque developed in a motor = *force* × *lever arm* = Fr, in which F is the side-push and r is the radius of the armature. Starting with the equation $W = E'I_at$, let us consider the work done during one complete revolution of the armature. In this case t is equal to the period T (time of one revolution). But the period is equal to the reciprocal of the frequency n (number of r.p.s.); that is $T = 1/n$. Then $W = F \times 2\pi r = E'I_a/n$. Since Fr = torque = $\mathfrak{I}$, we have

$$torque = \mathfrak{I} = E'I_a/2\pi n.$$

When E' is given in volts, I_a in amperes, and n is the number of r.p.s., torque $\mathfrak{I}$ is expressed in dyne-centimeters.

225. Power.—Power is work per unit of time $= W/t = E'I_a$. Then from the equation $\mathfrak{I} = E'I_a/2\pi n$, we may write

$$power = P = \mathfrak{I}2\pi n = watts.$$

Since 1 hp. = 33,000 ft.-lb./min. = 550 ft.-lb./sec. = 746 watts, it follows that 1 watt = 33,000/746 = 44.2 ft.-lb./min. = 0.737 ft.-lb./sec.

226. A.C. Motors.—The two essential elements of an a.c. motor are a stationary part called the *stator* and a rotating part called the *rotor*. There are various kinds of a.c. motors, depending on (*a*) the stator windings, (*b*) the kind of current used, that is, single phase, two phase, or three phase, (*c*) the construction of the rotor, (*d*) the method of starting, and so on. We shall consider here only two general types, known respectively as the *induction motor* and the *synchronous motor*. The principles involved in the construction and operation of these two types are included in one way or another in practically all the other kinds.

227. Induction Motors.—The working of an induction motor depends upon the possibility of producing a rotating magnetic field by means of

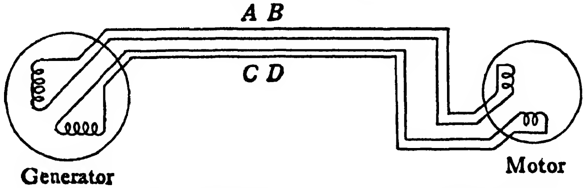

FIG. 151.—Two-phase generator and two-phase motor.

alternating currents. There are single-phase, two-phase, and three-phase induction motors. The simplest type, so far as explanation is concerned, is that of the two-phase motor. A two-phase induction motor is supplied with current from a two-phase generator (Fig. 151). The stator magnet consists of a ring having four poles (Fig. 152). In considering how the rotating field is produced, it is necessary to recall that when a two-phase current is at its maximum in line AB, it is zero in line CD (Fig. 151). In this case the poles at a and b (Fig. 152) are magnetized and those at c and d are dead.

If, for example, a is an N-pole, then b is an S-pole. At the end of a quarter turn of the generator armature the current in AB drops to zero and that in CD rises to a maximum. At this stage poles a and b are dead and c and d become magnetized. At the half turn, CD falls to zero and the current in AB again rises to a maximum, but in a reversed sense. The polarity of a and b is thus reversed. In a like manner the changing polarity continues around the entire circuit, producing a rotating field in the stator of the motor (Fig. 152) the N- and S-pole chasing each other around as the alternations in the lines occur. The rotor, not shown in Fig. 152, keeps step with the rotating field of the stator.

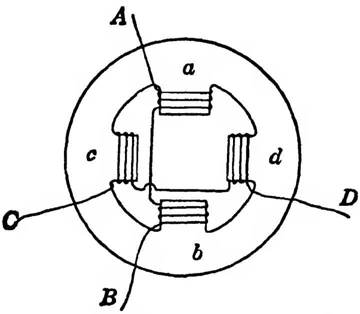

FIG. 152.—Strator magnet, rotating field.

Three things should be noted with reference to the induction motor: first, the rotating field is due to the successive alternations of a two-phase current; second, there are no slip rings or brushes; and third, no current from the line enters the rotor. The current in the rotor is an induced current, but it has no direct connection with the current from the generator.

Because of its simplicity of construction and ease in starting, the induction motor has a very extensive use in factories and in household motor appliances.

228. Synchronous Motors.—A synchronous motor is essentially an a.c. generator operating as a motor.

In order to start a synchronous motor it has to be speeded up by some outside source until it is in step or in synchronism with the phase conditions of the generator.

The synchronous motor is in general used only where constant speed is required; it is unsuited for loads requiring variable speed, such, for example, as those necessary in the operation of machine tools and similar mechanical devices.

A *rotary converter* is a synchronous motor to the shaft of which there is attached a d.c. generator. The converter, therefore, receives a.c. power as a synchronous motor and delivers d.c. power as a d.c. generator.

229. The Transformer.—A transformer consists of two or more sets of mutual induction coils and is used in general on a.c. lines for raising or lowering the voltage. The coils, primary P and secondary S, are usually wound on an iron core which is bent in the form of a ring (Fig. 153). The conventional transformer diagram is shown in Fig. 154. When the number of turns in the secondary N' is greater than the number of turns in the primary N, the transformer is of the step-up type; when, on the other hand, the number of turns in the secondary is less than the number in the primary, we have a step-down transformer.

Neglecting small losses due to hysteresis and eddy currents in the transformer, we may write

$$EI = E'I',$$

where E and I are primary e.m.f. and current, respectively, and E' and I' secondary e.m.f. and current. This equation tells us that, neglecting the small losses mentioned, the power delivered $E'I'$ is equal to the power received EI. This is true because when the e.m.f. steps up, for example, the current steps down in a like ratio. It may also be shown that the ratio between E and E' is very nearly equal to the ratio of the number of turns of wire in the primary and secondary coils; that is

$$\frac{E}{E'} = \frac{N}{N'}.$$

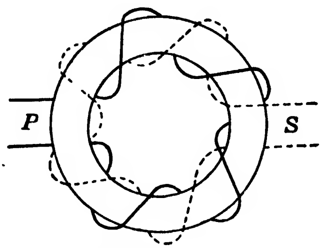

FIG. 153.—Faraday transformer ring.

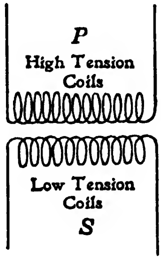

FIG. 154. Diagram of step-down transformer.

In commercial practice the ratios between N and N' most frequently used are 10/1 and 20/1. For example, in a transformer in which $N/N' = 20/1$ the high-tension current for lighting is transmitted at 2200 volts and stepped down to 110 volts where it is to be used; that is, $2200/E' = 20/1$, from which $E' = 110$.

The transformer is one of the most efficient machines devised by man, the efficiency of large transformers being usually 97 per cent or more.

Problems

690. In a two-pole, two-path motor having 100 conductors on the armature, the flux $\phi = 1{,}200{,}000$. Find the speed with which the armature would have to run in order to develop a counter e.m.f. of 108 volts.

691. An impressed e.m.f. of 110 volts is applied to a series motor, the armature of which has a resistance of 0.4 ohm, and the field a resistance of 8 ohms. What is the counter e.m.f. developed in the armature when the motor is taking a current of 5 amp.?

692. A shunt motor, having an armature resistance of 0.2 ohm, is running on a 110 volt circuit. At a given speed the counter e.m.f. is 107 volts. (*a*) What current does the armature take at this speed? (*b*) If the armature were suddenly stopped what current would flow through it?

693. Given a shunt motor to which is applied a pressure of 110 volts. The field resistance is 44 ohms; the armature resistance is 0.14 ohm. The current supplied to the motor is 50 amp. The stray power loss is 700 watts. Find (*a*) the true efficiency (commercial); (*b*) the armature efficiency.

694. A two-pole shunt motor, having a two-path armature, has a flux of 2,000,000 lines of induction through the armature, in which there are 144 conductors. The armature resistance is 1.67 ohms; the field resistance is 55 ohms. The armature current is 40 amp.; the field current is 2 amp. The speed is 900 r.p.m. Find (*a*) the counter e.m.f. developed; (*b*) the electrical work done in 4 hr.

695. If at this speed (problem 694) the commercial efficiency is 32 per cent, what is the stray power loss?

696. Find (*a*) the torque (problem 694); (*b*) the power developed, in horsepower.

697. A street-car motor of the series type has a resistance of 1.8 ohms. It is operated on 500 volts applied e.m.f. and takes 20 amp. of current. Compute (*a*) the counter e.m.f.; (*b*) the power developed in kilowatts.

698. Starting with the assumption that the current through *AB* (Fig. 152) is at its maximum value, flowing in at *A* and out at *B*, and that the current through *CD* is zero, trace the N-pole around the stator for one complete cycle of the two-phase currents.

699. (*a*) In an induction motor, the stator of which is shown in Fig. 152, does the current from the two-phase circuit enter the rotor? (*b*) What causes the rotor to revolve?

700. What is the essential difference between an induction motor and a synchronous motor?

701. (*a*) What is a rotary converter? (*b*) What kind of a current does it take? (*c*) What kind of a current does it deliver?

702. A transformer receives electrical energy under a pressure of 550 volts and steps it up to 66,000 volts. What is the high-voltage current when the low-voltage current is 6000 amp., the efficiency of the transformer being 97 per cent?

703. The ratio of the primary to the secondary turns of a transformer is 20:1. When the primary current is 2 amp., the primary power is 4000 watts. What are the corresponding values of the secondary e.m.f. and current, assuming the efficiency to be 95 per cent?

CHAPTER XIII

LIGHT

VELOCITY AND INTENSITY

230. Velocity of Light.—The velocity of light was first determined by the Danish astronomer Roemer, who concluded that light travels with a speed of 186,000 miles per sec. The velocity of light as determined by Michelson is 299,860 km per sec. In ordinary calculations the following values are used

$$v = 300{,}000 \text{ km/sec.} = 186{,}000 \text{ mi./sec.}$$

231. The Law of Inverse Squares.—Geometrically a light wave may be considered to move outward from its source in the form of a series of concentric spheres. Since the area of a sphere varies as the square of its radius (area of sphere = $4\pi r^2$), it follows that the area illuminated will vary as the square of the distance from its source; and, on the other hand, the quantity of light *per unit area* will vary *inversely* as the square of the distance from the source. For example, let L (Fig. 155) be a point source of light from which radiant energy is flowing out uniformly in all directions. Consider the solid angle intercepted by the areas S and S'. Assume, for illustration, that S is 1 ft. from L and S' is 2 ft. distant. The quantity of light falling upon S' is the same as that falling upon S. The quantity of light *per unit area* on S' is, however, only one-fourth that on S. Let I be the quantity of light per unit area on S and I' the quantity per unit area on S'.

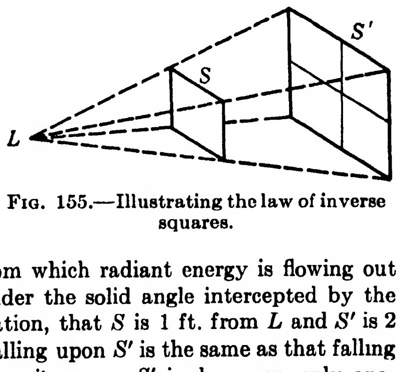

Fig. 155.—Illustrating the law of inverse squares.

Then

$$\frac{I}{I'} = \frac{d'^2}{d^2} = \frac{4}{1}.$$

This is an illustration of the application of the law of inverse squares.

232. Intensity of Light at the Source.—In considering the subject of intensity, two things should be kept in mind, (*a*) the intensity at the *source* and (*b*) the intensity of illumination of a *surface* at a given distance from the source. The intensity of light at the source is the quantity of radiant energy emitted per unit of time. The total flow of radiant energy from a luminous source, in all directions, is called the *luminous flux.*

The unit of intensity, with reference to the source, is the *candlepower* (cp.). The international standard is the light emitted by a sperm candle ⅞

inch in diameter and burning at the rate of 120 gr. per hr. While the sperm candle is the ultimate primary standard, it is seldom used in actual practice. Certain secondary standards have been devised, which are tested from time to time in terms of the standard sperm candle. These secondary standards are (*a*) the *flame standards* of Great Britain (the Pentane lamp), France (the Bourgie decimale), and Germany (the Hefner lamp); and (*b*) the *electric standards* of the United States. The *American unit of candlepower* is defined in terms of certain tested incandescent lamps, kept at the Bureau of Standards, Washington, D. C.

The *international unit of candlepower* = one American electrical unit = one Pentane unit = one Bourgie decimale = $^{10}/_9$ Hefner unit. The Hefner unit of candlepower is the light given by a horizontal beam from the Hefner lamp, burning pure amyl acetate, at normal atmospheric pressure (76 cm), in an atmosphere containing 8.8 liters of water vapor per cubic meter.

Intensity of the source is sometimes expressed in lumens. A *lumen* is the amount of light energy emitted per unit of time in one solid radian angle by a standard candle. To express it in equational form, 1 *lumen* = *luminous flux from a standard candle*/4π; that is

$$1\ \textit{candlepower} = 4\pi\ \textit{lumens}.$$

The candlepower of a given source of light is the ratio of the light energy emitted by it to that emitted by a standard candle, or its equivalent, in the same length of time.

Candlepower may be measured by means of a photometer.

233. Intensity of Illumination.—The intensity of illumination of a surface at a given distance from the source is the quantity of light falling upon the source per unit area. From the law of inverse squares, we have

$$\frac{I}{I'} = \frac{d'^2}{d^2}.$$

The unit of intensity is the *foot-candle*, which is the intensity of illumination at a distance of one foot from a light of one candlepower. Since according to the law of inverse squares intensity of illumination is inversely proportional to the square of the distance from the source,

$$\textit{foot-candles} = \frac{cp.}{ft.^2}$$

For example, at a distance of 1 ft. from a 32 cp. incandescent lamp, the intensity of illumination is 32 foot-candles; at a distance of 2 ft. it is $32/2^2$ = 8 foot-candles, and so on.

Intensity of illumination is usually measured by means of a foot-candle meter.

Problems

704. The nearest fixed star is about 3 light-years from us. What is the approximate value of this distance in miles?

705. A candle flame 2 in. in length is placed 6 in. in front of the small aperture in *A* (Fig. 156). Find (*a*) the length of the

image on the screen placed 2 ft. from the aperture; (*b*) 5 ft. from the aperture. (*c*) Compare the intensity of illumination of the two images.

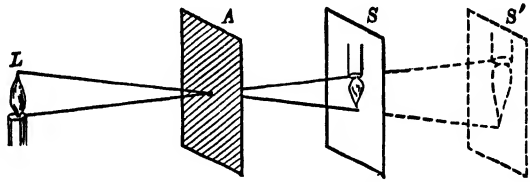

FIG. 156.—Illustrating inverted images.

706. A standard candle placed at a distance of 1 ft. from the screen of a Bunsen photometer gives the same intensity of illumination as that of an incandescent lamp placed at a distance of 4 ft. What is the candlepower of the lamp?

707. If a 2000-cp. street lamp actually gives 2000 cp. in a definite direction, at what distance from the lamp will the same amount of illumination be obtained as from a standard candle at a distance of 2 ft. from the illuminated surface?

708. Student *A* sits at a distance of 4 ft. from an electric light, in which position the illumination is 2 foot-candles. Student *B* sits at a distance of 8 ft. from the light. What is the foot-candle illumination at position *B*?

709. A cluster of electric lamps is suspended 4 ft. above a work table. A foot-candle illumination of 8 is required for fine shop work, and 12 for engraving work. (*a*) What must be the combined candlepower of the lamps if the illumination requirement for fine shop work is met? (*b*) What height above the table should the lights be placed to meet the foot-candle requirement for engraving?

710. A 40-watt incandescent lamp is rated at 1.25 watts per cp. What intensity of illumination in foot-candles is produced at a distance of 4 ft. from this lamp?

711. A gas-filled incandescent street lamp, rated at 0.5 watt per cp., gives an illumination of 2 foot-candles at a distance of 10 ft. What is the wattage of the lamp?

712. The intensity of a given source of light is $16/\pi$ lumens. What is the intensity of illumination in foot-candles at a distance of 4 ft. from the source?

REFLECTION

234. Law of Reflection.—The law of reflection states that the angle of incidence i is equal to the angle of reflection r, the two angles being in the same plane (Fig. 157).

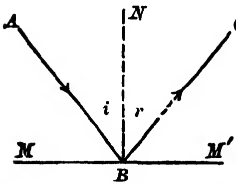

FIG. 157.—Angles of incidence and reflection,

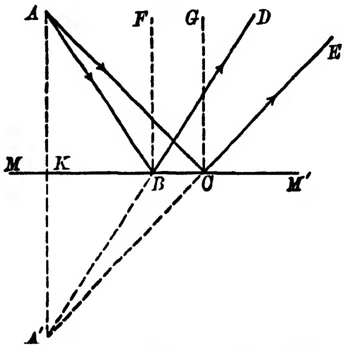

FIG. 158.—Image in plane mirror.

235. Plane Mirrors.—Images formed in plane mirrors are virtual. A *virtual image* is one formed by the apparent focusing of the rays of light from an object (Fig. 158). The virtual image of A lies at the point A', on the straight line AK, and as far back of the mirror MM' as the object is in front of it.

236. Successive Reflections.—We have given two plane mirrors M and M', set in position such that ϕ represents the angle between their reflecting

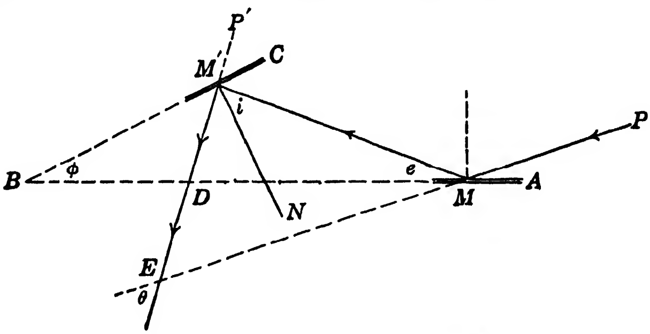

FIG. 159.—Illustrating principle of the sextant.

surfaces (Fig. 159). Suppose that a ray of light from a given source falls upon the mirror M and is reflected twice, once at M and again at M'. The angle formed by the incident ray PM and the reflected ray $= \theta =$ total deviation. Since $\theta = 180 - 2(e + i)$, and $\phi = 90 - (e + i)$ we have

$$\textit{total deviation} = \theta = 2\phi,$$

that is, for the case of two reflections, the total deviation is twice the angle included between the mirrors. This equation has an important application

in the use of the sextant, an instrument for measuring the angle subtended by two distant points (P and P'), or the angular elevation of a point above the horizon.

237. Images of Images.—When light is reflected successively from two plane mirrors, the image in the first mirror becomes the object in the second, and so on. If P (Fig. 160) be a luminous point between the mirrors AB and AC it may be shown geometrically that the images P_1, P_2, P_3, etc., lie on a circle, the radius of which is AP. Starting with the object P, images will appear on the circle successively until the arc DE is reached, after which no further reflections will occur. The arc DE lies behind both mirrors.

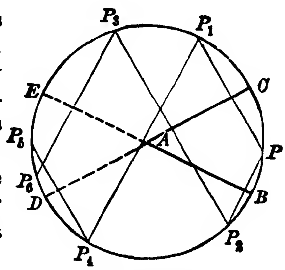

FIG. 160. —Images of images.

Let the angle BAC between the mirrors be ϕ, then $2\pi/\phi = n$. When n is a whole number and is even, then the images on the arc DE (P_5 and P_6), coincide, and the number of images $= n - 1$; when n is a whole number and is odd, the images on DE do not coincide, and the number of images $= n$.

Example.—Two plane mirrors, as AB and AC (Fig. 160), are placed face to face with their two edges together, at, A. The angle BAC is 72°. How many images of an object P are formed. *Solution:* Angle $\phi = 72°$, and therefore $n = 2\pi/\phi = {}^{360}\!/_{72} = 5$, an odd number. Hence the number of images formed $= n = 5$.

Problems

713. Prove geometrically that the virtual image A' (Fig. 158) lies on the straight line AK, and as far back of the mirror as the object A is in front of it.

714. A beam of light falls upon a plane mirror which lies in a horizontal position on the ground 20 ft. distant from the vertical wall of a building. The reflected ray strikes the wall at a point 10 ft. from the ground. Find the angle of incidence of the light on the mirror.

715. It may be shown geometrically that the virtual image of A (Fig. 158) lies on the straight line AK, and that this virtual image A' appears to be as far behind the mirror as the object A is in front of it. If the angle of incidence ABF is 40°, and the distance KB is 3 ft., what is the distance of the virtual image A' from the object A?

716. The distance KB (Fig. 158) is 3 ft.; BC is 10 in.; angle ABK is 50°. Find the angle BAC.

717. Consider Fig. 159. Angle ϕ is 20°; angle $MM'C$ is 36°. Find (a) the total deviation due to two reflections; (b) the angle AMP.

718. Show by means of a drawing that three images are formed when BAC (Fig. 160) is equal to 90°. Theoretically how many images are possible when the mirrors, AB and AC, are parallel?

719. Find the number of images formed by two plane mirrors placed with two of their edges together, and forming an angle (*a*) 60°; (*b*) 45°.

238. Spherical Mirrors.—Consider Fig. 161. The opening MM' is the aperture of the mirror. The *vertex* V is a point midway between M and M'. The *center of curvature* C is the center of the sphere of which the mirror is a part. The *principal axis* PP' is a straight line passing through the center of curvature C and the vertex V. The *principal focus* F is the point at which rays parallel to the principal axis come to a focus. The *radius of curvature* is $r = CV$. The *focal length* is $f = FV = r/2$. A *real image* is one which is formed by the actual focusing of rays of light.

The distance of the object AB from the vertex V, measured on the principal axis PP' is the *object distance* $= p$; the distance of the image ab from V, measured on the principal axis, is the *image distance* $= p'$.

239. Typical Mirror Cases.—There are seven typical cases involving the relation of object and image in spherical mirrors. These seven cases may be illustrated by the diagrams (Figs. 161–166).

Case I.—Object at an infinite distance. In this case all the rays are parallel to the principal axis. Image is real, a point, and it lies at the principal focus F, midway between V and C.

Case II.—Object at a finite distance, greater than the radius (Fig. 161). Image is real, inverted, smaller than the object, and it lies between F and C.

Case III.—Object at the center of curvature (Fig. 162). Image is real, inverted, same size as object, and it lies upon the object.

Case IV.—Object between F and C (Fig. 163). Image is real, inverted, larger than the object, and it lies beyond the center of curvature C.

Case V.—Object at the principal focus (Fig. 164). Image is real, infinitely large, and it lies at an infinite distance from the mirror.

Case VI.—Object between V and F (Fig. 165). Image is virtual, erect, larger than the object, and it lies on the *negative* side of the mirror.

Case VII.—Object in front of a convex mirror (Fig. 166). Image is virtual, erect, smaller than the object, and it lies on the *negative* side of the mirror, between F and V.

240. General Mirror Formula.—The general formula for spherical mirrors having a small angular opening is

$$\frac{1}{p} + \frac{1}{p'} = \frac{2}{r} = \frac{1}{f},$$

where p = object distance; p' = image distance; r = radius of mirror; f = focal length.

When the object is at an infinite distance $p = \infty$, $1/p = 0$, and hence $1/p' = 2/r = 1/f$; that is, $f = r/2$. This means that the *principal focus* of a spherical mirror, is at a point on the principal axis midway between V and C.

241. The Sign of the Factors p, p′, r, and f.—In dealing with the equation $\frac{1}{p} = \frac{1}{p'} = \frac{2}{r} = \frac{1}{f}$, it is of the utmost importance to be able to determine the

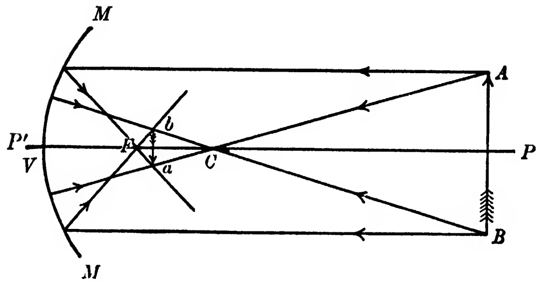

FIG. 161.

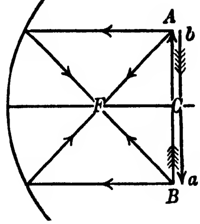

FIG. 162.

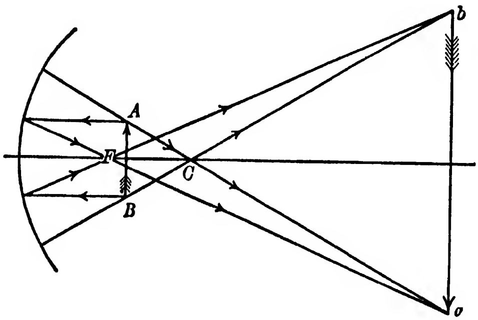

FIG. 163.

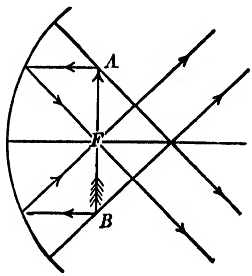

FIG. 164.

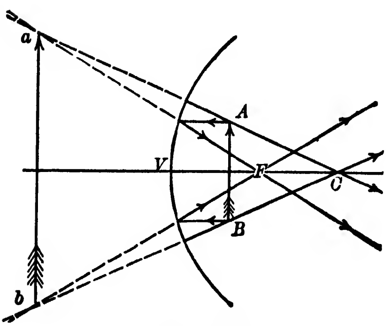

FIG. 165.

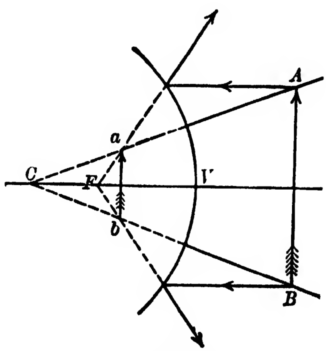

FIG. 166.

sign of the factors, p, p', r, and f. *Distances measured on the object side of a spherical mirror are positive* (+); *distances measured on the opposite side are negative* (−). It follows that *p is always positive.*

In the case of the *concave* mirror, when the object is at a distance from V greater then F (Fig. 161), p', r, and f are all on the object side, and hence are positive, and the equation is

$$\left(\frac{1}{+p}\right) + \left(\frac{1}{+p'}\right) = \left(\frac{2}{+r}\right) = \left(\frac{1}{+f}\right);$$

that is,

$$\frac{1}{p} + \frac{1}{p'} = \frac{2}{r} = \frac{1}{f}.$$

In the case of the *concave* mirror, when the object is placed between V and F, the image is virtual (Fig. 165). Here we have

$$\left(\frac{1}{+p}\right) + \left(\frac{1}{-p'}\right) = \left(\frac{2}{+r}\right) = \left(\frac{1}{+f}\right);$$

that is,

$$\frac{1}{p} - \frac{1}{p'} = \frac{2}{r} = \frac{1}{f}.$$

In the case of a *convex* mirror (Fig. 166), p', r, and f are negative, and the equation becomes

$$\left(\frac{1}{+p}\right) + \left(\frac{1}{-p'}\right) = \left(\frac{2}{-r}\right) = \left(\frac{1}{-f}\right),$$

or

$$\frac{1}{p} - \frac{1}{p'} = -\frac{2}{r} = -\frac{1}{f}.$$

242. Summary.—The following is a summary of the fundamental equations for concave and convex spherical mirrors:

Concave mirror, p greater than f,

$$\frac{1}{p} + \frac{1}{p'} = \frac{2}{r} = \frac{1}{f}. \tag{1}$$

Concave mirror, p less than f,

$$\frac{1}{p} - \frac{1}{p'} = \frac{2}{r} = \frac{1}{f}. \tag{2}$$

Convex mirror,

$$\frac{1}{p} - \frac{1}{p'} = -\frac{2}{r} = -\frac{1}{f}. \tag{3}$$

243. Size of Object and Image.—A study of the sketches illustrating any one of the cases of reflection (Figs. 161–166) reveals the fact that the *size of the object is to that of the image as their respective distances from the center of curvature of the mirror.*

Example.—An object 3 in. in length is placed 1 ft. from the vertex of a concave mirror the radius of which is 18 in. What is the size of the image?

Solution: We first make a sketch illustrating the relative positions of the half object ab and its image AB (Fig. 167), where $Vb = p = 12$, and $VB = p'$. The triangles ABC and abC are similar. Hence $AB:ab = BC:Cb$. Our next step is to find the values of BC and Cb. From equation $\frac{1}{p} + \frac{1}{p'} = \frac{2}{r}$

we write $\frac{1}{12} + \frac{1}{p'} = \frac{2}{18}$. Hence $p' = 36$ in. Then $Cb = 18 - 12 = 6$ in., and $CB = 36 - 18 = 18$ in., whence $3 : x = 6 : 18$, and $x = 9$ in.

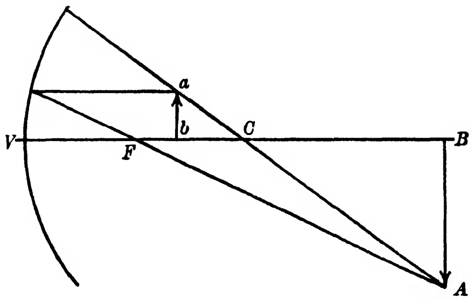

FIG. 167.

Problems

Make a sketch to illustrate the position and character of the image in the following cases for the spherical mirror; (problems 720–726); write the formula for each case, and note the signs (+ or −) of the quantities p', r, and f.

720. Object at an infinite distance from the concave side of the mirror.

721. Object at finite distance greater than radius.

722. Object at center of curvature.

723. Object between C and F.

724. Object at F.

725. Object between F and V.

726. Object in front of convex mirror.

727. The radius of a given spherical mirror is 1 ft. An object 3 in. in length is placed 24 in. from the mirror on the principal axis, concave side. Find the position of the image.

728. Find the size of the image (problem 727).

729. Object placed 9 in. from mirror (problem 727). Find position of image.

730. Find size of image (problem 729).

731. Object placed 3 in. from mirror (problem 727). Find position of image.

732. Find size of image (problem 731).

733. Object placed 1 ft. from mirror (problem 727) on principal axis, convex side. Find position of image.

734. Find size of image (problem 733).

735. The distance of the image from the vertex of the mirror when an object is placed 2 ft. from the mirror, on the principal axis, concave side, is 4.8 in. Find the focal length of the mirror.

736. An object is placed between the vertex of a mirror and the principal focus. The image is virtual, erect, and 16 in. from the mirror. The object is 8 in. from the mirror. Find the radius of curvature.

737. A concave mirror has a radius of curvature of 32 in. (*a*) Where must a person stand in front of it in order to see an image of one's face twice its natural size?

738. Make a drawing to any convenient scale of a concave mirror of 8 cm radius. Place an object 4 cm long, 20 cm from the mirror and find its image. Measure the distance of the image from the mirror and its length and compare these results with those found by computation.

739. (*a*) What kind of a mirror will produce an erect image of an object one-half of its natural size when the object is 10 in. from the mirror? (*b*) What is the radius of curvature of the mirror?

740. Given two mirrors having equal apertures. Considering aberration effects, which will make the hotter image of the sun, a concave mirror of 8-in. focal length, or one of 20-in. focal length? Why?

REFRACTION

244. Defining Terms.—*Refraction* is the bending of a ray of light out of its course due to its passage from one medium to another of different den-

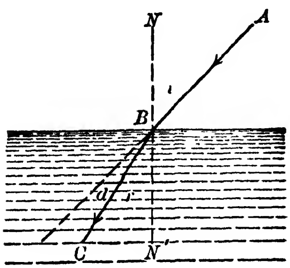

FIG. 168.—Refraction.

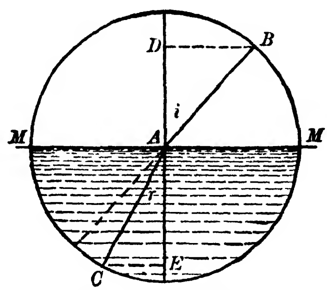

FIG. 169.—Illustrating sin *i* and sin *r*.

sity. Light passing from a rare medium (as air) to a dense medium is refracted toward the normal; light passing from a dense medium to a rare medium is refracted away from the normal.

Consider Fig. 168. Here a ray of light AB is incident at the point B. Angle i = the angle of incidence; r = angle of refraction; d = angle of deviation.

245. Index of Refraction.—Snell's law of refraction states that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant; that is, $\sin i/\sin r = \mu$, in which the constant μ is called the *index of refraction.*

According to Huygen's principle of refraction, it may be shown that the index of refraction μ with reference to two media is the ratio of the velocity of light in one medium to the velocity of light in the other; or in equational form; $\mu = v/v'$.

Absolute index of refraction is the ratio of $\sin i$ to $\sin r$ when the light passes from a vacuum to the given medium. *Relative index* of refraction is the ratio of $\sin i$ to $\sin r$ when the light passes from one medium to another. The relative index of refraction from air to water may, for example, be designated as μ_{aw}.

If we consider light to pass from a rare medium (as air) to a denser medium (water), Fig. 169, we may write $\sin BAD/\sin CAE = \mu_{aw}$; if, on the other hand, we consider the light to pass from the dense medium to the rare, $\sin CAE/\sin BAD = \mu_{wa}$, from which it follows that $\mu_{wa} = 1/\mu_{aw}$. In general, the index of refraction considered from the dense medium to the rare medium is equal to the reciprocal of the index of refraction from the rare medium to the dense medium.

246. To Trace a Ray of Light from One Medium to Another.—Suppose that we wish to trace a ray from air to water, the angle of incidence being 42° and the relative index of refraction being $\mu_{aw} = 4/3$. Erect a normal at the point of incidence, and determine the angle of refraction from the equation $\sin i/\sin r = \mu$. $\sin i = \sin 42° = 0.6691$. Then $0.6691/\sin r = 4/3$, from which $\sin r = 0.5017$. Hence $r = 30° 7'$.

247. Refraction through Plane Parallel Plates.—Light refracted through a medium bounded by plane parallel surfaces (Fig. 170) suffers no change in direction, but does undergo a lateral displacement. It may be shown that the lateral displacement PN is

$$PN = \frac{t \sin (i - r)}{\cos r}$$

where PN = lateral displacement; t = thickness of plate; i = angle of incidence; and r = angle of refraction.

248. Refraction through Several Media.—In the case of the refraction of light through several media, as for example, from air to water, to glass, to air (Fig. 171), we may write

$$\mu_{wg} = \frac{\mu_{wa}}{\mu_{ga}} = \frac{\mu_{ag}}{\mu_{aw}}.$$

Thus we may say that the relative index of refraction for any two media, as from water to glass (μ_{wg}) may be expressed in terms of the relative indices of the given media to some third medium, as air. Note that $\mu_{wg} = 1/\mu_{gw}$.

Example.—The relative index of refraction for a given specimen of glass $\mu_{ag} = 3/2$; the relative index from air to water is $\mu_{aw} = 4/3$. Find the relative index of refraction from water to glass. *Solution:* $\mu_{wg} = \mu_{ag}/\mu_{aw} = (3/2)/(4/3) = 9/8 = 1.125$.

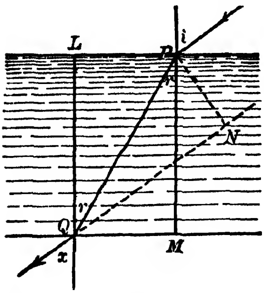

FIG. 170.—Deviation due to refraction.

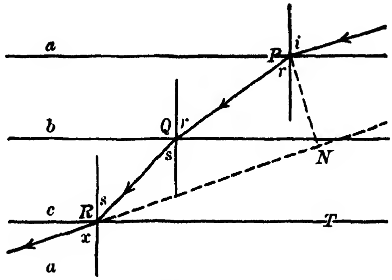

FIG. 171.—Refraction through several media.

249. Refraction through a Prism.—Angle A is the *refracting angle* of the prism (Fig. 172) and D is the *angle of deviation* $= \phi$. $ABE + ACE = 180°$. Also, angle A + angle $E = 180°$; that is, $E = 180° - A$. In the triangle BCE, $r + i' + E = 180°$, hence $i' = A - r$. The value of the angle of refraction r is obtained from the equation $\sin i/\sin r = \mu$; and that of r' from the equation $\sin i'/\sin r' = 1/\mu$.

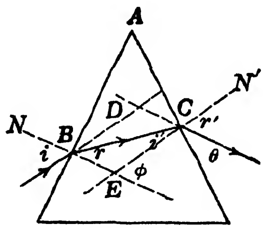

FIG. 172.

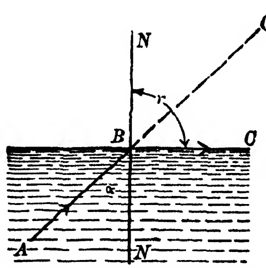

FIG. 173.—Critical angle.

250. Critical Angle.—Consider light as passing from the dense medium to the rare medium (Fig. 173). The critical angle $ABN' = \alpha$ is that angle of incidence in the dense medium such that the angle of refraction is 90°. Let α be the critical angle, then $\sin \alpha/\sin 90°$ may be written

$$\sin \alpha = \frac{1}{\mu},$$

that is, the sine of the critical angle is equal to the reciprocal of the index of refraction.

Example.—If an eye immersed in a fluid, the index of refraction of which is 1.42, looks out through the horizontal surface, what will be the greatest apparent zenith distance of a star, the light from which just grazes the surface. *Solution:* Light from the star C (Fig. 173) will be refracted from B to

A. The star will consequently appear at C'. The angle NBC' measures the "zenith distance." But $NBC' = ABN'$ = the critical angle α, and $\sin \alpha = 1/\mu = 1/(1.42)$. Hence $\alpha = 44° 46'$.

Problems

741. Taking Michelson's value for the velocity of light in air as 2999×10^7 cm/sec., find the velocity of light (*a*) in water, $\mu = 4/3$; (*b*) in glass, $\mu = 3/2$.

742. A ray of light falls upon crown glass making an angle of incidence equal to 36°. The thickness of the plate is 25 mm; its index of refraction is 1.5. Make a sketch to show the path of the ray through the plate. Find (*a*) the angle of refraction, and (*b*) the lateral displacement.

743. A piece of plate glass ($\mu = 1.64$) is placed parallel to the surface of a table. An object on the table viewed through the glass at an angle of incidence of 45° appears to be displaced parallel with the surface of the table by 0.7 cm. Find the thickness of the glass.

744. A plate of crown glass is 2 cm thick. A ray of light falls upon this plate, making an angle of incidence of 30°. Find the lateral displacement.

745. A ray of light which falls obliquely upon a piece of glass having plane parallel faces, suffers a lateral displacement of 1 cm. The angle of refraction is 20°. Index of refraction is 1.54. Find the thickness of the glass.

746. Consider Fig. 171. Let the medium *a* be air; let the medium *b* be a liquid, the index of refraction 1.6; let the medium *c* be glass, index 1.52. Find the index of refraction from *b* to *c*.

747. The index of refraction from air to crown glass is 1.512; from air to carbon disulphide, 1.68. Find the index of refraction (*a*) from CS_2 to crown glass; (*b*) from crown glass to CS_2.

748. Taking the index of refraction of carbon disulphide to be 1.68, find its critical angle.

749. The refracting angle of a glass prism ($\mu = 1.58$) is 40°. A ray of light falls upon one face of the prism, making an angle of incidence equal to 38°. Trace the ray through the prism, and find the angle which the emergent ray makes with the opposite face.

750. Find the critical angle between crown glass and carbon disulphide.

751. Find the velocity of light in a medium whose critical angle is 42°, taking the value for the velocity of light in air as 2999×10^7 cm/sec.

REFRACTION THROUGH LENSES

251. Defining Terms.—There are two general classes of lenses, convex and concave. A *convex lens* is one that is thicker at the middle than at the edges. A *concave lens* is one that is thinner at the middle than at the edges.

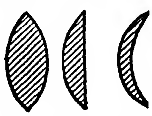

FIG. 174.—Convex lenses.

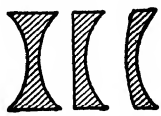

FIG. 175.—Concave lenses.

Convex lenses (Fig. 174) may be classified as double-convex, plano-convex, concave-convex. Concave lenses, likewise, are classified as double-, plano-, and convex-concave lenses (Fig. 175).

The line PP' (Fig. 176) is the principal axis of the lens; O is the optical center; F is the principal focus. In the case of thin lenses, the focal length f is measured from F to the lens; in the case of thick lenses, the focal length f is measured from F to the *principal plane* of the lens. In this text we shall deal only with thin lenses, unless specifically stated to the contrary.

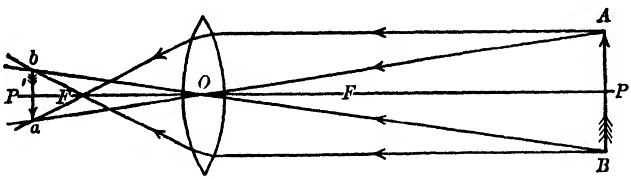

FIG. 176.

252. Typical Lens Cases.—There are five cases involving the relation of object to image in lenses. These five cases may be illustrated by the diagrams, Figs. 176–179.

Case I.—Convex lens, object at an infinite distance from the lens. Image is real, a point, and it lies at the principal focus. That this is true will appear from a consideration of Fig. 176. As the object AB moves away from the lens, the image ab approaches the principal focus F, becoming smaller and smaller as the object recedes. When AB is at an infinite distance from the lens, the point image lies upon F.

Case II.—Convex lens, object at a finite distance from the lens, greater than the focal length (Fig. 176). Image is real, inverted with respect to the object, and it lies beyond the principal focus F. AB and ab are called conjugate positions; that is, when the object is at AB the image is at ab, and, on the other hand, when the object is at ab the image is at AB.

Case III.—Convex lens, object at a distance from the lens equal to the focal length (Fig. 177). The image is at an infinite distance. This is the conjugate of Case *I*.

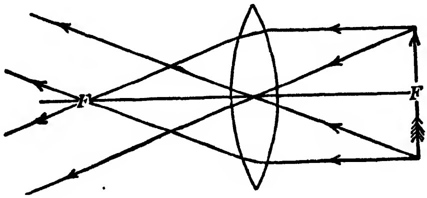

Fig. 177.

Case IV.—Convex lens, object at a distance from the lens less than the focal length (Fig. 178). Image is virtual, erect, larger than the object, and it lies on the object side of the lens.

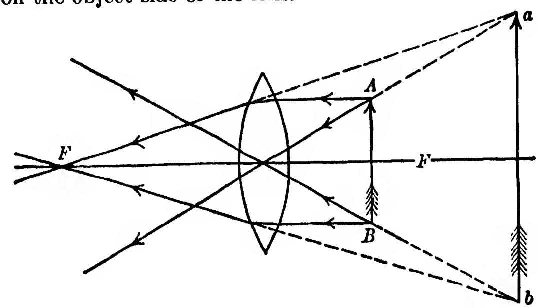

Fig. 178.

Case V.—Concave lens (Fig. 179). Image is virtual, erect, and smaller than the object.

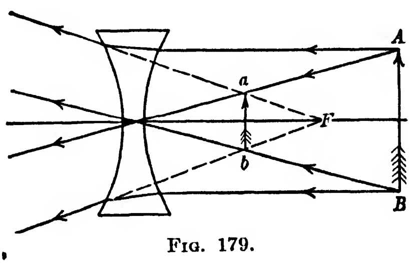

Fig. 179.

253. General Lens Formula.—For thin lenses it may be demonstrated that

$$\frac{1}{q} - \frac{1}{p} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right) = \frac{1}{f},$$

in which p = the object distance from the lens; q = image distance; μ = index of refraction of lens; r = radius of curvature of face of lens on the

object side, and r' = opposite face; f = focal length of the lens. This is the general lens equation, all special cases being derived from it.

In using this general lens equation it is of the utmost importance to be able to determine the signs of the various factors entering therein. *Distances measured on the object side of the lens are positive* (+); *distances measured on the opposite side are negative* (−). It follows then that p is always positive. In convex lenses, f is always negative; and in concave lenses f is always positive. For convenience we shall consider the *object side* of the lens as the *right-hand side.*

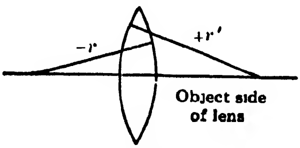

FIG. 180.

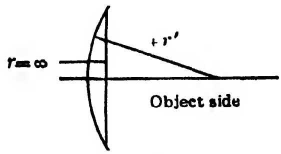

FIG. 181.

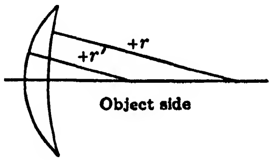

FIG. 182.

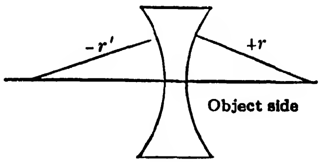

FIG. 183.

The signs of r and r' depend upon the type of lens considered. Fo example, in Fig. 180 we have $-r$ and $+r'$; in Fig. 181, $+r'$ and $r = \infty$; ii Fig. 182, $+r$ and $+r'$; in Fig. 183, $+r$ and $-r'$. It should be noted tha in the cases shown in Figs. 181 and 182 the signs of r and r' will be reversec if the lenses are reversed.

254. Special Lens Equations.—By means of the general lens equatioı the special equations for Cases *I* to *V* may be derived as follows:

Case I.—Object at an infinite distance from the lens. Since $p =$ $1/p = 0$, and hence

$$\frac{1}{q} = \frac{1}{f}.$$

Case II.—Remembering that the object side of a lens is considerea positive, we may write from Fig. 176 the following values: $-q$, $+p$, $+r'$, $-f$. Substituting these values in the general lens equation, we h

$$\left[\left(\frac{1}{-q}\right) - \left(\frac{1}{+p}\right)\right] = [\mu - 1]\left[\left(\frac{1}{-r}\right) - \left(\frac{1}{+r'}\right)\right] = \left[\left(\frac{1}{-f}\right)\right] = \frac{1}{q} +$$

$(\mu - 1)\left(\frac{1}{r} + \frac{1}{r'}\right) = \frac{1}{f}$, that is,

$$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}.$$

Case III.—Fig. 177. Here $p = f$, and consequently $\frac{1}{q} + \frac{1}{f} = \frac{1}{f}$, which $\frac{1}{q} = 0$. Hence $q = \infty$.

Case IV.—Fig. 178. In this case we have $+q$, $+p$, and $-f$. Inserting these values in the general equation $\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$ gives

$$\frac{1}{q} - \frac{1}{p} = -\frac{1}{f}.$$

Case V.—Fig. 179. Here both q and f are on the positive (object) side of the lens, hence their sign values are $+q$ and $+f$. The equation for this case is therefore

$$\frac{1}{q} - \frac{1}{p} = \frac{1}{f}.$$

255. Size of Image and Object.—From a consideration of the similar triangles ABO and abO (Fig. 176), or the corresponding triangles of any one of the typical lens cases, we may say that, in general, the size of the image is to that of the object as the image distance is to the object distance; that is, $AB{:}ab = p{:}q$.

210. Focal Length of a Thick Lens.—A practical method of finding the focal length of a thick lens is that employed in the use of the optical bench

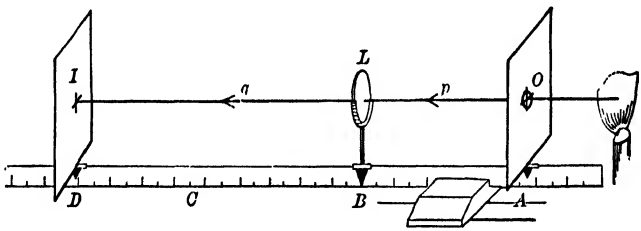

Fig. 184.—Optical bench.

(Fig. 184). The lens is first set in position B, giving a large and distinct image on the screen; it is now placed in position C, giving a small distinct image on the screen. Now $AB = CD$. If we let $AD = l$ and $BC = a$, we may show that

$$f = \frac{(l^2 - a^2)}{4l}.$$

Problems

Make drawings to illustrate the position and character of the image in each of the following cases for the double convex lens, and write the complete lens formula for each case.

752. Object at an infinite distance from the lens.

753. Object at a finite distance from the lens, greater than twice the focal distance.

754. Object at twice the focal distance.

755. Object between twice the focal distance and the focal distance.

756. Object at the focal distance.

757. Object at a point less than the focal distance.

758. Case of the bi-concave lens. Find by drawing the position and character of the image. Write the complete lens formula.

759. The focal length of a bi-convex lens is 10 in. An object 4 in. in length is placed on the principal axis at a distance of 2 ft. 6 in. from the lens. Find (*a*) the position, and (*b*) the size of the image. (*c*) Is the image real or virtual?

760. If the object (problem 759) is placed 6 in. from the lens, what will be (*a*) the position and (*b*) the size of the image? (*c*) Will the image be real or virtual?

761. The focal length of a bi-concave lens is 10 in. An object 10 in. in length is placed on the principal axis 30 in. from the lens. Find (*a*) the position and (*b*) the size of the image. (*c*) Is the image real or virtual?

762. An object is placed 60 in. from a bi-convex lens. A real image appears at a distance of 20 in. on the opposite side of the lens. Find the focal length of the lens.

763. The focal length of a bi-convex lens is 1 ft. (*a*) Where must an object be placed so that the image will be 18 in. from the lens and real; (*b*) 18 in. from the lens and virtual?

764. If an object 4 in. in length be placed 3 ft. from a bi-convex lens, the focal length of which is 1 ft., what will be the position and size of the image?

765. What will be the size of the image when the object (problem 764) is placed (*a*) 18 in. from the lens; (*b*) 12 in. from the lens?

766. How far from a bi-convex lens of focal length 10 in. must a candle flame be placed in order that a distinct image shall appear on a screen 30 in. distant?

767. An object 4 in. in length placed 20 in. from a bi-convex lens gives a real image 4 in. in length. What is the focal length of the lens?

768. Where must the object (problem 767) be placed so that the image shall be (*a*) 8 in. in length and real; (*b*) 8 in. in length and virtual?

769. The radii of curvature of a flint glass lens ($\mu = 1.6$) are as follows: $r = 10$ cm; $r' = 15$ cm. Determine the signs of r

and r', and find the focal length of this lens, when it is (*a*) bi-convex; (*b*) bi-concave.

770. A concave-convex lens has radii as follows: $r = 12$ cm; $r' = 10$ cm. Its index of refraction is 1.5. Find its focal length.

771. (*a*) Find the focal length of a plano-convex lens, if the radius of the curved surface is 12 cm, and the index of refraction is 1.6. (*b*) If this lens were to produce an image having the same size as the object, how far would it have to be from the object?

772. A concave-convex lens ($\mu = 1.5$) has radii as follows: $r = 12$ cm; $r' = 10$ cm. (*a*) Which face of the lens (concave or convex side) is toward the object? (*b*) Find the focal length.

773. A pocket magnifying glass has a focal length of 2 in. Find the size of the image when the object is placed between the lens and the focus and 0.5 in. from the focus.

774. The Washington monument is 550 ft. high. A photograph of it was taken at a distance of a quarter of a mile from the monument. The lens was 6 in. from the plate. Find the length of the picture of the monument.

775. A telescope lens is to be made of crown glass, index of refraction 1.53. The radii are to be equal, and the focal length is to be 15 ft. Determine the radii.

776. The object glass of the Yerkes telescope is 40 in. in diameter; its focal length is 62 ft. Taking the angular diameter of the sun as 32′, and assuming that the image is practically at the focus, find the size of the sun's image produced by this lens. How does the diameter of the lens affect the brilliancy of the image?

777. A convex lens placed at a distance of 25 cm from a candle flame forms a distinct image upon a screen. When the lens is moved 50 cm further from the candle a second image is formed upon the screen. Find (*a*) the focal length of the lens, and (*b*) the distance of the screen from the candle.

778. An object and a screen are 250 cm apart. Where must a lens having a focal length of 40 cm be placed to produce a clear image on the screen? Show that there are two solutions and find the relative size of image and object in each case.

779. The focal length of a lens is 80 cm. The indices of glass and water are taken as $3/2$ and $4/3$, respectively. Find (*a*) the value of the term $\left(\frac{1}{r} - \frac{1}{r'}\right)$; (*b*) the index of refraction of glass with reference to water; (*c*) the focal length of the lens in water.

METHODS OF DETERMINING INDICES OF REFRACTION

256. Spectrometer Method.—The spectrometer is used to determine the indices of refraction of transparent media in the form of prisms. To determine the index of refraction of a liquid by this method it is necessary to place the given liquid in a prismatic vessel having transparent faces.

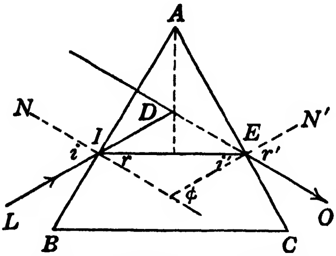

FIG. 185.—Deviation through a prism.

Let A be the refracting angle of the prism (Fig. 185) and D the angle of deviation. Now it may be shown that D is a minimum when the ray of light IE passing through the prism is symmetrical to the faces of the prism; that is, when $i = r'$. If the prism be set for the position of *minimum deviation,* we may write

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}.$$

257. Microscope Method.—Let AO be the real depth of a given medium, and let AI be its apparent depth, as viewed with the naked eye, or through a microscope, which is provided with a measuring scale for the determination of the distances AO and AI. The index of refraction in this case is

$$\mu = \frac{AO}{AI},$$

when AO = real depth; AI = apparent depth.

Let us think of two media, A and B, (air and water, for example) as considered in the above equation. If we let the real depth of B be $AO = d$, and the apparent depth $AI = d'$, then

$$d' = \frac{d}{\mu_{ab}},$$

where μ_{ab} is the index of refraction of B with respect to A. If we consider three media, A, B, C, (air, water, and glass) the real depths of B and C being db and dc respectively, then the apparent depth of $B + C$ with respect to A is

$$d'' = \frac{db}{\mu_{ab}} + \frac{dc}{\mu_{ac}}.$$

Example 1.—A rectangular piece of glass ($\mu = 3/2$) is immersed in water ($\mu = 4/3$), the surface of the glass being parallel to the surface of the water. The thickness of the glass is 3 cm, and the depth of the water above the upper face of the glass is 4 cm. What is the apparent depth of the water and glass combined, when viewed vertically downward from the air? *Solution:*

$$d'' = \frac{d_w}{\mu_{aw}} + \frac{d_g}{\mu_{ag}} = \frac{4}{(4/3)} + \frac{3}{(3/2)} = 3 + 2 = 5 \text{ cm}.$$

Example 2.—Find the apparent depth of the glass (Example 1) as viewed from water. *Solution:* the index of refraction of glass with respect to water (Art. 248) is $\mu_{wg} = \frac{\mu_{ag}}{\mu_{aw}} = \frac{(3/2)}{(4/3)} = \frac{9}{8}$. Then apparent depth $d' = \frac{d}{\mu_{wg}} = \frac{3}{(9/8)}$ $= 2.67$ cm.

258. The Lens Method.—By means of the equation $1/f = (\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right)$ we may determine μ in terms of f, r, and r'. The focal length f is usually determined by the optical bench method, and the radii r and r' are determined by means of a spherometer.

Problems

The indices of refraction of a number of the more commonly occurring refracting media are given in Table XXXIV, Appendix.

780. The refracting angle of a prism is 60°; for the position of minimum deviation, angle D is 36°. Find the index of refraction of the prism.

781. A hollow glass prism, refracting angle 60°, is filled with carbon disulphide (CS_2), index of refraction 1.64. Find the angle of minimum deviation for this prism.

782. An 18-in. layer of a dense liquid L, which is immiscible in water, and which has an index of refraction of 1.5, is covered with water to the depth of 24 in. Consider the index of refraction of water to be $4/3$. Find (*a*) the apparent distance of the surface of the liquid L below the surface of the water; (*b*) the apparent distance from the surface of the water to the bottom of liquid L, as observed by an eye in the air above the water.

783. Find the apparent depth of the liquid L (problem 782) as observed by an eye immersed in the water.

784. (*a*) A microscope is focused upon a printed page. A block of glass 12 mm thick is laid over the print and it is now found that the microscope must be raised 4.5 mm in order to produce a clear image. Find the index of refraction of the glass. (*b*) Of what sort of glass would you judge the lens was made?

785. A given lens has a focal length of 10 cm. Its radii of curvature are: $r = 10$ cm; $r' = 15$ cm. Find the index of refraction if (*a*) the lens is bi-concave; (*b*) bi-convex.

786. A concave-convex lens, having a focal length of 1.2 m, has radii of curvature as follows: $r = 12$ cm; $r' = 10$ cm. (*a*) Find the index of refraction of this lens. (*b*) Make a sketch to illustrate which face of the lens is considered as being on the object side.

787. A bi-convex lens ($\mu = 1.6$) has a focal length of 12 cm; $r = 12$ cm. Find r'.

OPTICAL INSTRUMENTS

259. Dispersion. Fraunhofer Lines.—If a beam of sunlight be passed through a prism, it suffers dispersion, the relative positions of the characteristic colors red, orange, yellow, green, blue, indigo, violet being represented by the Fraunhofer lines, from B to H, respectively (Fig. 186).

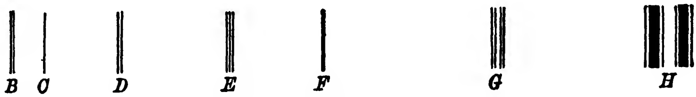

FIG. 186.—Fraunhofer lines.

260. Dispersive Power. Angular Dispersion.—Let μ_B, μ_C, μ_D . . . μ_H be the indices of refraction for the corresponding Fraunhofer lines from B to H. Then *total dispersion* $= \mu_H - \mu_B$; *partial dispersion* $= \mu_C - \mu_B$, $\mu_D - \mu_C$, etc.; *mean dispersion* $= \mu_F - \mu_C$; and *relative dispersion* or *dispersive power* $= (\mu_F/\mu_C)/(\mu_D - 1)$.

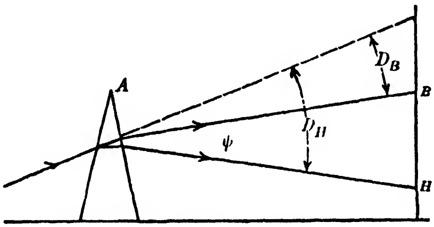

FIG. 187.—Angular dispersion, ψ.

Consider a prism of small refracting angle (Fig. 187). It may be shown that, for a small refracting angle, the angle of deviation for a given line, the H line say, is $D = A(\mu_H - 1)$ and the angle of deviation for any other line, the B line, is $D' = A\ (\mu_B - 1)$. Then the angular dispersion $= \psi = D - D' = A(\mu_H - \mu_B)$. This equation tells us that for a given refracting medium, the angular dispersion ψ for any two lines may be varied by varying the refracting angle A. It is thus possible for the optician to produce at will prism combinations which will give either deviation without dispersion, or dispersion without deviation.

When the refracting angle A is relatively large, that is, above 20°, the above equation does not strictly hold. In this case it is necessary to use the equations $\mu_H = \sin \frac{1}{2}(A + D)/\sin \frac{1}{2}A$, and $\mu_B = \sin \frac{1}{2}(A + D')/\sin \frac{1}{2}A$. Then $\psi = D - D'$.

261. Conditions for Achromatism.—Suppose that we wish to fit two prisms (or lenses) together so as to achromatize certain colors, the B and H lines say. It is necessary to select refracting angles A and A' such that the angular dispersion ψ shall be the same for both refracting media; that is, $\psi = A(\mu_H - \mu_B) = A'(\mu_H' - \mu_B')$. The condition for achromatism is, then,

$$\frac{A}{A'} = \frac{(\mu_H' - \mu_B')}{(\mu_H - \mu_B)}.$$

262. The Projection Lantern.—The essential parts of the projection lantern (Fig. 188) are (*a*) the source of light; (*b*) condensing lens C, the

function of which is to "condense" the divergent rays from the source upon the slide S; (c) the focusing or objective lens O. Since it is desired to form on the screen a magnified real image, the object S must be placed at a distance from O greater than the focal length of the objective lens O. The relation of the distance of the object from the lens O to the distance of the screen from O is

$$\frac{1}{f} = \frac{1}{q} + \frac{1}{p}.$$

in which f = focal length of lens O; p = distance from S to the objective lens O; q = distance from O to the screen.

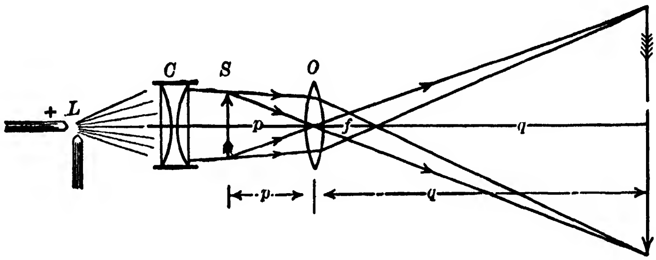

FIG. 188.—Section of projection lantern.

263. The Human Eye.—Mechanically considered, the human eye is a photographic camera, having an automatic focusing system in the muscles that control the crystalline lens, and a sensitive plate (the retina) which reports the image to the brain. The adjustment of the lens system of the eye, such that a distinct image is formed on the retina, is called accommodation.

A normal eye can accommodate over all distances from 6 in. (near point) to infinity (far point). *Distance of distinct vision* is the distance from the eye at which ordinary print can be most easily read. The *distance of distinct vision* for the normal eye = 10 in. = 25 cm.

Example.—A far-sighted person can see distinctly objects 30 in. away but none nearer. Determine the focal length of glasses (convex lenses) that will enable him to read at a distance of 10 in. *Solution:* The problem in this case is to find a lens of a focal length such that an object at the distance of distinct vision (10 in.) shall appear to the eye to be at a distance of 30 in.; that is q = 30, and p = 10. Then $\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$, and $\frac{1}{30} - \frac{1}{10} = \frac{1}{f}$, hence f = 15 in.

264. Magnification.—The *magnification* (*magnifying power*) of an instrument is the ratio of the apparent size of an object as seen through the instrument under given conditions to the apparent magnitude as perceived by the eye. Since, however, it is not always convenient or possible to measure the apparent size of an object as seen through the instrument, it is the usual practice to express the magnifying power of an instrument in terms of certain constants of the instrument.

The apparent size of a linear object is measured by the visual angle V which it subtends (Fig. 189). The visual angle may be expressed as

$$V = \frac{L}{d} = \frac{l}{b} = \frac{l}{L} = \frac{b}{d}.$$

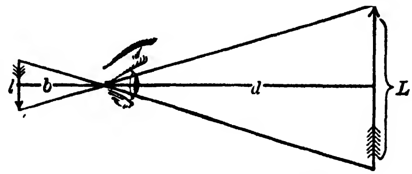

FIG. 189.—Visual angle.

265. The Simple Microscope.—In the case of the simple microscope (Fig. 190) the object AB is placed between the principal focus F and the lens, (see Case IV, page 205). The image $A'B'$ is virtual, erect, and larger than the object. Let us consider that the eye is located in such a position

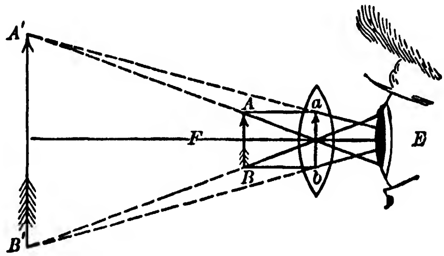

FIG. 190.—Simple microscope.

that the crystalline lens lies at the principal focus of the lens. We desire to determine the magnifying power of the simple microscope in terms of the visual angle subtended by AB, with and without the use of the lens L. Assuming that $ab = AB$, the visual angle with the lens $= AB/f$; the visual angle without the lens is $AB/25$ cm $= AB/10$ in. in which 25 cm or 10 in. is taken as the distance of distinct vision. Then

$$magnification = \frac{(AB/f)}{(AB/25\,\text{cm})} = \frac{25\,\text{cm}}{f} = \frac{10\,\text{in.}}{f},$$

where f = focal length of the lens.

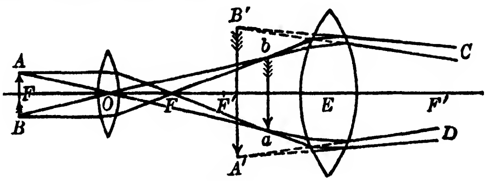

FIG. 191.—Compound microscope.

266. The Compound Microscope.—In Fig. 191 we have shown in outline the relative positions of the two lenses composing the refractive system

of a compound microscope. Lens O is the objective; E is the eyepiece. The object AB is placed just outside the principal focus F of the lens O. These are two images; a real image ab and a virtual image $A'B'$.

The magnification (approximate) due to lens O is $ab/AB = L/F$ where L is the approximate length of the microscopic tube, and F is the focal length of the objective lens O. The magnification due to the lens E is $25 \text{ cm}/f = 10 \text{ in.}/f$, where f is the focal length of the eyepiece, lens E. The total magnification due to both lenses is

$$magnification = \left(\frac{L}{F}\right) \times \left(\frac{25}{f}\right) = \frac{25L}{Ff}.$$

267. The Telescope.—In determining the magnifying power of the astronomical telescope in terms of the focal lengths of the lenses O and E (Fig. 192) it is necessary again to make use of certain approximations. Let

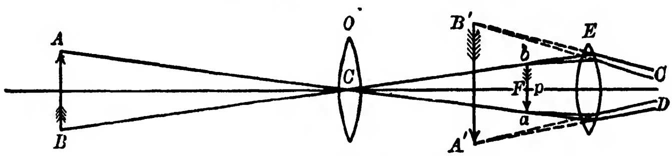

FIG. 192.- Astronomical telescope.

the object AB be a star at a great distance from lens O. We assume that the image distance Cp is practically equal to F, the focal length of lens O. The angular measure of the apparent magnitude of the object AB is ab/F. Also, the angular measure of ab, considered with reference to lens E and the eye, is approximately ab/f. The magnifying power, then, of the instrument is the ratio of these two angles; that is,

$$magnification = \frac{(ab/f)}{(ab/F)} = \frac{F}{f}.$$

268. Combination of Two Lenses.—In the construction of optical instruments it is frequently necessary to use a combination of lenses. Let L and L'

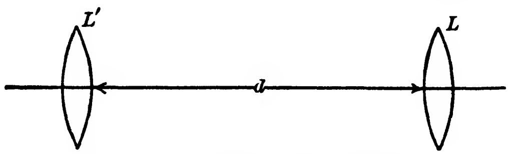

FIG. 193.—Lenses placed coaxially.

(Fig. 193) represent two lenses (convex or concave) placed so that their axes coincide. Let d = distance between the lenses; f = focal length of L; and f' = focal length of L'. Consider first the relation of object to image in the case of L. From the general lens equation we may write $\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$, where p is the object distance, and q is the image distance, with reference to L. Consider now that the image formed by L becomes the object for L'; then $q \pm d$ = distance of image from L' = p' = object dis-

tance with respect to L'. If we let q' be the image distance with respect to L', then $\frac{1}{q'} - \frac{1}{p'} = \frac{1}{f'}$. In dealing with these two cases, it is important to note that *distances measured on the side of the lens from which the light comes are positive, and distances measured on the other side are negative.*

When the lenses touch, $d = 0$, and we may write $\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$, where F is the focal length of the combination.

Example 1.—Two convex lenses of focal lengths $f = 20$, and $f' = 30$ are placed symmetrically on an axis at a distance 10 cm apart (Fig. 194). An

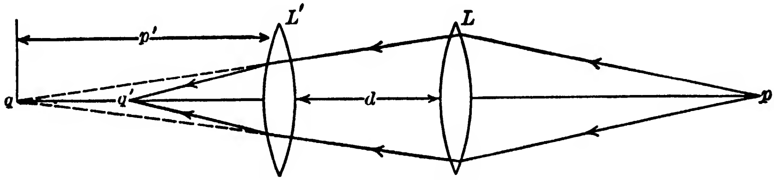

FIG. 194.—Image due to two convex lenses.

object is placed 100 cm in front of lens L. Find the position of the image due to the combination.

Solution: Let us consider first the image formed by L. Here p is $+$, f is $-$, and q is $-$, hence the equation $\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$ becomes $\frac{1}{q} + \frac{1}{100} = \frac{1}{20}$, from which $q = 25$. Since q is measured on the negative side of the lens, $(q + d) = p' = -25 + 10 = -15$. In the case of lens L', p', q', and f' are all on the negative side, and hence we write $-\frac{1}{q'} + \frac{1}{15} = \frac{1}{30}$, and hence $q' = 10$ cm, measured to the left of L'.

Example 2.—Given a convex lens ($f = 4$ in.) and a concave lens ($f' = 4$ in.) placed coaxially 4 in. apart, to find the position and character of the image of an object placed 6 in. from lens L (Fig. 195).

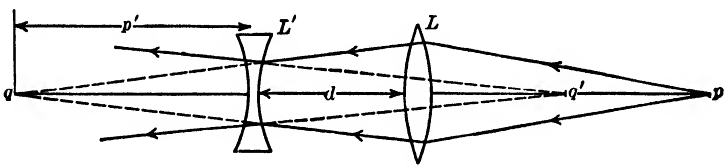

FIG. 195.—Image due to combination of concave and convex lenses.

Solution: We shall first find the image due to L. In this case p is positive and q and f are negative, hence $\frac{1}{q} + \frac{1}{6} = \frac{1}{4}$, whence $q = 12$ in., measured on the negative side of L. In the case of the concave lens, q' and f' are positive while p' is negative. The term $(q + d) = p' = -8$. The general equation $\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$ therefore becomes $\frac{1}{q'} + \frac{1}{8} = \frac{1}{4}$, from which $q' = 8$ in., and is

measured to the right of L'. This means that the image is virtual, and lies between the object and the lens L.

Problems

The indices of refraction for the B, C, D, F, and H lines of flint glass, crown glass, water, and carbon disulphide are given in Table XXXV, Appendix.

788. Find for flint glass (*a*) the total dispersion; (*b*) mean dispersion; (*c*) dispersive power.

789. How does the dispersive power of carbon disulphide compare with that of flint glass?

790. The refracting angle A of a flint glass prism is 20°. Find the angular dispersion between (*a*) the B and H lines; (*b*) the C and D lines.

791. The angular dispersion in a crown glass prism for the B and H lines is 22′ 27.84″. Find the refracting angle.

792. The refracting angle of CS_2 prism is 40°. Find the length of the spectrum (B to H) formed on a screen 10 ft. distant from the prism, assuming that the screen is at right angles to the B line.

793. How far from a flint-glass prism having a refracting angle of 30° should a screen be placed so that a spectral image 2 ft. in width shall be shown?

794. A plano-convex lens whose radius is 60 cm is made of glass having an index of refraction of 1.53 for red rays, and 1.55 for blue rays. Find how much farther from the lens is the principal focus for red than is that for blue.

795. Find the angle of a flint glass prism that will achromatize the region from B to H in a crown-glass prism whose angle is 4°.

796. A projection lantern is to produce a magnification of 50 diameters at a distance of 50 ft. Find the distance of the lens from the slide, and the focal length of the lens.

797. The projection lens of a lantern has a focal length of 1 ft. How far back of the lens must a slide be placed in order to focus clearly upon the screen 24 ft. from the lantern?

798. Make drawing of a section of an eye, and a spectacle lens necessary for correction of (*a*) far sight; (*b*) near sight.

799. If the greatest distance of distinct vision for a myopic (near-sighted) eye is 10 cm, what is the focal length of spectacle lenses suitable for reading at a distance of 25 cm?

800. If the nearest distance for distinct vision for a far-sighted person is 35 in., what should be the focal length of the spectacles he would require for reading at a distance of 10 in.?

801. If the greatest distance of distinct vision for a myopic eye is 4 in. what should be the focal length of the proper reading spectacles?

802. The focal length of the objective of a 12-in. refracting telescope is 20 ft. Determine the focal length of the eyepiece in order that the magnifying power may be 80.

803. Given two convex lenses of focal lengths 23 in. and 1 in., respectively. Make drawing to illustrate the use of these lenses as a telescope, and compute the magnifying power.

804. Explain the principle of the compound microscope, with the aid of a diagram, and state from what data you would calculate its magnifying power.

805. A convex and a concave lens, each 10 in. in focal length, are placed coaxially at a distance of 3 in. apart. Find the position of the image when the object is at a distance of 15 in. beyond the convex lens.

806. Solve problem 805 assuming the object to be placed 15 in. beyond the concave lens.

807. Two thin convex lenses, having a common axis, touch. The focal length of one is 20 cm; that of the other is 15 cm. (*a*) Find the focal length of the combination. (*b*) What is the position of the image when the object lies on the principal axis of the combination and at a distance of 30 cm from their point of contact?

CHAPTER XIV

RADIATION

NATURE AND LAWS OF RADIATION

269. Radiant Energy.—When a disturbance occurs in an elastic medium, waves spread radially from the source in all directions. The resulting wave motion is always accompanied by a transfer of energy through space. This form of energy is called *radiant energy.* While the transfer of energy by means of water waves or sound waves is in a sense a form of radiation, it is customary, in speaking of radiation, to confine our attention to that form of radiation which is associated with the transfer of energy through the medium of ether waves, such, for example, as radiant heat, light, and electromagnetic effects.

The only essential difference between ether waves, whether they be those of heat, light, or electricity, is that of wave length. Ether waves may vary in length from 0.000,000,000,003 cm for cosmic rays to 500,000,000 cm (about 3000 miles) for the electromagnetic waves which are given off by the ordinary 60-cycle commercial alternating current.

270. The Radiation Spectrum.—The following table will serve to illustrate the range of ether waves with respect to wave length.

RADIATION SPECTRUM

Kind of Wave	λ in Cm
Commercial 60-cycle a.c	500,000,000.0
Radio waves, from	3,000,000 0
to	2,000.0
Infra-red, heat rays, from	0 03
to	0 000,078
Visible spectrum, from red	0.000,076
to violet	0.000,040
Ultra-violet, from	0.000,040
to	0.000,001,5
X-rays, from	0.000,000,12
to	0.000,000,001
Gamma rays	0.000,000,000,7
Cosmic rays, from	0.000.000,000,005
to	0 000,000,000,003

271. Wave-length Units.—Radiation wave lengths may be expressed in terms of various units. For example, those of the 60-cycle alternating current are given in terms of kilometers or miles, while those of the commercial radio systems are given in meters. The shorter wave lengths may be expressed in terms of centimeters, millimeters, microns, or Ångströms. A

micron, designated by the letter μ, $= 1/1000$ mm. An Ångström, introduced by the Swedish physicist, Ångström, is $A = 1/10{,}000{,}000$ mm $= 1/100{,}000{,}000$ cm. As defined by the International Solar Union at Paris, 1907, "one Ångström is 1/6438.4696 of the wave length of the red cadmium ray in air at normal pressure and 15°C.," which is practically equivalent to 1/100,000,000 cm.

272. Laws of Radiation.—While radiation waves of different length may require the use of different methods of measurement, they are all subject to the same general laws, some of the more important of which are the following:

1. *Kirchhoff's Law of Radiation.*—For a given temperature, the ratio of the radiating power to the absorbing power of all bodies is the same, and the value of this ratio depends on the temperature and the wave length. By *radiating power* is meant the quantity of energy radiated from unit area of a given surface in unit time, and by *absorbing power* is meant the fraction of the incident energy that is absorbed by the body. Radiating power is sometimes called *emissive power*, and is designated by E.

Kirchhoff's law may be stated thus,

$$\frac{E}{A} = e,$$

where E is the emissive (radiating) power, A is absorbing power, and e is the emissive power of a perfectly black body, for a given temperature and wave length. A perfectly black body is one that absorbs all the radiant energy that falls upon it; that is, none of the incident energy is reflected. For experimental purposes a near approximation to a perfectly black body is a small hole in a hollow sphere, the interior of which is blackened. The energy of a wave train falling upon the hole is practically all absorbed, since the chances of any of the energy's being reflected out through the hole are very small.

2. *Stefan's Law.*—This law, sometimes given as the Stefan-Boltzman law, states that the total radiation of a perfectly black body is directly proportional to the fourth power of its absolute temperature, or

$$E = \sigma T^4,$$

where E is the emissive power = total energy radiated per unit area per unit time; T is absolute temperature; and σ is a proportionality factor known as Stefan's constant. In c.g.s. units $\sigma = 5.71 \times 10^{-5}$ erg/cm²/sec./deg.⁴

In actual practice radiating bodies are in general surrounded by other bodies, for example, the walls of the room in which the experiment is being carried out. The given body radiates energy at the temperature T_1 and receives energy at T_2. The form that the equation takes in laboratory practice, then, is

$$E = \sigma(T_1^4 - T_2^4).$$

3. *Wien's Displacement Law.*—Suppose, for illustration, that a piece of carbon is heated. It emits, first, black heat waves; then as its temperature is raised, red heat waves; and finally at a high temperature, white heat waves. That is to say, as the temperature rises the radiation wave lengths

become shorter and shorter. Wien's law states that changes of wave length with change of temperature occur in such a way that the product of λ_m (wave length for maximum radiation) and T is a constant; that is,

$$\lambda_m T = K.$$

when λ_m is measured in microns ($\mu = 1/1000$ mm), the constant $K = 2890$; when λ_m is measured in cm, $K = 0.2890$.

4. *Planck's Law.*—Wien's displacement law, as given under (3), does not give the full form of the energy wave-length curve. In 1896 Wien proposed a second law which gave the form of the energy curve for the region of the visible spectrum, but which did not fit the experimental results obtained for longer waves. Planck modified Wien's second law and gave it a form which agrees well with the experimental results for wave lengths throughout the entire radiation spectrum. Planck's law may be written in equational form as

$$E_\lambda = \frac{C_1 \lambda^{-5}}{(e^{C_2/\lambda T} - 1)},$$

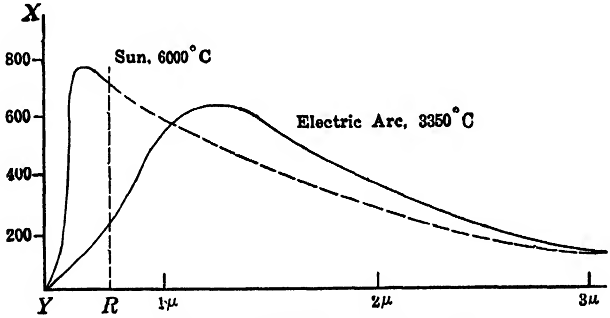

FIG. 196.—Radiation-energy curves.

where E_λ is the energy corresponding to the wave length λ, C_1 and C_2 are constants, and e is the base of the natural (Napierian) system of logarithms. The important constant is C_2 which, according to Planck's computation, may be written $C_2 = ch/k$, where c is the speed of light $= 2.999 \times 10^{10}$ cm/sec., h is the universal constant $= 6.56^{-27}$ erg per vibration, and $k = 1.372 \times 10^{-16}$ erg per degree C.

The energy wave-length curves shown in Fig. 196 for sunlight and the electric arc represent graphs which conform to Planck's equation.

273. The Quantum Theory.—The physical interpretation of Planck's equation furnished the starting point for his quantum theory. While the black body to which his formula applies does not exist in nature, it can easily be realized in the laboratory. The ordinary conception of radiant energy is that it can be transmitted from one body to another in a continuous manner by wave motion. Planck showed that this assumption is not borne out by experimental facts, when applied to energy transference as applying to Kirchhoff's black-body enclosure. He, therefore, made the bold assumption that radiant energy is emitted in small, discrete units which he called *quanta*

(singular, quantum). A *quantum* is a quantity of energy which is designated by the letter ϵ and which is equal to the product of h and ν, or

$$\epsilon = h\nu,$$

where ϵ is a quantum of energy measured in fractions of an erg, h is Planck's universal constant $= 6.56^{-27}$ erg per vibration, and ν is the frequency (number of vibrations per second) of the vibrating system.

274. Temperature Measurement by Radiation.—By means of the equations given in Art. 272, it is possible to calculate temperatures which are too high for ordinary methods of measurement or for the determination of the temperatures of celestial bodies. Temperatures so determined are called *black-body temperatures*. The black-body temperature of the sun is calculated to be 6000°C., and that of the electric arc 3350°C. which is in close agreement with the value of 3500°C. as determined by experimental methods.

Problems

In the solution of problems involving wave length λ and frequency n or ν, it should be borne in mind that in the use of the equation *velocity* = *wave length* $\times$ *frequency* the velocity of ether waves, whether they be radio waves or x-rays, is the same as the velocity of light, namely, 186,000 mi./sec. $= 2.999 \times 10^{10}$ cm/sec. In problems of this sort it is permissible to take the velocity of light as 3×10^{10} cm/sec.

808. Taking the wave length of the D-line of the solar spectrum (Table XXXVII, Appendix) as 0.000,589 mm, compute the wave length in (*a*) microns, (*b*) Ångströms.

809. A body is radiating energy in accordance with the law of inverse squares. Wall A is 1 m from the radiating source and wall B is 6 m from the source. How does the intensity of the radiation falling upon A compare with that upon B?

810. A spherical body is maintained at a temperature of 227°C. Assuming that the laws of black-body radiation apply, how much energy will be radiated per cm² in 1 min.?

811. Find the net loss of energy from the body (problem 810), the temperature of the room being 27°C.

812. The wave length for maximum emission from a given body is 0.589μ. What is the temperature of the body in absolute degrees?

813. Compute the numerical value of the constant C_2 in Planck's equation.

814. (*a*) What is the frequency ν of light corresponding to the D-line of the solar spectrum (problem 808)? (*b*) What is the amount of energy in a quantum of this light?

ELECTRICAL DISCHARGES

275. Cathode Rays.—*Cathode rays* are streams of negatively charged particles (electrons) which are shot off from the cathode, C of Fig. 197, of a highly exhausted tube upon the application of a high potential difference to the electrodes. These rays manifest themselves in various ways as follows: (a) They produce brilliant fluorescent effects in the walls of the tube and in substances within the tube upon which they fall. (b) They heat bodies upon which they fall. (c) These rays exert pressure upon bodies, producing, under certain conditions, mechanical effects. (d) Cathode rays are attracted by positively charged bodies and are repelled by negatively charged bodies. (e) And finally, cathode rays are deflected by a magnet, the plane of deflection being at right angles to that produced by electrostatic charges.

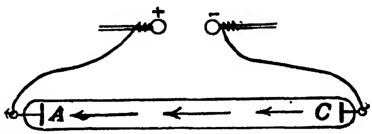

FIG. 197.—Cathode-ray tube.

276. Fundamental Properties of Electrons.—An electron is the elementary unit of negative electricity. All electrons carry negative charges of the same amount. An electron in motion constitutes a current, which is surrounded by a magnetic field.

a. Speed of Electrons.—By subjecting a fine stream of cathode rays to two fields of force, one of magnetic intensity H and the other of electrostatic intensity E, J. J. Thomson succeeded in deriving a workable equation for the determination of the velocity v of an electron under given conditions. His equation is

$$v = 3 \times 10^{10}\frac{E}{H}\text{ cm/sec.}$$

The speed of an electron is variable, depending upon the conditions under which it moves. For example, in the ordinary cathode tube v is about 10^9 cm/sec. In the Coolidge tube, however, the speed approaches that of light, namely, 3×10^{10} cm/sec.

b. Charge on an Electron.—Millikan in his classical experiment in this field proved that the charge upon an electron is

$$e = 1.592 \times 10^{-19}\text{ coulomb.}$$

c. Ratio of the Charge to the Mass.—It has been determined that the ratio of the charge on an electron to its mass is

$$\frac{e}{m} = 1.769 \times 10^8\text{ coulombs/gram.}$$

d. Mass of an Electron.—From the two preceding equations we may write

$$m = 9 \times 10^{-28}\text{ gram.}$$

277. X-rays.—When a stream of electrons in an evacuated tube falls upon a target, a form of radiation is set up in the ether, known as x-rays. The kinetic energy of the electrons is transformed into the energy of the x-rays; moreover, there is always a constant ratio between the amount of kinetic energy of the electron which is converted into the energy of the

x-rays and the frequency of the rays produced. This may be expressed in terms of Planck's quantum relation as

$$\epsilon = h\nu,$$

where ϵ is the amount of energy (one quantum) lost by the electron, h is the universal constant $= 6.56 \times 10^{-27}$, and ν is the frequency of the resulting x-rays.

The penetrating power of x-rays is determined largely by the pressure of the gas within the tube. A high degree of exhaustion, and consequent low pressure of the gas in the tube, gives rays of high penetrating power, known as *hard rays;* somewhat lower exhaustion gives rays of less penetrating power, which are known as *soft rays.*

X-rays are essentially of the nature of light waves, differing only in that the wave length of the x-ray is very much shorter than that of a light wave. The quantum of a given x-ray is $\epsilon = h\nu$, where ν is the frequency of the ray $= 3 \times 10^{10}/\lambda$.

278. Radioactivity.—In 1896 Henri Becquerel discovered that the element uranium emits penetrating rays that are capable of affecting a photographic plate, and two years later Madame Curie showed that thorium sends out radiations of the same kind. These rays are called *Becquerel rays.* Shortly after the discovery of the Becquerel rays, Madame Curie began a series of brilliant researches which led to the discovery of *radium,* an element which is more than a million times as radioactive as uranium.

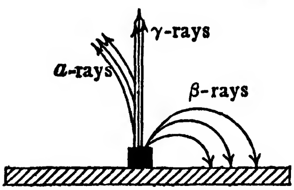

FIG. 198.—Alpha, beta, and gamma rays.

Any substance which spontaneously emits radiations similar to those which come from uranium is called radioactive, and it is said to possess the property of *radioactivity.*

In 1899 Rutherford found that Becquerel rays consist of three kinds of emanations, which he named alpha (α), beta (β), and gamma (γ) rays. When Becquerel rays are subjected to a strong magnetic field, the α-rays are deflected in one direction and β-rays in another, while the γ-rays are unaffected (Fig. 198). The α-rays possess relatively small penetrating power; they consist of positively charged helium atoms. The β-rays are composed of electrons traveling at a very high speed—a speed that under certain conditions may approach the velocity of light. The γ-rays are x-rays of very short wave length.

279. Mass of the β-particle.—The β-particle is an electron. Now it has been found that the mass of an electron appears to depend upon the speed with which it is moving, the greater the velocity the greater the apparent mass. It has been shown experimentally and also from a consideration of the theory of relativity that if m is the mass of a relatively slow-moving electron, one moving in a cathode tube for example, then the apparent mass m' of an electron moving at a greater speed is given by the equation

$$m' = \frac{m}{\sqrt{1 - (v/c)^2}},$$

where v is the velocity of the moving particle, and c is the velocity of light $= 3 \times 10^{10}$ cm/sec.

Example.—If we let $m = 9 \times 10^{-28}$ g, what will be the apparent mass m' when the velocity is 2.1×10^{10} cm/sec.? *Solution:* The ratio $\frac{v}{c} = \frac{2.1 \times 10^{10}}{3 \times 10^{10}} = \frac{7}{10}$. Then $m' = \frac{9 \times 10^{-28}}{\sqrt{1 - (7/10)^2}} = 12.6 \times 10^{-28}$ g. That is to say, the apparent mass m' of the electron at high speed is 1.4 times as great as that when it is in relatively slow motion. This presents a new conception of mass. We have been taught to think of mass as a measure of the inertia of a body. If mass is a function of the speed of a particle as well as the quantity of matter which it contains, it follows that inertia is a property of energy as well as of matter.

280. The Life Period of Radioactive Elements.—All radioactive elements tend to disintegrate in time into lower and more stable elements. For example, it has been computed that a gram of radium will be reduced by radiation to half a gram in 1700 years. In estimating the life period of a radioactive element, two terms are in common use, namely, the half-life period and the average-life period. The *half-life period* is the time required for a given mass of a radioactive element to be reduced to one half its former mass. The half-life period of radium is 1700 years. Half-life periods vary from several million years to a few seconds. The *average-life period* has been calculated to be 1.44 times the half-life period. Thus the average-life period of radium is $1700 \times 1.44 = 2448$ years.

If we consider helium (He) to be an end product in a given radioactive series, it is obviously possible to compute the age of a given mineral in terms of the amount of helium which it contains.

Example.—A given specimen of a certain mineral, containing 10 per cent of uranium and 60 per cent of thorium, yields 8 per cent of helium. On the assumption that a gram of uranium in equilibrium with its products produces 1.1×10^{-7} cc of helium per year, and a gram of thorium in equilibrium with its products produces 0.31×10^{-7} cc of helium per year, what is the age of the specimen? *Solution:* One gram of the specimen contains 10 per cent of uranium $= 0.1$ g, and 60 per cent of thorium $= 0.6$ g. Then 0.1 g of U produces $0.1 \times 1.1 \times 10^{-7} = 0.11 \times 10^{-7}$ cc of He, and 0.6 g of Th produces $0.6 \times 0.31 \times 10^{-7} = 0.186$ cc of He. Total amount of He produced $= (0.1 \times 10^{-7}) + (0.186 \times 10^{-7}) = 0.286 \times 10^{-7}$ cc of He. Then the time required to produce 8 g of He $= 8/(0.286 \times 10^{-7}) = 280{,}000{,}000$ years.

By methods similar in principle to that given above, it is calculated that the age of the earth is about 2,000,000,000 years. The term "age of the earth" means the time that has elapsed since the solid crust was formed.

Problems

815. (*a*) What is the fundamental difference between a cathode ray and an x-ray? (*b*) What is the distinction between a cathode ray and a β-ray?

816. A neutral body is charged negatively by the addition of 5×10^9 electrons. What is the charge (quantity of electricity) upon it, in coulombs?

817. Compute the charge on the body (problem 816) in (*a*) electromagnetic units, (*b*) electrostatic units.

818. In a given cathode tube, 1885×10^{13} electrons strike the anode per second. (*a*) What is the charge in coulombs imparted to the anode in 1 sec.? (*b*) The electronic current in the tube is equivalent to what fraction of an ampere?

819. What is the total mass of the electrons discharged from the cathode (problem 818) per second?

820. (*a*) In what units is the quantum expressed? (*b*) Explain why a quantum of radiant energy of one wave length differs in amount from a quantum of radiant energy of another wave length.

821. Find the quantity of energy in a quantum (*a*) of x-rays whose wave length $\lambda_1 = 12 \times 10^{-8}$ cm (Art. 270); (*b*) of x-rays, $\lambda_2 = 1 \times 10^{-9}$.

822. Assuming that the mass m of an electron at rest is 9×10^{-28} g, and the velocity of light c is 3×10^{10} cm/sec., find the apparent mass m' (*a*) when its velocity v is 1.5×10^{10} cm/sec.; (*b*) when $v = 2.4 \times 10^{10}$ cm/sec.; (*c*) when $v = 2.7 \times 10^{10}$; (*d*) when $v = c$.

823. Suppose that we have 4 g of a radioactive active element, the half-life period of which is 1 year. How many grams of the element will remain at the end of 4 years?

824. A given specimen of a certain mineral, which contains 8 per cent of uranium, is found upon analysis to yield 2.2 cc of helium per gram of the mineral. Calculate the minimum age of the mineral.

ELECTROMAGNETIC WAVES

281. Oscillatory Electric Discharge.—If we have given a charged Leyden jar (Fig. 199) of capacitance C, the outer coat of which is connected to a coil of inductance L and resistance R, an oscillatory discharge will take place between A and B, provided R^2 is less than $4L/C$. In this case the frequency of the oscillatory discharge is

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L}}.$$

When R^2 is negligible in comparison with $4L/C$, we have

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

If R^2 is larger than $4L/C$, the discharge is aperiodic; that is, no electrical oscillation occurs.

282. Maxwell's Theory.—In 1864 Maxwell advanced the theory that by means of electrical disturbances it ought to be possible to create electromagnetic waves in the ether similar to light waves. In 1888 this theory was confirmed experimentally by Hertz, who not only proved that electrical oscillations, such as occur in the discharge of a Leyden jar, give rise to electromagnetic waves in the ether, but, by means of a simple device (Fig. 200) succeeded in detecting their presence at a considerable distance from the source. When the loop L is properly tuned by sliding the wire S back and forth, an oscillatory discharge between the knobs AB gives rise to a corresponding, though much weaker, discharge between a and b.

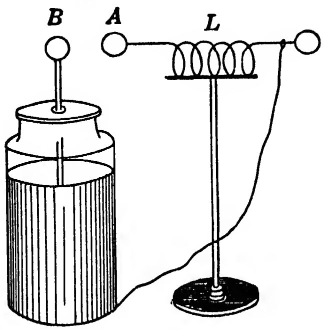

FIG. 199.—Oscillatory discharge.

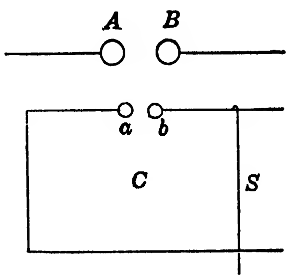

FIG. 200.—The Hertz spark-gap detector.

283. Nature of Electromagnetic Waves.—When an electrostatic charge is at rest no magnetic effects are produced, since stationary charges have no magnetic fields. As soon as the charge moves, however, as in a series of oscillatory discharges, a magnetic field is produced, because a moving charge is equivalent to an electric current. When the discharge takes place, there are snapped off into space, as it were, oscillatory electrostatic and magnetic disturbances. Electromagnetic waves, therefore, consist of a combination of moving oscillatory electrostatic and magnetic fields, the vibrations of which are in planes at right angles to each other.

Electromagnetic waves consist of transverse vibrations, and are capable of being reflected, refracted, and polarized the same as light waves. In fact electromagnetic waves possess all the properties of light waves, differing only in the fact that they are enormously longer than light waves.

284. Velocity, Wave Length, and Frequency.—Electric waves travel through space with the speed of light, namely, 3×10^{10} cm/sec. = 300,000,000 m per sec. = 186,000 miles per sec.

In accordance with the general wave formula, *velocity* = *wave length* × *frequency*, or

$$v = \lambda n = 300{,}000{,}000 \times n \text{ m/sec.},$$

from which

$$\lambda = \frac{300{,}000{,}000}{n} \text{ meters},$$

and

$$n = \frac{300,000,000}{\lambda} \text{ vibrations/sec.}$$

For example, if the wave length of a given radio wave is reported as 300 m, then its frequency is

$$n = \frac{300,000,000}{300} = 1,000,000.$$

The U. S. Bureau of Standards at Washington has recommended to broadcasting corporations that wave frequency be expressed in "kilocycles," the corresponding wave length in meters being given in parenthesis. For example, the term "1000 (300)" means a radio wave train having a frequency of 1000 kilocycles and a wave length of 300 meters. A *kilocycle* is 1000 cycles, that is,

$$\textit{kilocycles} = \frac{\textit{frequency}}{1000}$$

285. Electrical Tuning.—Two vibrating systems are in resonance—that is in tune—when their frequencies are the same. For example, an oscillatory system of frequency n is in tune with another whose frequency is n' when $n = n'$. The frequency of a given system depends upon the inductance L and the capacitance C in accordance with the equation

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}},$$

or since the period T is the reciprocal of the frequency

$$T = 2\pi\sqrt{LC},$$

where n is the number of vibrations per second, T the time of one vibration in seconds, L the inductance in henrys, and C the capacitance in farads.

286. Radio Waves.—Radio waves are those electromagnetic waves which are used in wireless telegraphy, radio telephony, and television. Radio waves are of three kinds, namely, damped, continuous, and modulated.

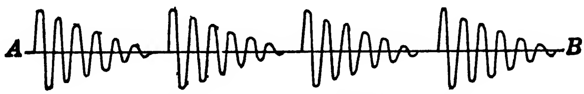

FIG. 201.—Train of damped waves.

a. Damped Waves.—When a spark occurs between the terminals of a Hertz oscillator, for example, there is sent out in the ether, a wave motion the vibrations of which are said to be *damped;* that is, the vibrations die out quickly, each one being less in amplitude than the one preceding it (Fig. 201). This damping effect is due mainly to resistance encountered by the oscillating current in the spark gap. Damped oscillations are suitable for wireless telegraphy, *but cannot be used in radio telephony.* For this purpose undamped oscillations giving continuous waves are necessary.

b. Continuous Waves.—The first necessary condition for the reproduction of the tones of the human voice is that the oscillator at the sending station shall

send out a continuous series of waves of uniform intensity (Fig. 202). These are sometimes called *carrier waves*. The frequency of such waves, however, is so great as to make it impossible for the telephone receiver to respond to them, and even if the telephone could respond, no sound would be heard because the audibility range of the human ear is not great enough.

c. Modulated Waves.—The second condition is that these continuous waves be modulated in intensity by the voice of the person speaking into the transmitting device at the sending station. A series of voice modulated

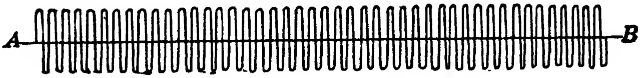

FIG. 202.—Undamped wave train.

waves is shown in Fig. 203. The frequency of the human voice is very low as compared with the radio frequency. Many radio waves are therefore included in, and affected by, each wave of the voice current. *The light lines represent the radio waves. The dotted lines represent the superimposed voice waves.* The effect of voice modulation is, then, to send out electromagnetic waves in pulses, the frequencies of the pulses being low enough to be heard by the person at the receiving end.

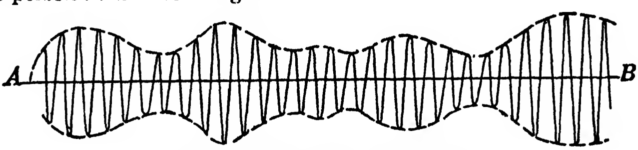

FIG. 203.—Modulated wave train.

The number of vibrations per second of the full lines (Fig. 203) is the *radio frequency;* the number of vibrations per second of the dotted lines, the *audio frequency.* The radio frequency may be as high as 1,000,000; the audio frequency in general lies well below 20,000, usually in the neighborhood of 1000.

287. Wireless, Radio, and Television.—*a. Wireless telegraphy* is the transmission of messages by a system of electric "dots and dashes." The

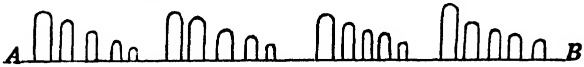

FIG. 204.—Train of rectified damped waves.

essential parts of a wireless sending set are an induction coil which is operated by means of a key and a sending aerial. Suppose that 1000 sparks per second are produced by the sending oscillator, each spark sending out waves of frequency $n = 1{,}000{,}000$. Then when the sending key is closed, a series of 1000 wave trains per second move out into space with the speed of light. These trains of waves consist of damped waves.

In the receiving circuit the incident waves are rectified by means of a vacuum-tube rectifier, into the half-wave form shown in Fig. 204. If the incoming wave were not rectified no effect would be produced upon the

listening ear, first, because the resultant effect upon the diaphragm of the receiver would be zero, since there would be as much tendency for the diaphragm to move one way as the other; and second, because the frequency of the oscillations would be above the range of audibility. The human ear is not capable of hearing sounds for a frequency above 20,000 to 30,000 per second. If the oscillating currents which are induced in the receiving system are rectified, each wave train will produce an effect like that of a *single pulse* in one direction. As a result the listening ear hears a tone which corresponds to a given number of vibrations per second. As the key at the sending station is closed and opened, the series of wave trains is broken up into short and long groups which produce in the receiving telephone a series of short and long notes corresponding to the dots and dashes of ordinary telegraphy.

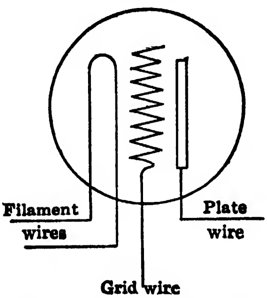

FIG. 205.—Diagram of an electron tube.

b. Radio telephony is the transmission of speech by means of electromagnetic waves. Radio telephony differs from wireless telegraphy in certain very important particulars. So far as reception is concerned, wireless telegraphy is simply a series of short and long notes (dots and dashes) of a given pitch. In radio it is necessary to vary the amplitude of the waves so as to produce the various tones of the human voice. The essential conditions for radio telephony may be stated briefly as follows: (*a*) the oscillator at the sending station must be so designed as to send out a *continuous succession of waves of uniform intensity* (Fig. 202); and (*b*), in place of a sending key

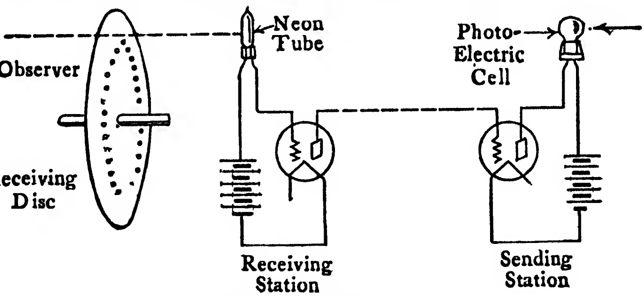

FIG. 206.—Television set-up.

such as is used in wireless telegraphy, there must be substituted a telephone transmitter connected to the sending antenna in such a manner that the talking currents in the transmitter circuit will cause the oscillations to vary in intensity, producing *modulated waves* (Fig. 203).

Rectification of the receiver currents is produced by the usual vacuum-tube rectifier (Fig. 205).

c. Television or "radio sight" is the process of transmitting pictures by radio. As in voice transmission, there is a sending set and a receiving set. The essential parts of the sending set consist of a powerful source of light

for illuminating the object to be "televised," a perforated rotating disc, a *photo-electric cell*, and a radio transmitter (Fig. 206). The sending scanning disc, not shown in Fig. 205, is located to the right of the photo-electric cell. A powerful beam of light is passed through the rotating scanning disc to the object, from which the reflected light is focused upon the photo-electric cell.

The essential parts of the receiving set are a perforated disc similar in all respects to the one used in the sending station, a *neon lamp*, and an amplifier. The receiving disc contains the same number of holes spaced in exactly the same way as the sending disc, and it must rotate at the same speed as that of the sending disc.

Problems

825. (*a*) What must be the relation between R, L, and C of a given system in order that the electrical discharge shall be oscillatory? (*b*) In a given discharging system such as that shown in Fig. 199, the inductance is 1 mil-henry, the capacitance 0.01 mf., and the resistance 10 ohms. Will the discharge be periodic or aperiodic?

826. The wave length of a given electromagnetic wave is 1200 m. What is its frequency in (*a*) vibrations per second; (*b*) kilocycles?

827. What is the period T of a 1000-kilocycle radio wave?

828. What is the frequency n of a system in which the inductance is 0.004 henry and the capacitance 0.009 mf.?

829. A given system has an inductance of 0.001 henry. What must be the capacitance of this system in order to be in tune with that of problem 828?

830. (*a*) What is the main cause of damping in oscillatory waves? (*b*) What is the distinction between damped waves and modulated waves?

831. What is the distinction between radio frequency and audio frequency? (*b*) What is the number of radio waves from A to B, Fig. 203? Audio waves?

832. What are electric "dots and dashes" and how are they produced?

833. (*a*) In radio circuits why is it necessary to rectify the oscillating currents which are induced in the receiving system? (*b*) What is the function of the grid in a radio rectifying tube, such as that shown in Fig. 205?

834. What are the characteristic properties of a photo-electric cell, and what is its use in the sending station of a television system?

835. What are the characteristic properties of a neon tube, and what is its use in a television receiving set?

APPENDIX

TABLES

Many of the tables of equivalents and physical constants are given in the body of the text in connection with their respective subjects, their location being indicated by appropriate page numbers.

Table I.—Metric and English Units of Length, Capacity and Mass, pages 2–4.

Table II.—Important Equivalents and Equations, page 5.

Table III.—Greek-letter Symbols Used in Mathematical Equations

Symbol	Name	Symbol	Name	Symbol	Name
Α, α	Alpha	Ι, ι	Iota	Ρ, ρ	Rho
Β, β	Beta	Κ, κ	Kappa	Σ, σ	Sigma
Γ, γ	Gamma	Λ, λ	Lambda	Τ, τ	Tau
Δ, δ	Delta	Μ, μ	Mu	Υ, υ	Upsilon
Ε, ϵ	Epsilon	Ν, ν	Nu	Φ, ϕ	Phi
Ζ, ζ	Zeta	Ξ ξ	Xi	Χ, χ	Chi
Η, η	Eta	Ο, o	Omicron	Ψ, ψ	Psi
Θ, θ	Theta	Π, π	Pi	Ω, ω	Omega

Table IV.—Values of g in cm/sec./sec., page 12.

Table V.—Moments of Inertia.

1. Thin circular hoop, axis through center, perpendicular to face, radius R, $I_o = MR^2$.
2. Circular plate or cylinder, axis through center, perpendicular to face, radius R, $I = MR^2/2$.
3. Circular ring, axis through center, perpendicular to face, outer radius R, inner radius R', $I_o = M(R^2 - R'^2)/2$.
4. Sphere, axis through center, radius R, $I_o = MR^2 2/5$.
5. Uniform thin rod, axis through middle, length l, $I_o = Ml^2/12$.
6. Rectangular figure, axis through center, perpendicular to plane, width a, length b, $I_o = M(a^2 + b^2)/12$.
7. Moment of inertia about axis parallel to axis through center of gravity, $I = I_o + Md^2$.
8. Moment of force, $Fd = I\alpha = Ia/r$.
9. Period of vibration, $T = 2\pi\sqrt{I/Mgh}$.

Table VI.—Densities in g/cm³

Alcohol	0.8	Nickel	8.9
Aluminum	2.6	Nitric acid	1.56
Brass	8.4	Osmium	22.5
Copper	8.9	Paraffin	0.9
Diamond	3.5	Platinum	21.5
Ether	0.74	Potassium	0 86
Glass, crown	2.6	Silver	10 5
Glass, flint	3 7	Sodium	0.97
Glycerine	1 26	Steel	7.8
Gold	19.3	Sulphuric acid	1.84
Gasoline	0 7	Tungsten	17.5
Hydrochloric acid	1 27	Water, 0°C	0.999
Ice	0 9	Water, 4°C	1.00
Iron, cast	7 1	Water, sea	1.03
Iron, wrought	7 8	Wood, maple	0.7
Kerosene	0 8	Wood, pine	0.5
Lead	11 4	Wood, lignum vitae	1 3
Mercury	13.6	Zinc	17 5

Density of gases at 0°C. and 76 cm

Air	0 001,29 g/cm³	0 08 lb./cu. ft.
Helium	0 000,17 g/cm³	0 0112 lb./cu. ft.
Hydrogen	0.000,09 g/cm³	0.006 lb./cu. ft.

Table VII.—Surface Tensions, page 77.

Table VIII.—Diffusion Constant, page 80.

Table IX.—Elasticity Constants

	Young's modulus		Simple rigidity		Volume elasticity
	Dynes per cm²	Lb. per in. sq.	Dynes per cm²	Lb. per in. sq.	Dynes per cm²
Aluminum	7×10^{11}	9.5×10^{6}	3×10^{11}	4×10^{6}	$7\ 5\times10^{11}$
Brass	10×10^{11}	14×10^{6}	4×10^{11}	5.4×10^{6}	10.5×10^{11}
Copper	12×10^{11}	17×10^{6}	4.5×10^{11}	6.5×10^{6}	12.5×10^{11}
Cast iron	13×10^{11}	18×10^{6}	$5\ 6\times10^{11}$	7.8×10^{6}	9.5×10^{11}
Wrought iron	20×10^{11}	28×10^{6}	7.7×10^{11}	11×10^{6}	16.5×10^{11}
Steel	22×10^{11}	31×10^{6}	8×10^{11}	12×10^{6}	18.5×10^{11}
Water	....	....	....	....	0.22×10^{11}

TABLE X.—COEFFICIENTS OF LINEAR EXPANSION

Aluminum	0.000,023	Iron and steel	0.000,012
Brass	0.000,018	Lead	0.000,028
Copper	0.000,017	Platinum	0.000,008
Glass	0.000,008	Silver	0.000,019
Gold	0.000,014	Zinc	0.000,029

TABLE XI.—COEFFICIENTS OF VOLUME EXPANSION, SOLIDS

NOTE.—The coefficients of volume expansion of the solids named in Table X may be found by multiplying the linear coefficients by three; that is $\beta = 3\alpha$.

TABLE XII.—COEFFICIENTS OF VOLUME EXPANSION, LIQUIDS

NOTE.—$V_t = V_0(1 + \beta t + \beta' t^2 + B'' t^3)$.

Substance	β	β'	β''
Alcohol, ethyl	$1,020 \times 10^{-6}$	220×10^{-8}	
Alcohol, methyl	$1,134 \times 10^{-6}$	136×10^{-8}	87×10^{-10}
Benzol	$1,176 \times 10^{-6}$	128×10^{-8}	80×10^{-10}
Ether	$1,500 \times 10^{-6}$	350×10^{-8}	40×10^{-10}
Pentane	$1,465 \times 10^{-6}$	310×10^{-8}	16×10^{-10}

TABLE XIII.—SPECIFIC HEATS

Air, constant pressure	0.237	Iron	0 116
Alcohol	0.602	Lead	0 03
Aluminum	0.22	Marble	0 21
Brass	0.094	Mercury	0 033
Copper	0.094	Silver	0 056
Glycerine	0.55	Steam, 100°C., 76 cm	0 48
Glass	0.2	Water	1.0
Ice	0.5	Zinc	0.094

TABLE XIV.—SPECIFIC HEAT SUPERHEATED STEAM

Pressure, lb. per sq. in.	14.2	50	100	200
Temp. 300°F	0.46	0 51		
Temp. 400°F	0.46	0.50	0 56	0 68
Temp. 500°F	0.46	0.49	0 53	0 59
Temp. 600°F	0.46	0.49	0 51	0 55

TABLE XV.—MELTING POINTS

	Degrees C.		Degrees C.
Aluminum	657	Lead	327
Beeswax	62	Mercury	−38.8
Butter	33	Paraffin	45 to 50
Copper	1084	Platinum	1778
Glass	1000 to 1400	Rose's fusible metal	94
Gold	1063	Solder, soft	225
Ice	0	Sulphur	115
Iridium	2200	Tin	232
Iron	1100 to 1200	Tungsten	2950

TABLE XVI.—BOILING POINTS

	Degrees C.		Degrees C.
Alcohol	78	Glycerine	290
Ammonia	−34	Mercury	357
Benzene	80	Toluene	110
Carbon disulphide	46	Turpentine	160
Chloroform	61	Water	100

TABLE XVII.—BOILING POINTS OF WATER UNDER DIFFERENT PRESSURES

Centimeters	Degrees	Centimeters	Degrees
73	98.88	76	100.00
74	99.26	77	100.37
75	99.63	78	100.73

TABLE XVIII.—EXTREMELY LOW FREEZING AND BOILING POINTS

	Freezing point, Degrees C.	Boiling point, Degrees C.
Nitrogen	−210	−194.0
Oxygen	−227	−181.0
Hydrogen	−260	−252.5
Helium	....	−268.8

TABLE XIX.—NUMBER OF GRAMS OF WATER VAPOR REQUIRED TO SATURATE THE AIR, PER CUBIC METER

Degrees C.	Grams	Degrees C.	Grams	Degrees C.	Grams
0	4.835	10	9.330	20	17.118
1	5.176	11	9.935	21	18.143
2	5.538	12	10.574	22	19.222
3	5.922	13	11.249	23	20.355
4	6.330	14	11.961	24	21.546
5	6.761	15	12.712	25	22.796
6	7.219	16	13.505	26	24.109
7	7.703	17	14.339	27	25.487
8	8.216	18	15.218	28	26.933
9	8.757	19	16.144	29	28.450

Table XX.—Heats of Combustion in Calories per Gram

Alcohol, ethyl	7,400	Hydrogen	34,700
Anthracite coal	8,000	Illuminating gas	6,000
Dynamite	1,300	Sulphur	2,200
Gunpowder	700	Wood	4,000

Table XXI.—Heats of Combustion in B.t.u.

	B.t.u. per lb.		B.t.u. per lb.
Alcohol, ethyl	12,000	Fuel oil	18,000
Alcohol, wood	9,000	Gasoline	19,000
Carbon burned to CO_2	14,650	Kerosene	18,000
Carbon burned to CO	4,400	Hydrogen burned to H_2O	62,000
Coal, bituminous	14,000		B.t.u. per cu. ft.
Coal, hard	13,000		
Charcoal	16,000		
Coke	12,000	Natural gas	1,000
Sulphur	4,000	Manufactured gas	600
Wood	8,000		

Table XXII.—Thermal Efficiencies, page 101.

Table XXIII.—Thermal Conductivity

Note.—In this table k is the number of calories which will pass per second through 1 cm^2 of a plate 1 cm thick, the difference of temperature on the two sides being 1°C.

	k		k
Air	0.00005	Ice	0 005
Aluminum	0 5	Mercury	0.016
Copper	0 9	Platinum	0.17
Cotton	0 00004	Pine wood	0.0002
Felt	0.00009	Silver	1.0
Glass	0.0015	Snow	0.0001
Gold	0 7	Water	0 0015
Iron	0.15	Zinc	0.25

Table XXIV.—Thermal Conductivity

Note.—In this table k is the number of B.t.u. which will pass per hour through 1 sq. ft. of a plate 1 in. thick, the difference of temperature on the two sides being 1°F.

	k		k
Brickwork	5.0	Lead	113.0
Copper	515.0	Plaster	4.0
Glass	7.0	Stone	17 0
Iron	233.0	Wood	0.75

Table XXV.—Coefficients of Absorption of Sound, page 118.

Table XXVI.—Dielectric Constants, k

Vacuum	1.0000	Mica	6
Air	1.0006	Paper	2
Glass, crown	7.0	Paraffin	2
Glass, flint	8.0	Sulphur	3
Gutta percha	4.0	Water	81

Table XXVII.—Resistivity Values at 0°C.

Conductor	Ohm-cm	Ohms per mil-foot	Conductor	Ohm-cm	Ohms per mil-foot
Aluminum	2.6×10^{-6}	17 5	Lead	19.3×10^{-6}	115.0
Constantan	49×10^{-6}	29 5	Manganin 20°C	42×10^{-6}	260.0
Copper	1.6×10^{-6}	9.5	Mercury	94×10^{-6}	566.0
German silver	26.6×10^{-6}	125 7	Nickel	12×10^{-6}	75.0
Gold	$2\ 2\times10^{-6}$	12 6	Platinum	9×10^{-6}	54.0
Iron	$9\ 7\times10^{-6}$	58.3	Silver	1.5×10^{-6}	9.5

Table XXVIII.—Wire Gauge Values, American (B. & S.)

Gauge No.	Diam. in mm	Diam. in mils	Sq. of diam. mils	Gauge No.	Diam. in mm	Diam. in mils	Sq. of diam. mils
0000	11.684	460.00	211,600.0	19	0.899	35.39	1252.4
000	10.405	409.64	167,805.0	20	0.812	31.96	1021.5
00	9.266	364.80	133,079.4	21	0.723	28.46	810.1
0	8.254	324.95	105,592.5	22	0.644	25.35	642.7
1	7.348	289 30	83,694.2	23	0.573	22.57	509.5
2	6.544	257.63	66,373.0	24	0.511	20.10	404.0
3	5.827	229.42	52,634.0	25	0.455	17.90	320.4
4	5.189	204.31	41,742.0	26	0.405	15.94	254.0
5	4.621	181.94	33,102.0	27	0.361	14.19	201.5
6	4.115	162.02	26,250.5	28	0.321	12.64	159.8
7	3.665	144.28	20,816.0	29	0.286	11.26	126.7
8	3.264	128.49	16,509.0	30	0.255	10.03	100.6
9	2.907	114.43	13,094.0	31	0.227	8.93	79.7
10	2.588	101.89	10,381.0	32	0.202	7.95	63.2
11	2.305	90.74	8,234.0	33	0.180	7.08	50.1
12	2.053	80.81	6,529.9	34	0.160	6.30	39.7
13	1.828	71.96	5,178.4	35	0.143	5.61	31.5
14	1.628	64.01	4,106.8	36	0.127	5.00	25.0
15	1.450	57.07	3,256.7	37	0.113	4.45	19.8
16	1.291	50.82	2,582.9	38	0.101	3.96	15.7
17	1.150	45.26	2,048.2	39	0.090	3.53	12.5
18	1.024	40.30	1,624.3	40	0.080	3.14	9.9

Table XXIX.—Thermoelectric Powers in Microvolts per Degree

Bismuth	−89	Silver	+ 3
Constantan	−36.5	Zinc	+ 3.7
Cobalt	−22	Copper	+ 3.8
German silver	−12	Iron	+ 17.5
Platinum	− 1	Antimony	+ 24
Lead	0	Selenium	+807

Table XXX.—Atomic Weights

Element	Atomic weight	Valence	Element	Atomic weight	Valence
Aluminum	27.0	3	Lithium	7.0	1
Antimony	120.0	3, 5	Magnesium	24.3	2
Arsenic	75.0	3, 5	Manganese	55.0	2, 3, 7
Barium	137.4	2	Mercury	200.0	1, 2
Bismuth	208.0	3, 5	Nickel	58 7	2, 3
Boron	11.0	3	Nitrogen	14.0	3, 5
Bromine	80.0	1	Oxygen	16.0	2
Cadmium	112.4	2	Palladium	106.7	2, 4
Calcium	40.0	2	Phosphorus	31.0	3, 5
Carbon	12.0	4	Platinum	195.2	2, 4
Chlorine	35.45	1	Potassium	39.1	1
Chromium	52.0	2, 3, 6	Radium	226.4	2
Cobalt	59.0	2, 3	Selenium	79.0	2, 4, 6
Copper	63.57	1, 2	Silicon	28.0	4
Fluorine	19.0	1	Silver	107.88	1
Gold	197.2	1, 3	Sodium	23.0	1
Helium	4.0		Strontium	87.6	2
Hydrogen	1 0	1	Sulphur	32.0	2, 4, 6
Iodine	126.9	1	Tin	119.0	2, 4
Iridium	193.0	4	Tungsten	184.0	6
Iron	55.8	2, 3	Uranium	238.5	4, 6
Lead	207.0	2, 4	Zinc	65.37	2

Table XXXI.—Practical, Electrostatic, and Electromagnetic Units of Current, Quantity, Resistance, Potential, Capacitance, and Inductance, page 164.

Table XXXII.—Fundamental Mirror Equations, page 198.

Table XXXIII.—Fundamental Lens Equations, page 206.

Table XXXIV.—Indices of Refraction

Water	1.33	Benzene	1.50
Carbon disulphide	1.64	Crown glass	1.52
Turpentine	1.47	Flint glass	1.62
Alcohol	1.36	Diamond	2.47

TABLE XXXV.—INDICES OF REFRACTION FOR CERTAIN LINES

	B	*C*	*D*	*F*	*H*
Flint glass	1.6127	1.6144	1.6193	1.6315	1.6527
Crown glass	1.5301	1.5311	1.5339	1.5404	1.5509
Water (18°.7 C.)	1.3310	1.3320	1.3336	1.3380	1.3448
CS_2 (18°.7 C.)	1.6182	1.6219	1.6308	1.6555	1.7020

Table XXXVI.—RADIATION SPECTRUM, page 219.

TABLE XXXVII.—WAVE LENGTH AND FREQUENCY OF THE COLORS OF THE SOLAR SPECTRUM

Color	Line	λ in mm	Frequency in cm/sec.
Red	A	0.000760	395,000,000,000,000
Orange	C	0.000656	458,000,000,000,000
Yellow	D	0.000589	510,000,000,000,000
Green	E	0.000527	570,000,000,000,000
Blue	F	0.000486	618,000,000,000,000
Indigo	G	0.000431	697,000,000,000,000
Violet	H	0.000397	760,000,000,000,000

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